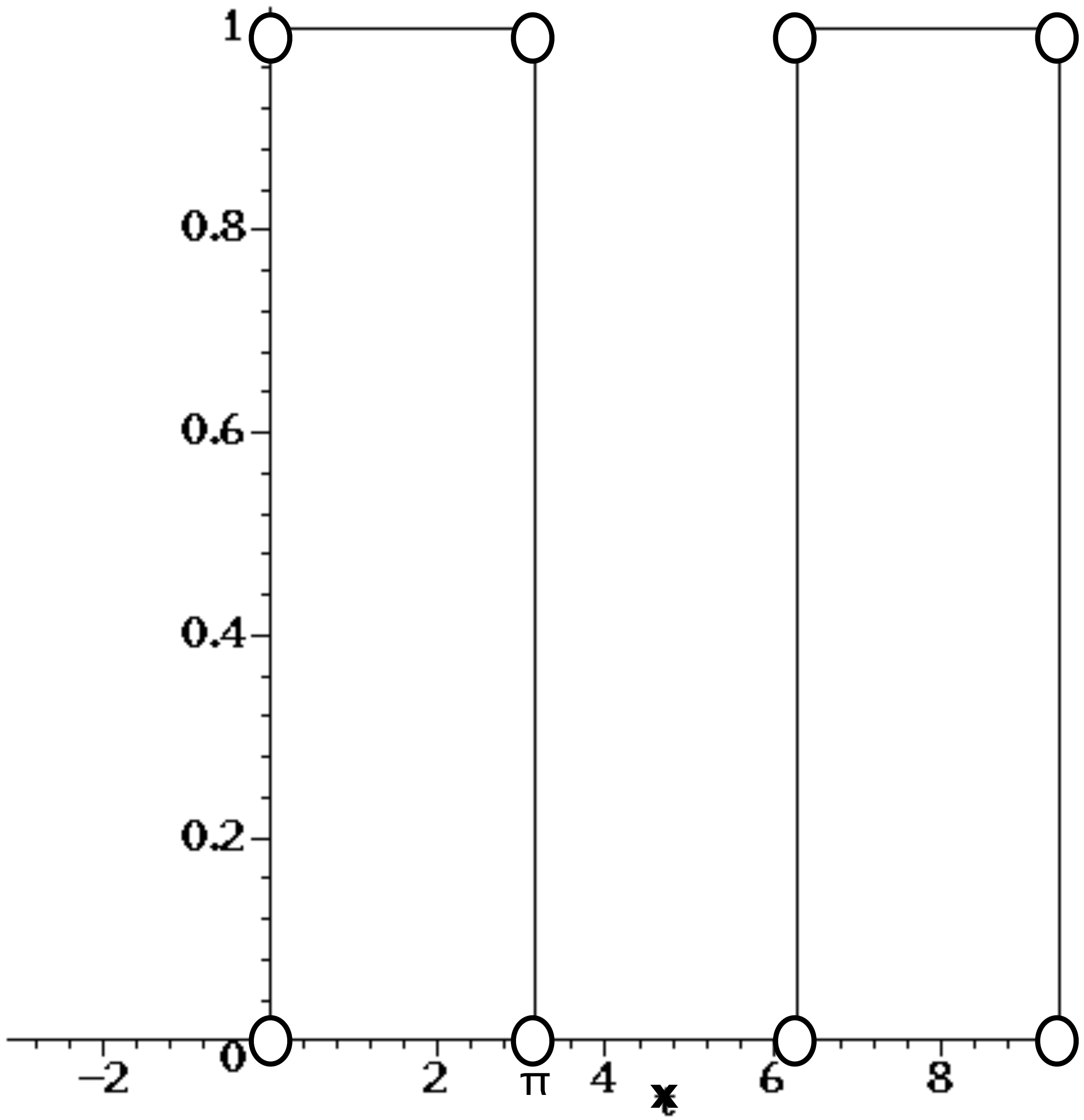
Séries de Fourier

M. Nielsen

Física Matemática I

$$f(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & -\pi < x < 0 \end{cases}$$

com período 2π



Considere a função degrau:

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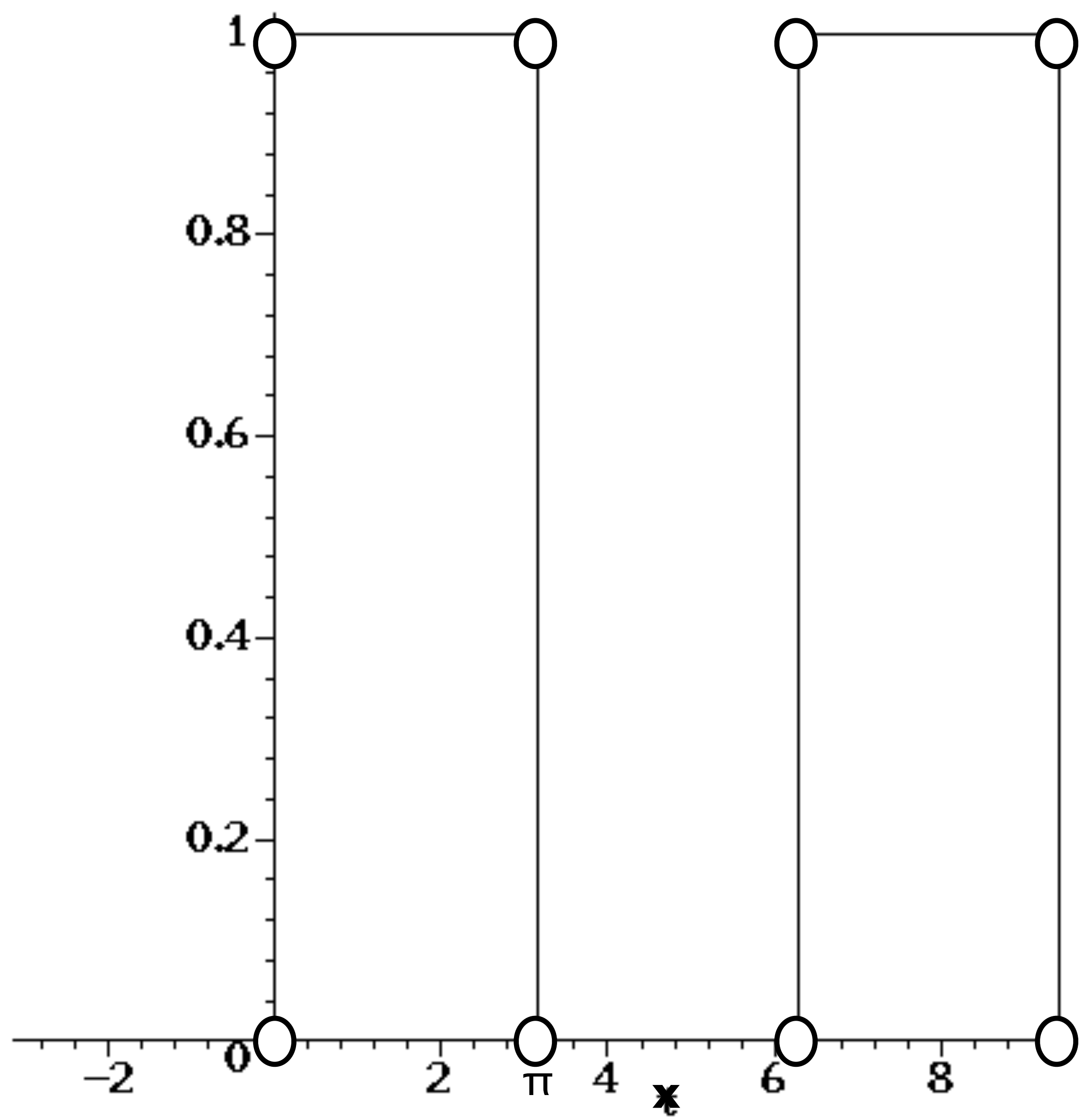

Essa função é seccionalmente diferenciável, e então pode ser expandida numa série de Fourier:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right]$$

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$$a_n = \frac{1}{L} \int_{-L}^{L} dx f(x) \cos \frac{n\pi x}{L}$$

$$b_n = \frac{1}{L} \int_{-L}^{L} dx f(x) \sin \frac{n\pi x}{L}$$

onde L=π

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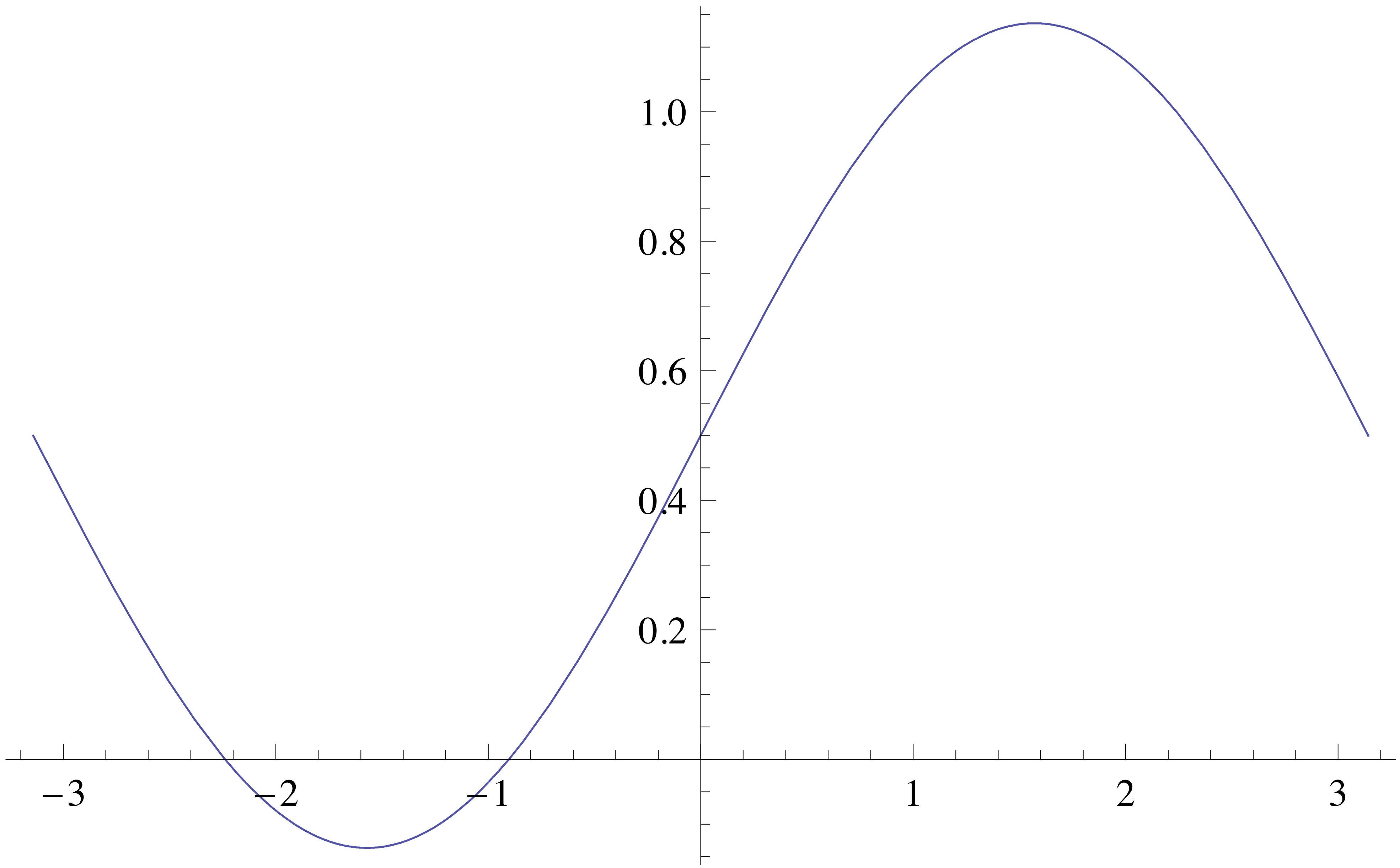
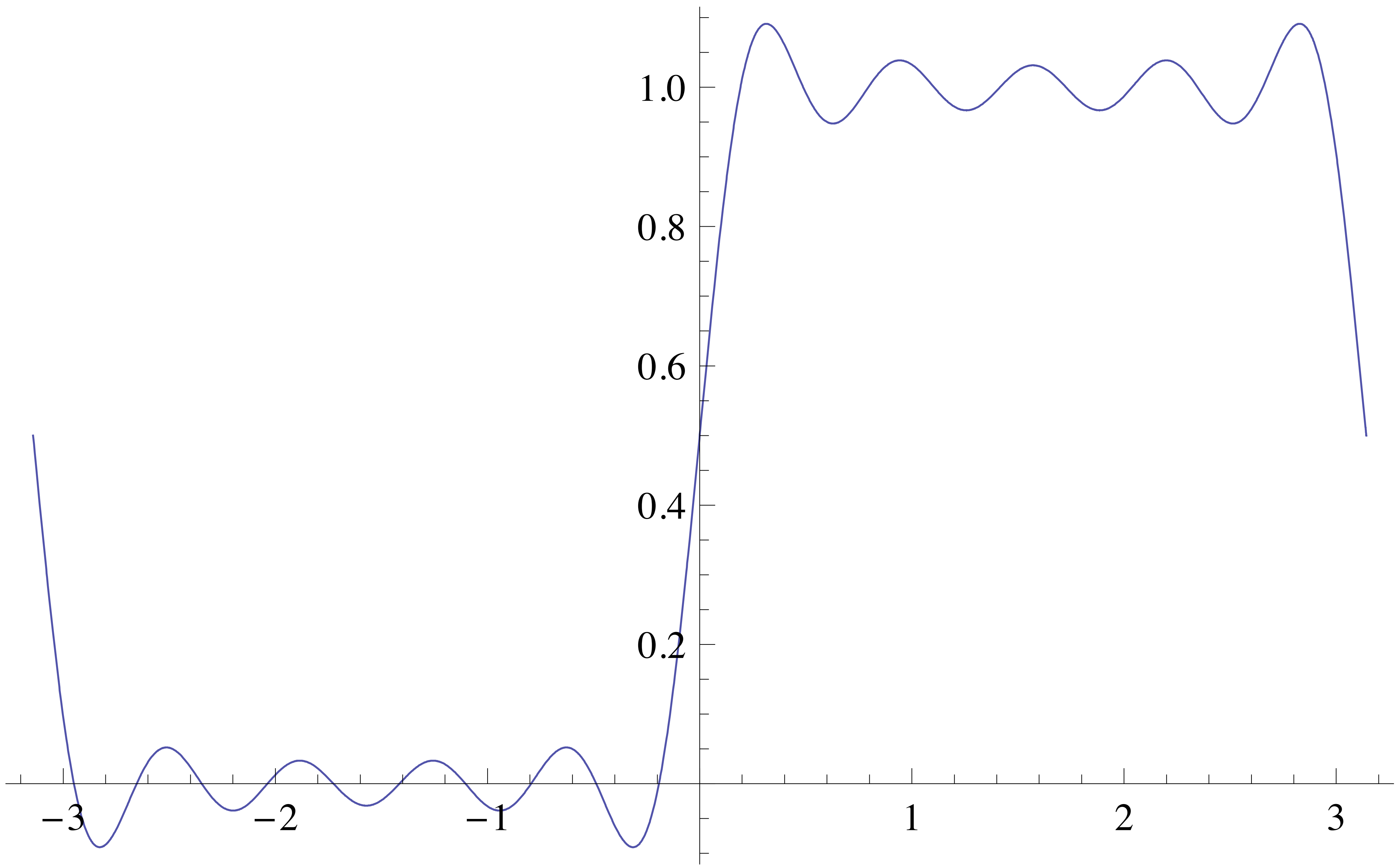
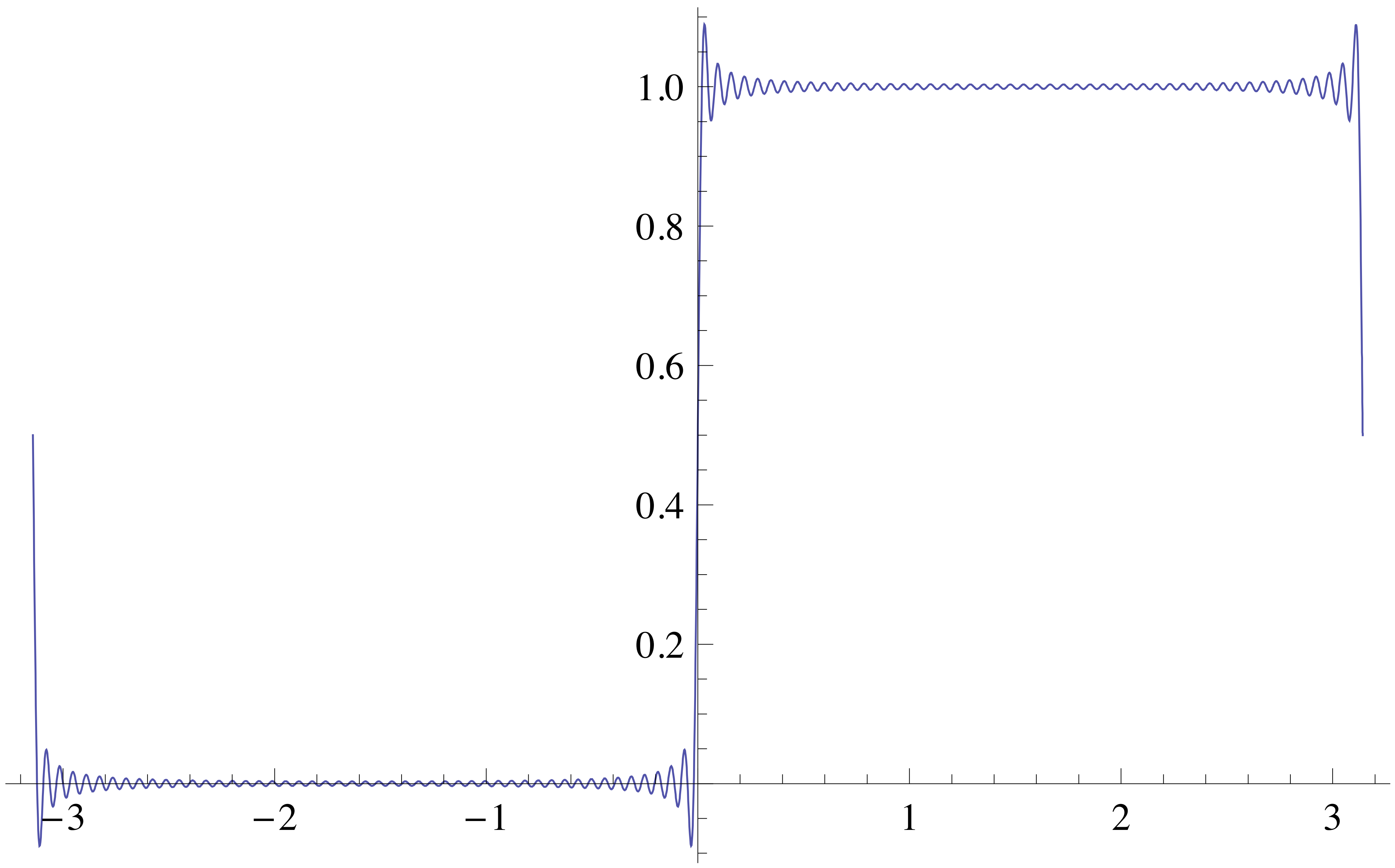
considerando apenas um termo na soma:



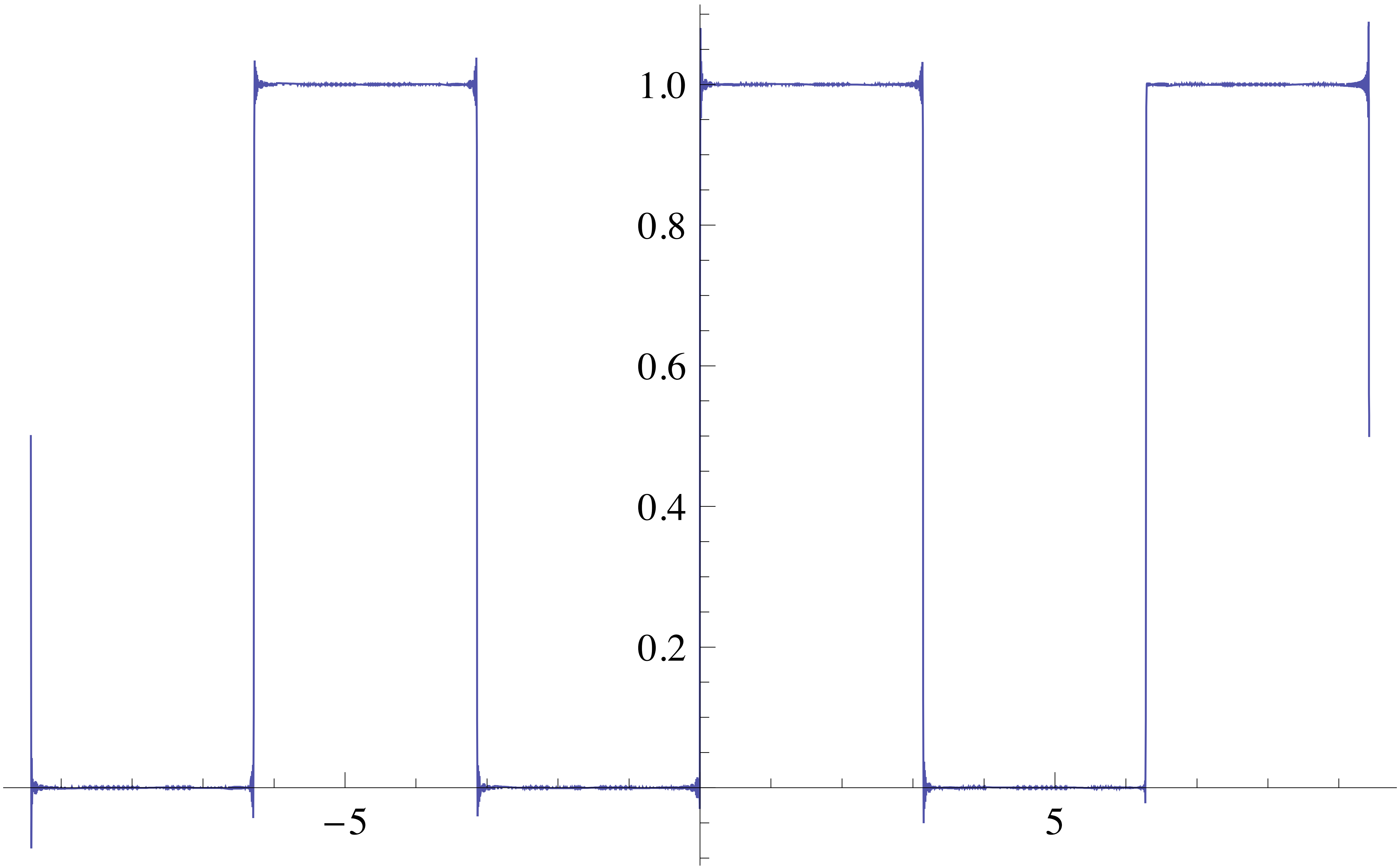
considerando dez termos na soma:

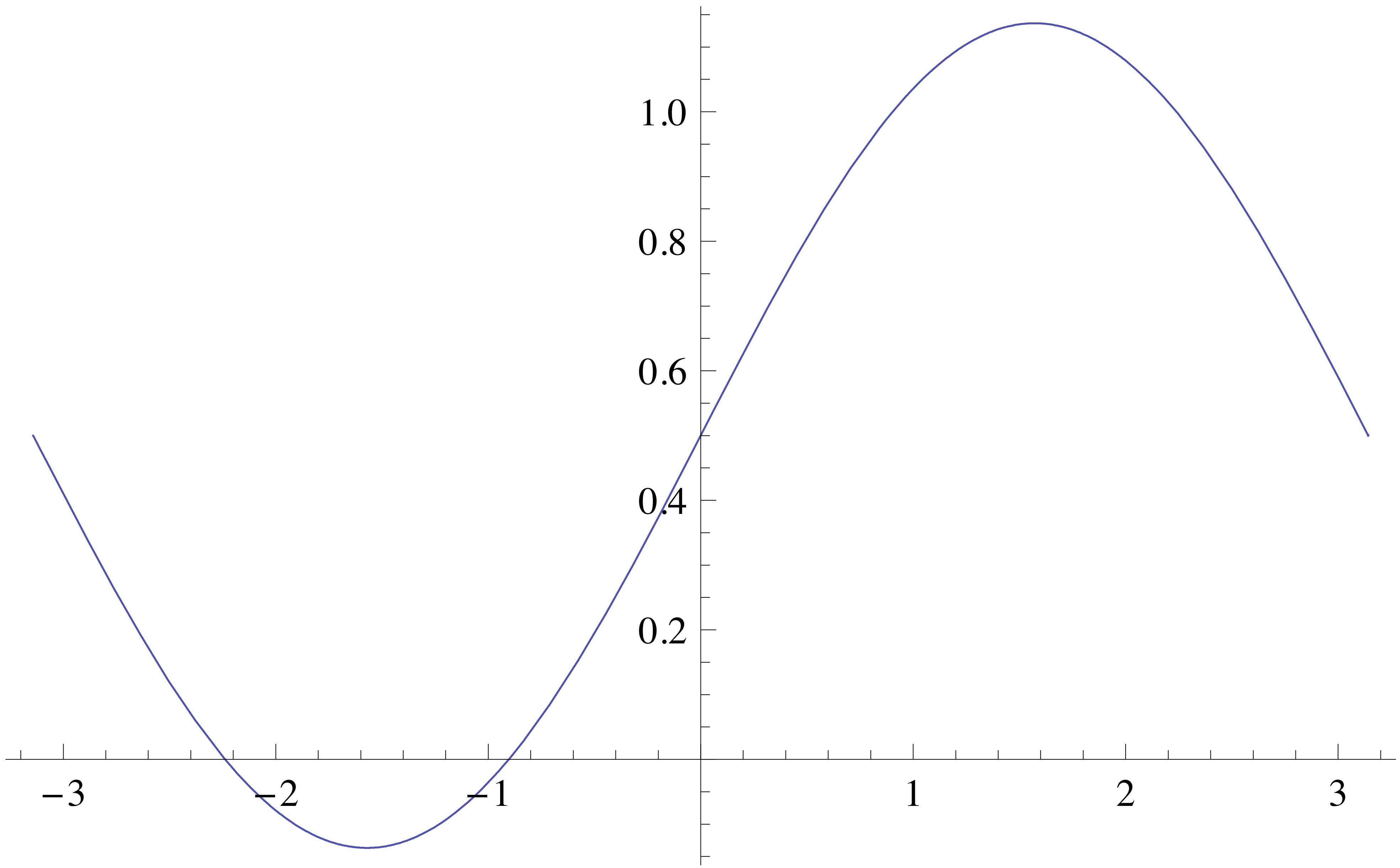


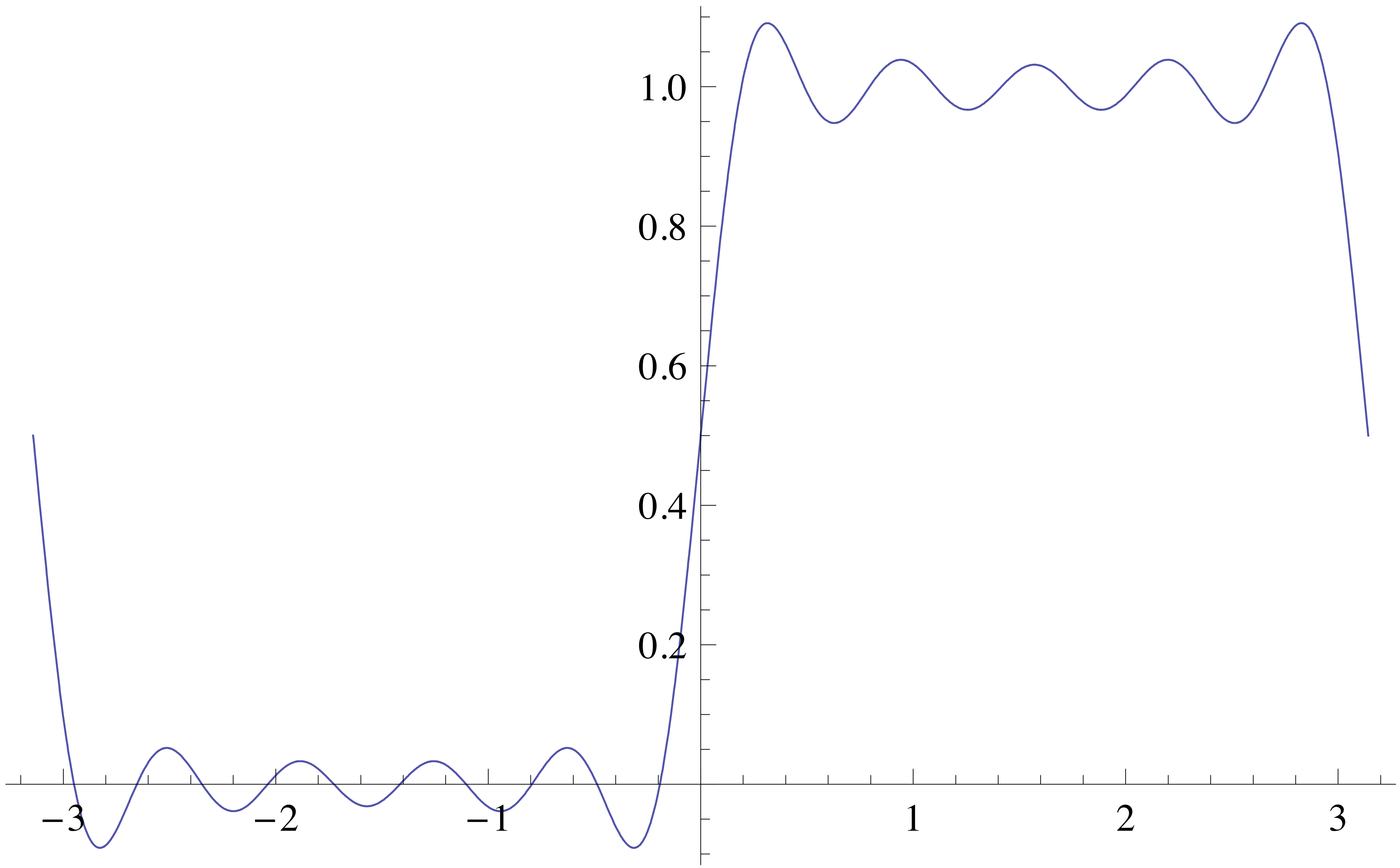
considerando cem termos na soma:



considerando quinhentos termos na soma:




1.0
0.8
0.6
0.4
0.2
−3
−2
−1
1
2
3



1.0
0.8
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0.2
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−2
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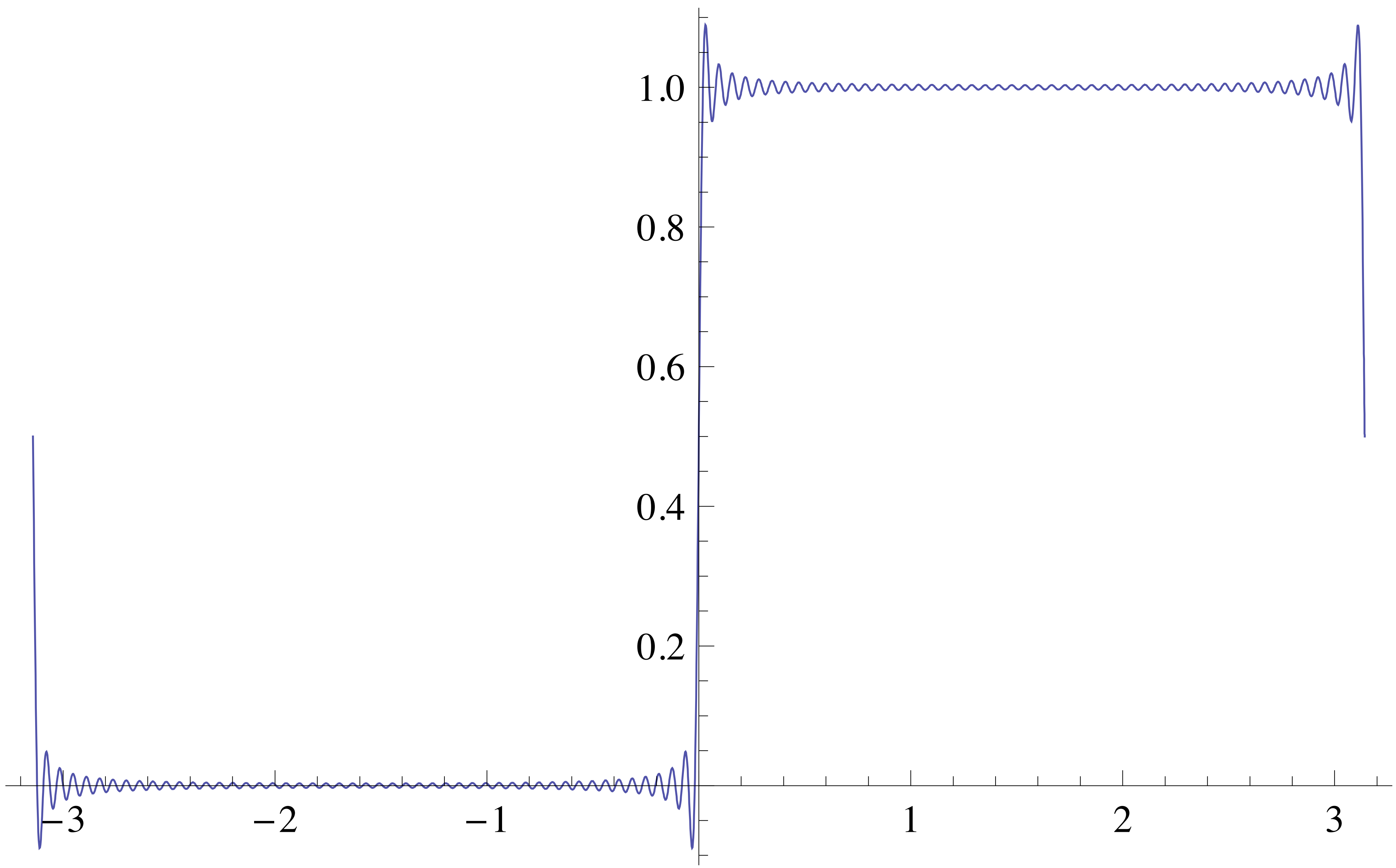

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$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1 - \cos n\pi}{n\pi} \sin nx$$

$$\cos n\pi = (-1)^n \quad \Rightarrow \quad 1 - \cos n\pi = 1 - (-1)^n = \begin{cases} 0, & n = 2k \\ 2, & n = 2k+1 \end{cases}$$

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série de Leibniz

Considere agora a função f(x)=x, -1<x<1, de período 2: L=1

$$b_n = \int_{-1}^{1} dx\ x \sin(n\pi x) = 2\int_{0}^{1} dx\ x \sin(n\pi x) =$$

$$= -2x\ \frac{\cos(n\pi x)}{n\pi}\Big|_0^1 + 2\int_0^1 dx \frac{\cos(n\pi x)}{n\pi} = -2\frac{\cos n\pi}{n\pi}$$

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considerando apenas três termos na soma:

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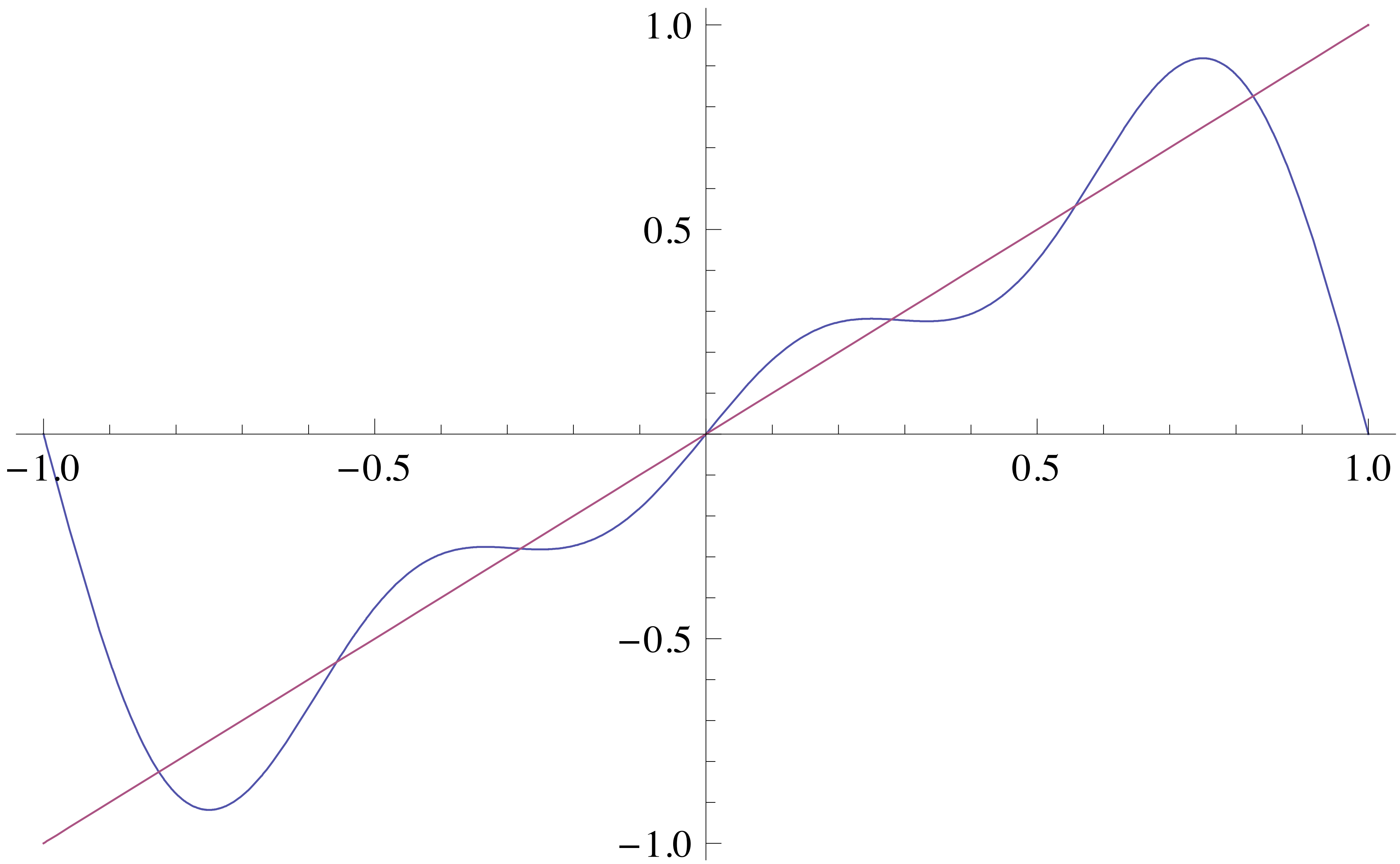

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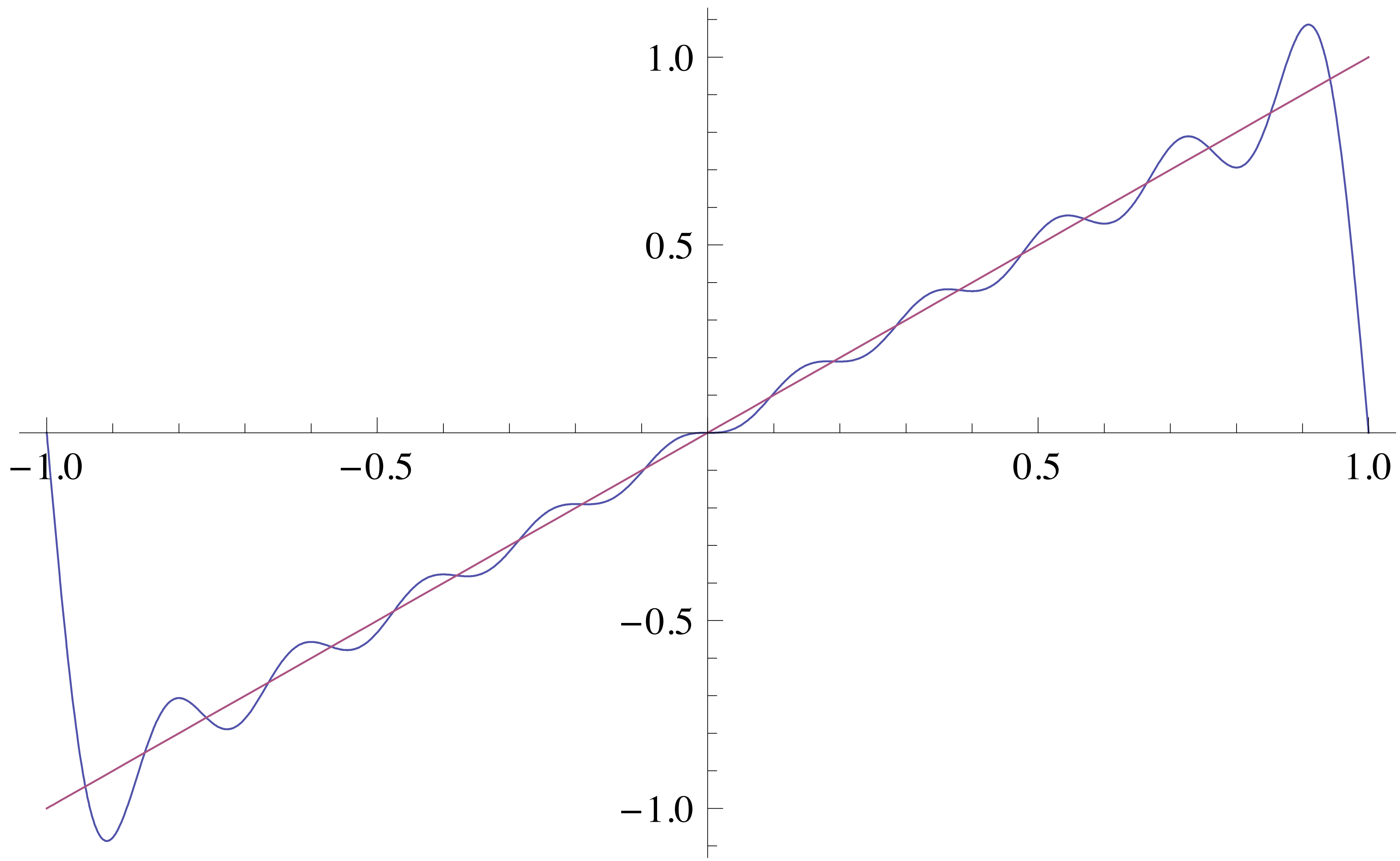

considerando cem termos na soma:

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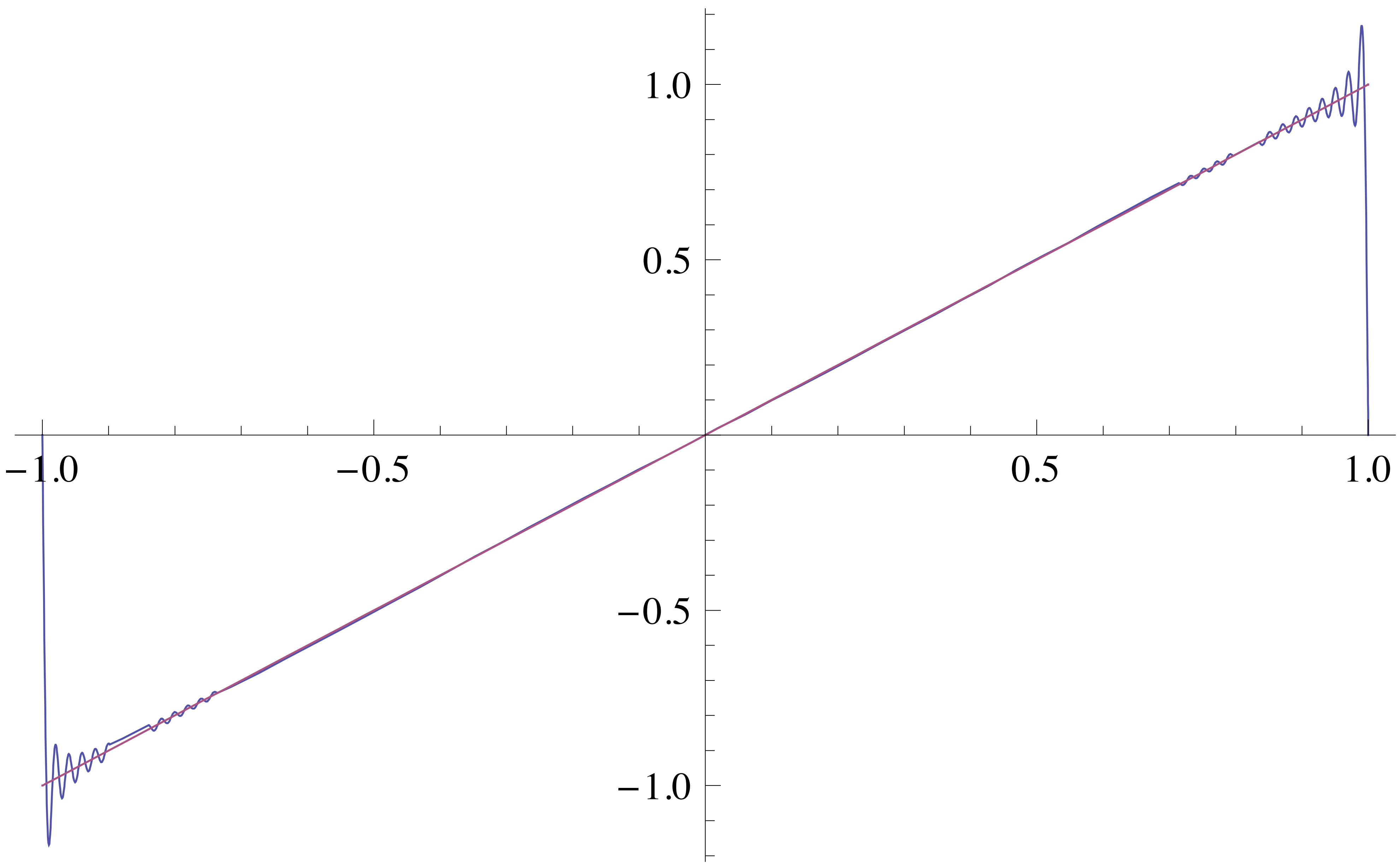

considerando mil termos na soma:

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