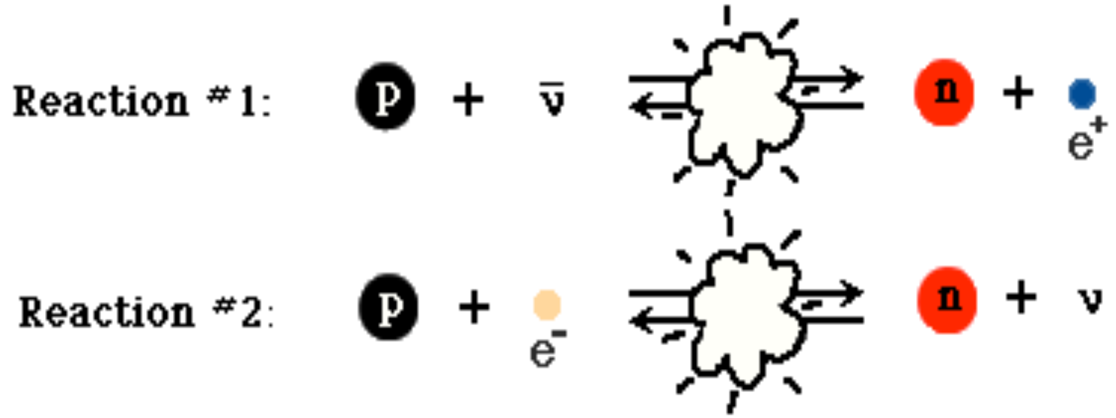
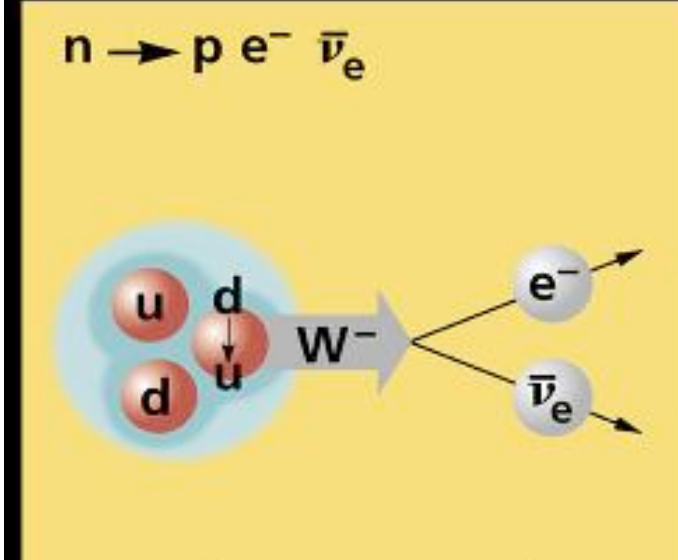
proton/neutron conversions



(The double arrows indicate these reactions go both ways.)



A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

BOSONS

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W⁻	80.4	−1
W⁺	80.4	+1
Z⁰	91.187	0

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c²	Electric charge
ν_e electron neutrino	<1×10⁻⁸	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	−1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	−1	1.672	3/2

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	−1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

BLOCO 2

GRANDEZAS MACROSCÓPICAS DO NUCLEO

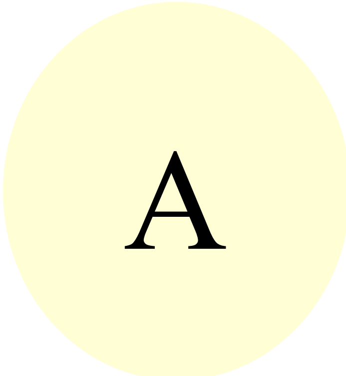
ESPALHAMENTO RUTHERFORD
- PARÂMETRO DE IMPACTO
- ÂNGULO DE DEFLEXÃO
- BARREIRA COULOMBIANA

RAIO NUCLEAR
- ESPALHAMENTO DE ELETRONS
- DIFUSIVIDADE

MASSA NUCLAR:
- ENERGIA DE LIGAÇÃO
- EXCESSO DE MASSA
- ENERGIA DE SEPARAÇÃO
- VALOR "Q" de REAÇÃO
- MODELO DA GOTA LÍQUIDA

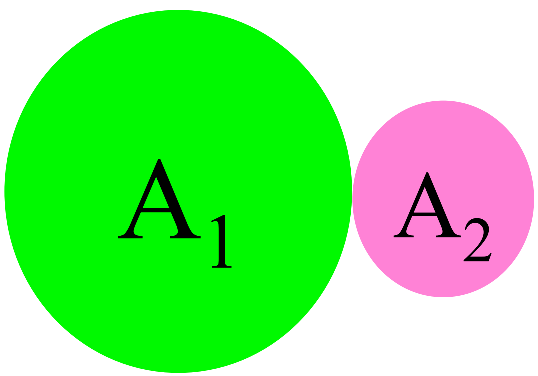
CINEMÁTICA DE REAÇÃO - CINEMÁTICA INVERSA

raio nuclear



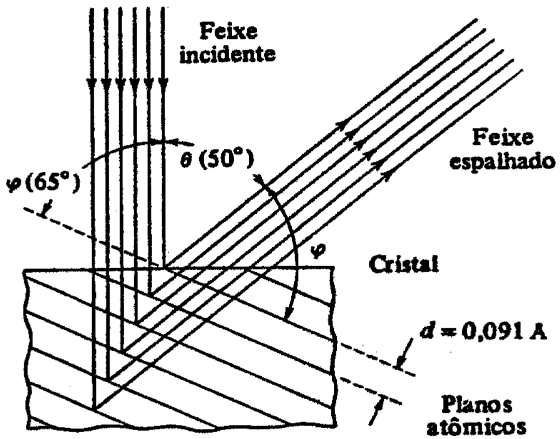
$$R=r_oA^{1/3}$$

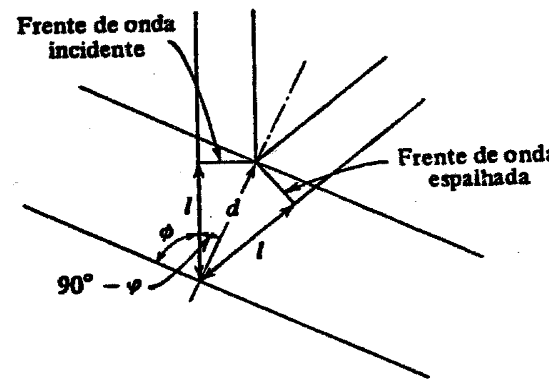
r_0~**1.25fm**

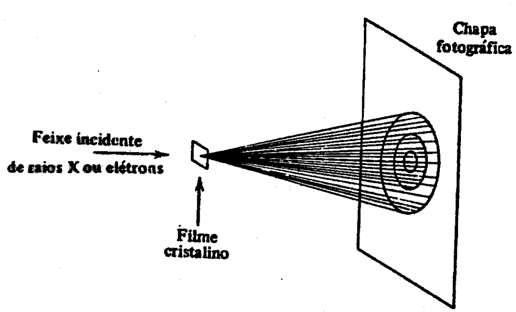


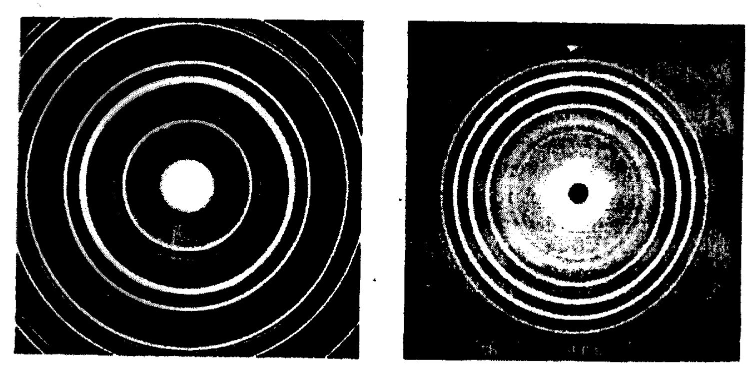
$$R=r_o(A_1^{1/3}+A_2^{1/3})$$

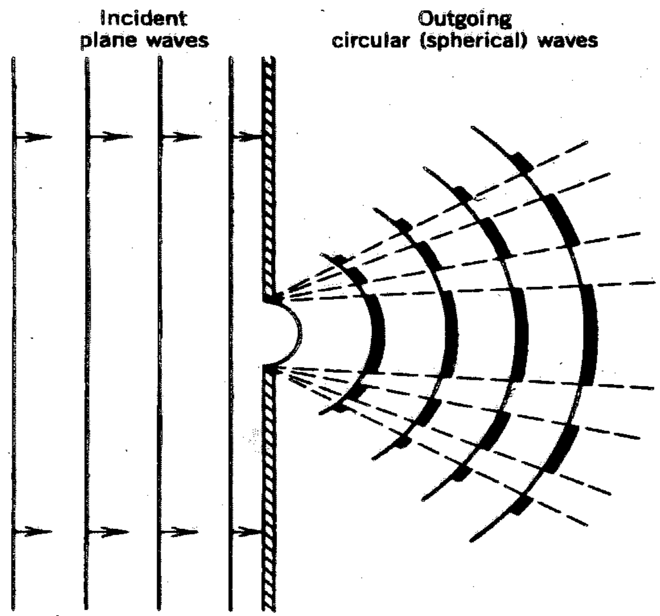
O Postulado de de Broglie Propriedades Ondulatórias das Partículas











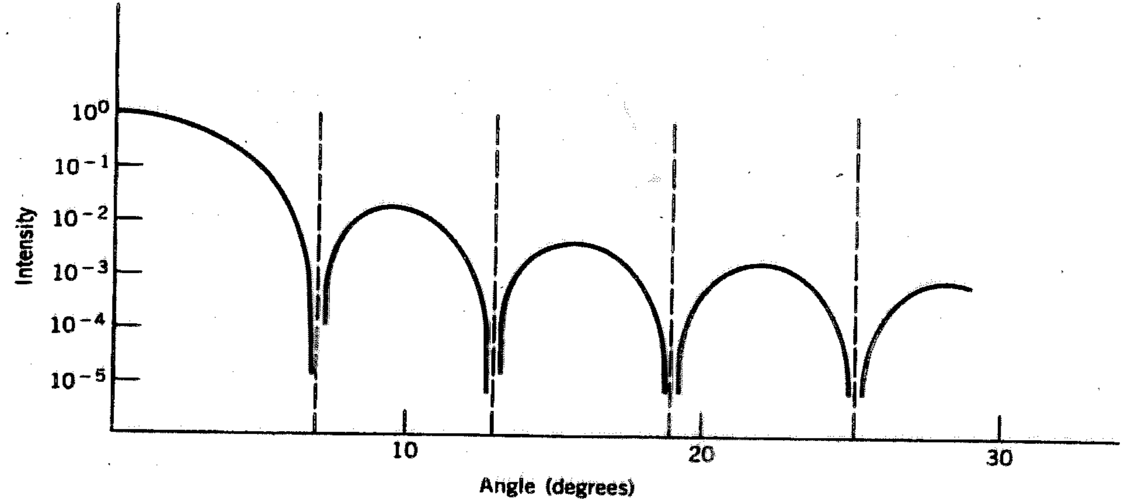
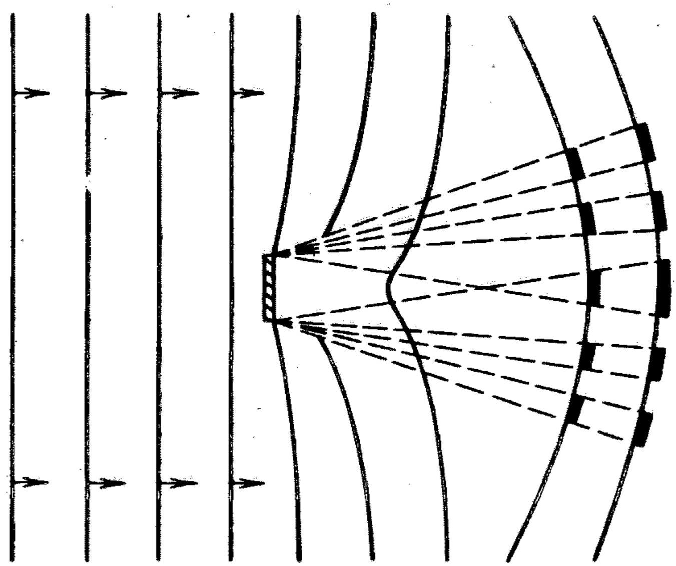


Figure 11.13 Diffraction pattern of light incident on a circular aperture; a circular disk gives a similar pattern. The minima have intensity of zero. The curve is drawn for a wavelength equal to ten times the diameter of the aperture or disk.



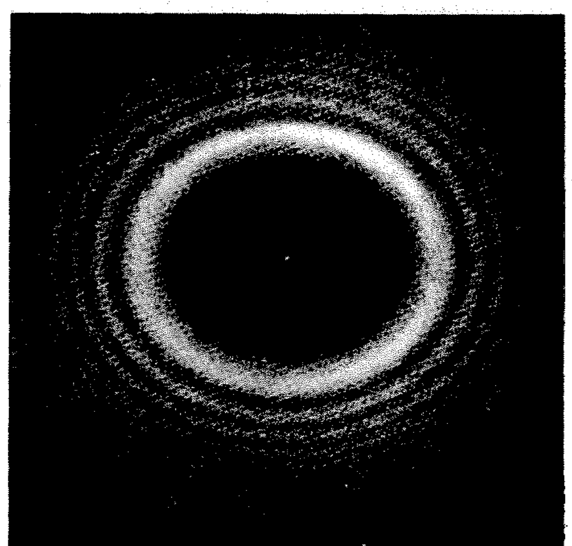


Figure 4.3 Representation of scattering by (top) a small opening and (bottom) a small obstacle. The shading of the wavefronts shows regions of large and small intensity. On the right are shown photographs of diffraction by a circular opening and an opaque circular disk. Source of photographs: M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomena* (Berlin: Springer-Verlag, 1962).

3.3 The nuclear electric charge distribution

ref: williams

We need a model. The methods used to measure charge distribution find the time average so we can define a time-independent charge density. For the present we shall assume spherically symmetric nuclei so that we can define a radial charge density, $\rho(r)$. Two models are given in Fig. 3.1. Model I with its sharp-edged charge distribution is very unlikely but can be tested. Model II softens the hard edges by assuming a charge distribution with a mathematical form normally associated with the Fermi–Dirac statistics but which, applied to nuclei, is called the **Saxon–Woods** form. So the challenge to experiments is to see if either model makes predictions which fit the data and if so, to determine the parameters ρ_0, a and d, and if not, to find a better model and its parameters.

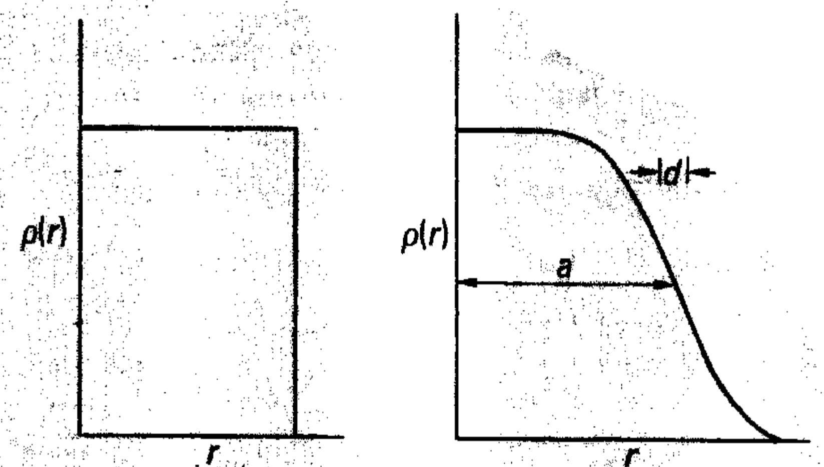
Fig. 3.1 Two models of the radial electric charge distribution of nuclei.

(a) Model I: $\rho(r) = \rho_0,\ r < a,$

$\rho(r) = 0,\ r > a.$

(b) Model II: $$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-a}{d}\right)}.$$

The $r = a \pm d/2$ points have densities 62.2% and 37.8% of the central density respectively. The 90% to 10% thickness is $4.39d$. This shape is called the Saxon–Woods form. The charge density, ρ_0, is fixed by normalizing to the total nuclear charge $Z|e|$.



(a) (b)

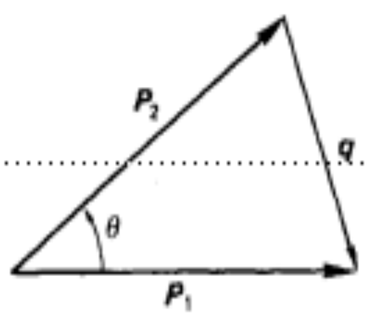


Fig. 1.7 The momentum transfer $\boldsymbol{q}$ ir elastic scattering at a fixed target. Th vectors $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ represent the incid‹ and scattered particle momenta respectively ($|\boldsymbol{P}_1| = |\boldsymbol{P}_2| = P$). If the angle of scatter is θ, the geometry gi‹

$q = 2P\sin(\theta/2)$.

3.4 The nuclear electric form-factor

How do we deal with the effect of an extended nuclear charge on the Mott scattering of electrons? The answer is to do as in classical optics where we derive the Fraunhofer diffraction pattern of an aperture in a screen by taking the Fourier transform of that aperture. For electron scattering the aperture is replaced by a spherical distribution of charge. We take the nucleus to have charge Ze where e is the charge on the proton. If that charge was point-like at $r = 0$ we can imagine that it gives rise to a scattered wave amplitude $Zef(\theta)$ at large distances at an angle θ, defined so that

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} = Z^2 e^2 |f(\theta)|^2. \tag{3.1}$$

If that charge is spread out then an element of charge $d(Ze)$ at a point $\mathbf{r}$ will give rise to a contribution to the amplitude of $e^{i\delta} f(\theta) d(Ze)$ where δ is the extra 'optical' phase introduced by wave scattering by the element of charge at the point $\boldsymbol{r}$ compared to zero phase for scattering at $r = 0$.

Consider now Fig. 3.2. The incident and scattered electron have momentum $\boldsymbol{p}$ and $\boldsymbol{p}'$ with $p = |\boldsymbol{p}| = |\boldsymbol{p}'|$. The momentum transfer $\boldsymbol{q}$ ($|q| = 2p\sin(\theta/2)$, see Fig. 1.7) is along OZ, O being the nuclear centre. The 'optical ray' P_1OP_1' is taken to have zero relative path length. The ray P_2SP_2' has equal angles of incidence and reflection at the plane AXA' which is perpendicular to OZ. Therefore the path length d is the same for all rays parallel to P_1OP_1' and reflected at any point in AXA'. This path length is given by $d = 2OX\sin(\theta/2)$. The phase, δ, is $2\pi d/\lambda$, where λ is the de Broglie wavelength. Now the reduced wavelength $\lambda/2\pi = \hbar/p$ so

$$\delta = pd/\hbar = \frac{2p\sin(\theta/2)}{\hbar} OX = \frac{q}{\hbar} OX.$$

Return to the point S: let the charge density be $\rho(r)$ when $r = OS$ (we assume spherical symmetry). If we have polar coordinate r, α, β, where the polar axis is along Z and $S\hat{O}Z = \alpha$, then a volume element $\mathrm{d}V$ at S is $r^2 \sin\alpha \, \mathrm{d}r \, \mathrm{d}\alpha \, \mathrm{d}\beta$ and the charge is $\rho(r)\mathrm{d}V$. Then this charge element gives rise to an amplitude

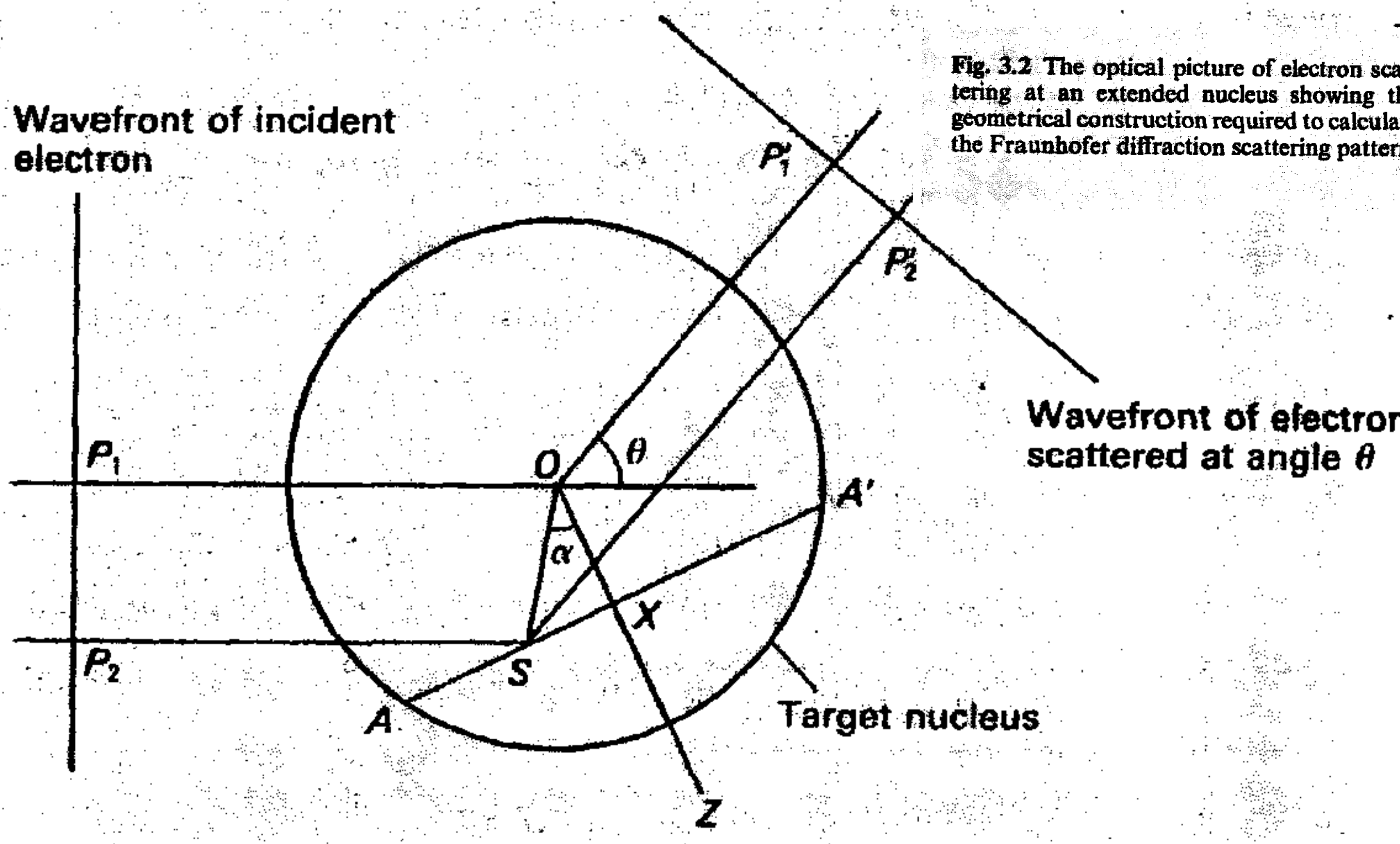


Fig. 3.2 The optical picture of electron scattering at an extended nucleus showing the geometrical construction required to calculate the Fraunhofer diffraction scattering pattern.

$$\text{charge} \times f(\theta) \times e^{i\delta}$$

$$= \rho(r) r^2 \sin\alpha \, dr \, d\alpha \, d\beta \, f(\theta) e^{i\boldsymbol{q}.\boldsymbol{r}/\hbar}.$$

The exponent contains $\boldsymbol{q}.\boldsymbol{r}$ because $q(OX) = qr\cos\alpha = \boldsymbol{q}.\boldsymbol{r}$. Then the total scattered amplitude

$$A(\theta) = f(\theta) \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \rho(r) r^2 \sin\alpha \, dr \, d\alpha \, d\beta \, e^{i\boldsymbol{q}.\boldsymbol{r}/\hbar}, \tag{3.2}$$

the integration being over the whole nucleus. The integration over the azimuthal angle β around OZ is trivial because the point S just traverses the plane AXA' for which $\boldsymbol{q}.\boldsymbol{r}$ is constant.

$$A(\theta) = f(\theta) \int_0^{\pi} \int_0^{\infty} 2\pi\rho(r) r^2 \sin\alpha \, dr \, d\alpha \, e^{i\boldsymbol{q}.\boldsymbol{r}/\hbar}.$$

Now we have for the total charge

$$Ze = \int_0^{\pi} \int_0^{\infty} 2\pi\rho(r) r^2 \sin\alpha \, dr \, d\alpha,$$

so we see that we can write

$$A(\theta) = Zef(\theta)\frac{\iint \rho(r)r^2 \sin\alpha\, dr\, d\alpha\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar}}{\iint \rho(r)r^2 \sin\alpha\, dr\, d\alpha} = Zef(\theta)F(\theta).$$

Thus the Mott (or Rutherford) scattering amplitude $Zef(\theta)$ is changed by a factor $F(\theta)$ and the scattering cross-section becomes

$$\frac{d\sigma}{d\Omega} = |F(\theta)|^2 \frac{d\sigma}{d\Omega}\bigg|_{\text{Mott}}.$$

$F(\theta)$ is called a **form factor**. (This is a name inherited from the description of atomic X-ray scattering.) Somewhat formally it could be written

$$F(\theta) = \frac{\int \rho(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar} dV}{\int \rho(\boldsymbol{r}) dV} = \frac{1}{Ze}\int \rho(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar} dV.$$

Clearly as $\theta \to 0$, $q \to 0$ and $F(0) = 1$. Since the form factor is more properly a function of q than of θ it has become usual to write $F(q^2)$ rather than $F(\theta)$ and we note $F(q^2)$ is the Fourier transform of the charge distribution (remember the optics of Fraunhofer diffraction). These ideas and formulae are summarized in Table 3.1.

The effect of an extended nuclear charge is to reduce the differential cross-section for elastic electron scattering from that for a point-like nucleus by a factor which is the square of the form factor:

$$\frac{d\sigma}{d\Omega} \rightarrow |F(q^2)|^2 \frac{d\sigma}{d\Omega},$$

where the form factor

$$F(q^2) = \frac{1}{Ze} \int\int\int \rho(r)\, e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\, dV,$$

and $\mathbf{q}$ is the momentum transfer, $q = |\mathbf{q}|$ and $\rho(\mathbf{r})$ is the charge density. The volume integral is to be taken over the entire nucleus. If the nucleus is spherically symmetric, then

$$F(q^2) = \frac{4\pi\hbar}{Zeq} \int \rho(r) r \sin\left(\frac{qr}{\hbar}\right) dr.$$

point-like nucleus; as q increases the oscillatory nature of the exponential in eq. 3.2 for an extended nucleus reduces $|F(q^2)|$ from 1 and the scattering is reduced. This is not unexpected; an extended electric charge has greater difficulty in taking up the momentum transfer than does the point-like arrangement of the same total charge.

2. Since we know the rough size of nuclei, $R < 10^{-14}$ m ($\simeq 10$ fm), we can now estimate the q needed to see a significant reduction in scattering intensity due to size. We would want $\boldsymbol{q}.\boldsymbol{R}/\hbar$ to be of order 1, hence $q \simeq \hbar/R$. Remember $\hbar c = 197$ MeV fm, so we see that $q > 20$ MeV/c. To reach this at 30° scattering requires incident electrons of 40 MeV. In fact this is hardly adequate since nuclear radii are somewhat less than 10 fm and we want to see detail with a resolution of better than 1 fm. Therefore we should be aiming for $q \simeq \hbar/(1 \text{ fm}) \simeq 200$ MeV/c. The first detailed measurements were made with electrons of 150 MeV but later work has increased the energies used to 500 MeV.

Now what does $F(q^2)$ look like? As an exercise you are asked to show (Problem 3.1) that the form factor for model I is

$$F(q^2) = \frac{3\{\sin x - x\cos x\}}{x^3}, \qquad x = qa/\hbar.$$

This looks rather unmanageable but in fact is the spherical Bessel function $j_1(x)$. The square of this function is the factor by which the point-like Mott differential cross-section is reduced. This factor is plotted in Fig. 3.3 for the case of $a = 4.1$ fm ($^{58}_{28}$Ni nucleus) and the abscissa is marked in units of q MeV/c and of θ degrees for 450 MeV incident electrons. We notice immediately the diffraction zeros near 27°, 48°, 69°, and 95°. This is typical of an object with sharp edges. In Fig. 3.4a we give the measured differential cross-section: it shows diffraction minima at q values expected from model I. On the same figure is a fitted curve using a model close to our model II. The softer edges of the distribution fill in the diffraction minima and give results closer to the data than the model I prediction.

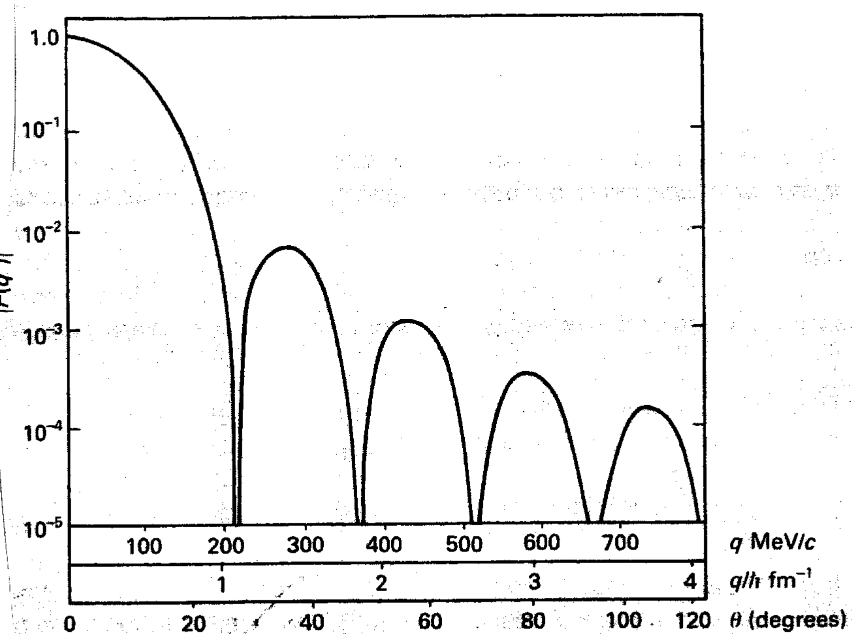
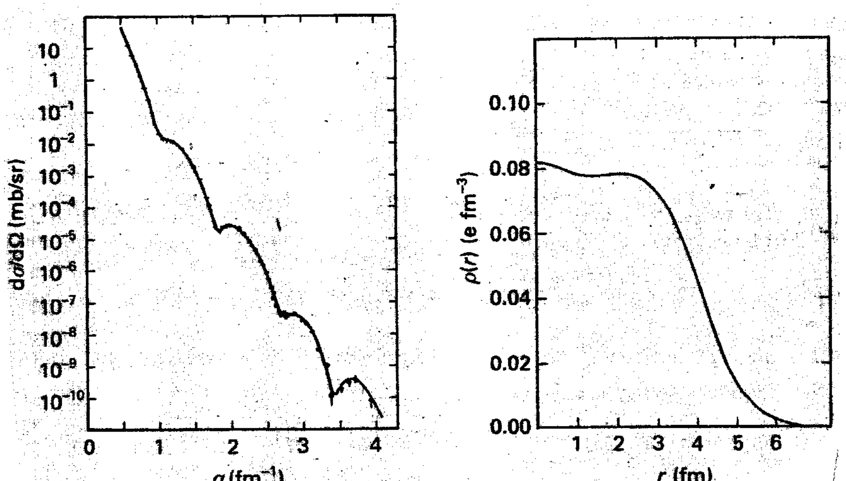
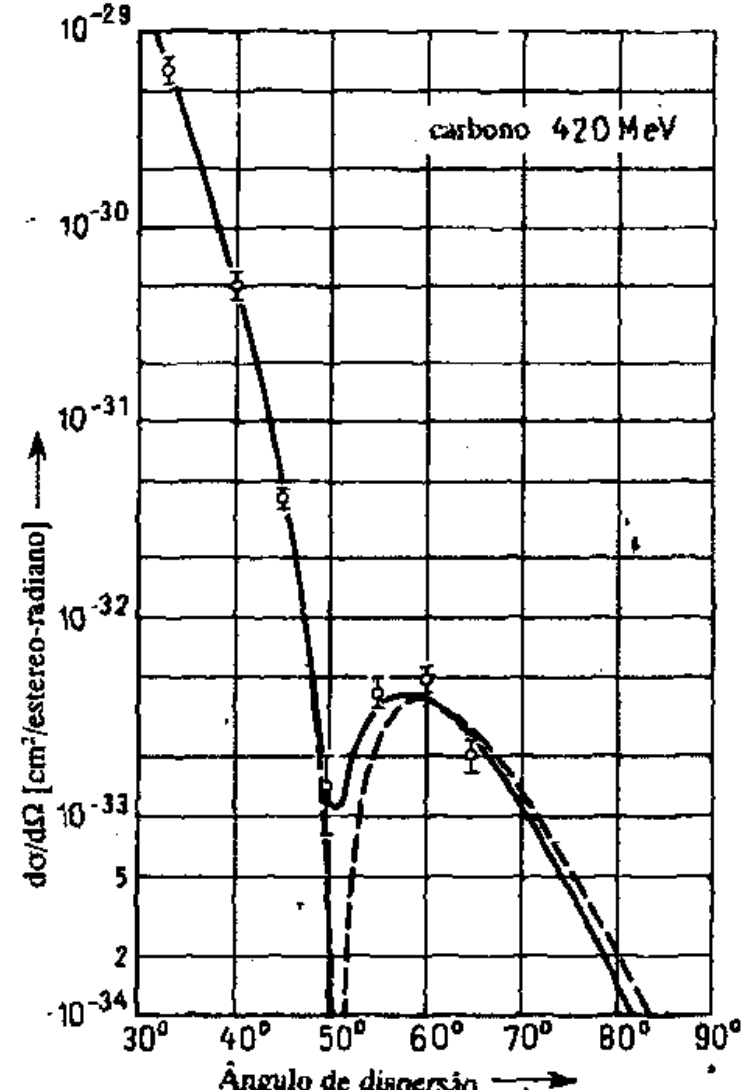


Fig. 3.3 The square of the form factor $|F(q^2)|^2$ as a function of q for a model I nucleus having $a = 4.1$ fm. The abscissa is also marked in inverse fermis ($q/\hbar$) and in degrees for an angle of scatter at a fixed nucleus for incident electrons of 450 MeV. Note that the ordinate is logarithmic.





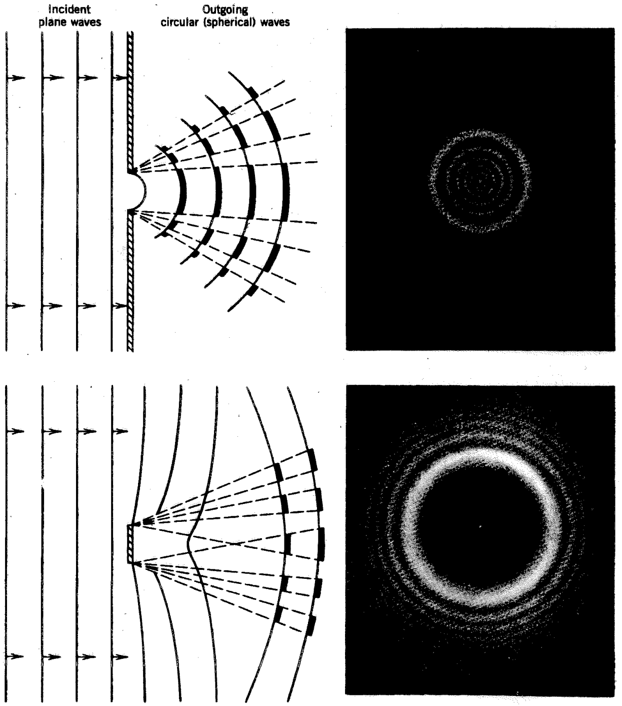


Figure 4.3 Representation of scattering by (top) a small opening and (bottom) a small obstacle. The shading of the wavefronts shows regions of large and small intensity. On the right are shown photographs of diffraction by a circular opening and an opaque circular disk. Source of photographs: M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomena* (Berlin: Springer-Verlag, 1962).

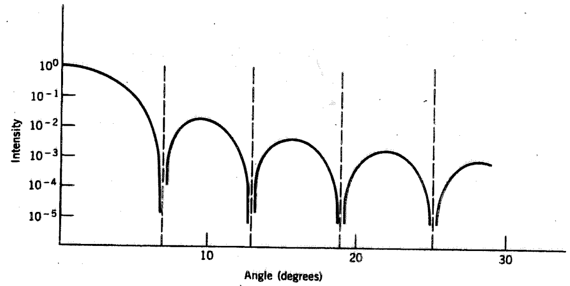


Figure 11.13 Diffraction pattern of light incident on a circular aperture; a circular disk gives a similar pattern. The minima have intensity of zero. The curve is drawn for a wavelength equal to ten times the diameter of the aperture or disk.

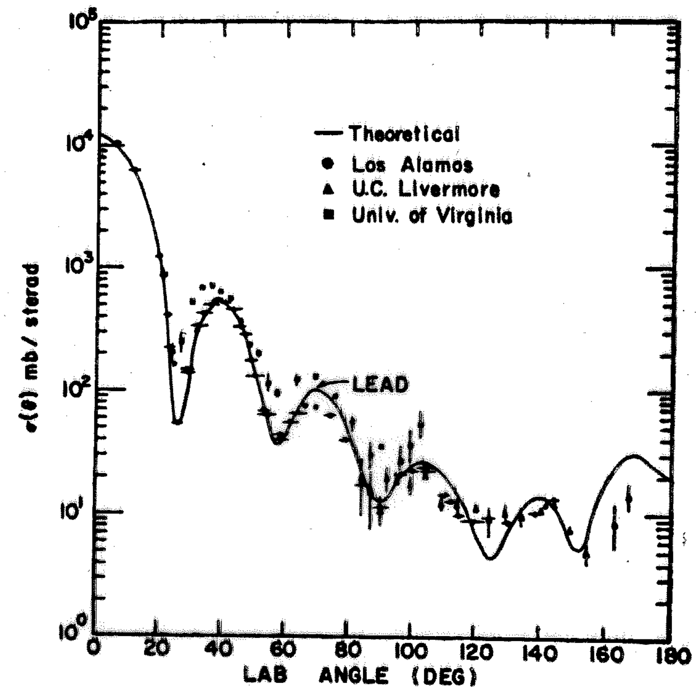


Figure 11.14 Elastic scattering of 14-MeV neutrons from Pb. From S. Fernbach, *Rev. Mod. Phys.* **30**, 414 (1958).

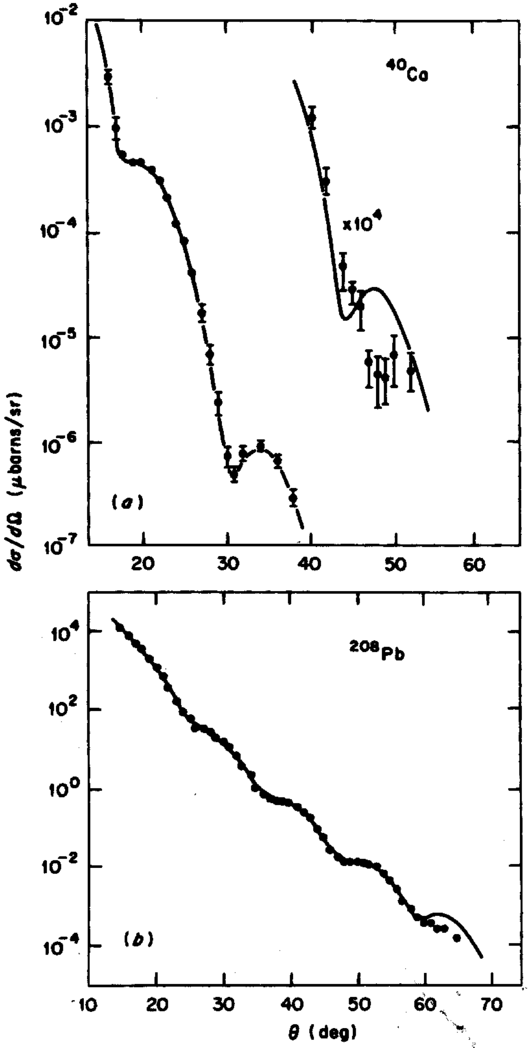


Figure 2.15 The differential cross-sections for electrons scattering elastically from (upper) ^{40}Ca at 750 MeV (after Bellicard *et al.*, 1967) and (lower) ^{208}Pb at 502 MeV. The curves are theoretical fits to the data (after Heisenberg *et al.*, 1969)

The charge distribution of the simplest nucleus of all, the proton, has been measured by electron scattering and found to have a root mean square radius of about 0.8 fm. This size can be related to the cloud of virtual mesons which surround the 'bare' proton. For the same reason, the neutron itself is found to have a charge distribution of finite extent; although its total charge is zero, it has a short-ranged distribution of positive charge and a longer-ranged distribution of negative charge, with a net root mean square radius of 0.36 fm.

In addition to the charge distribution we may also determine the distribution

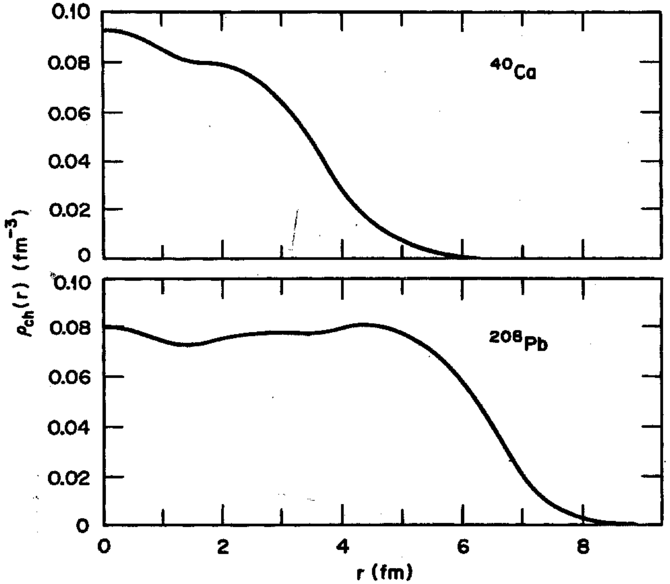


Figure 2.16 The charge distributions of ^{40}Ca and ^{208}Pb nuclei deduced by theoretical fits to the measurements such as those shown in *Figure 2.15*. The shapes at small radii are obtained by fitting the data for the larger angles (that is, for the larger momentum transfers.) (After Friar and Negele, 1973; Sinha *et al.*, 1973)

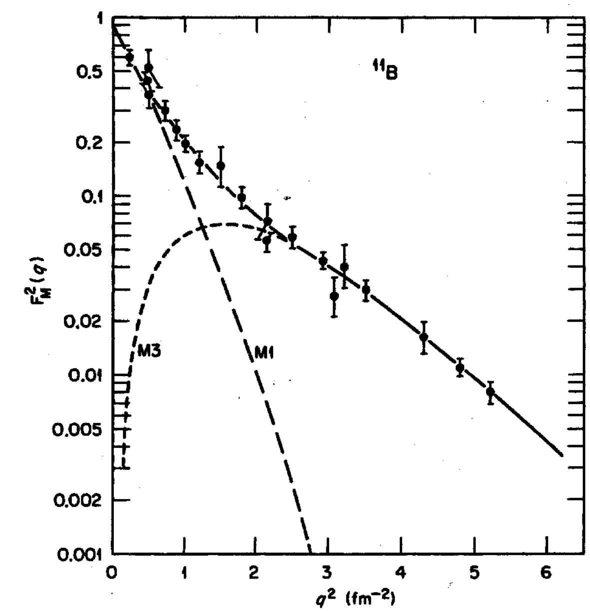
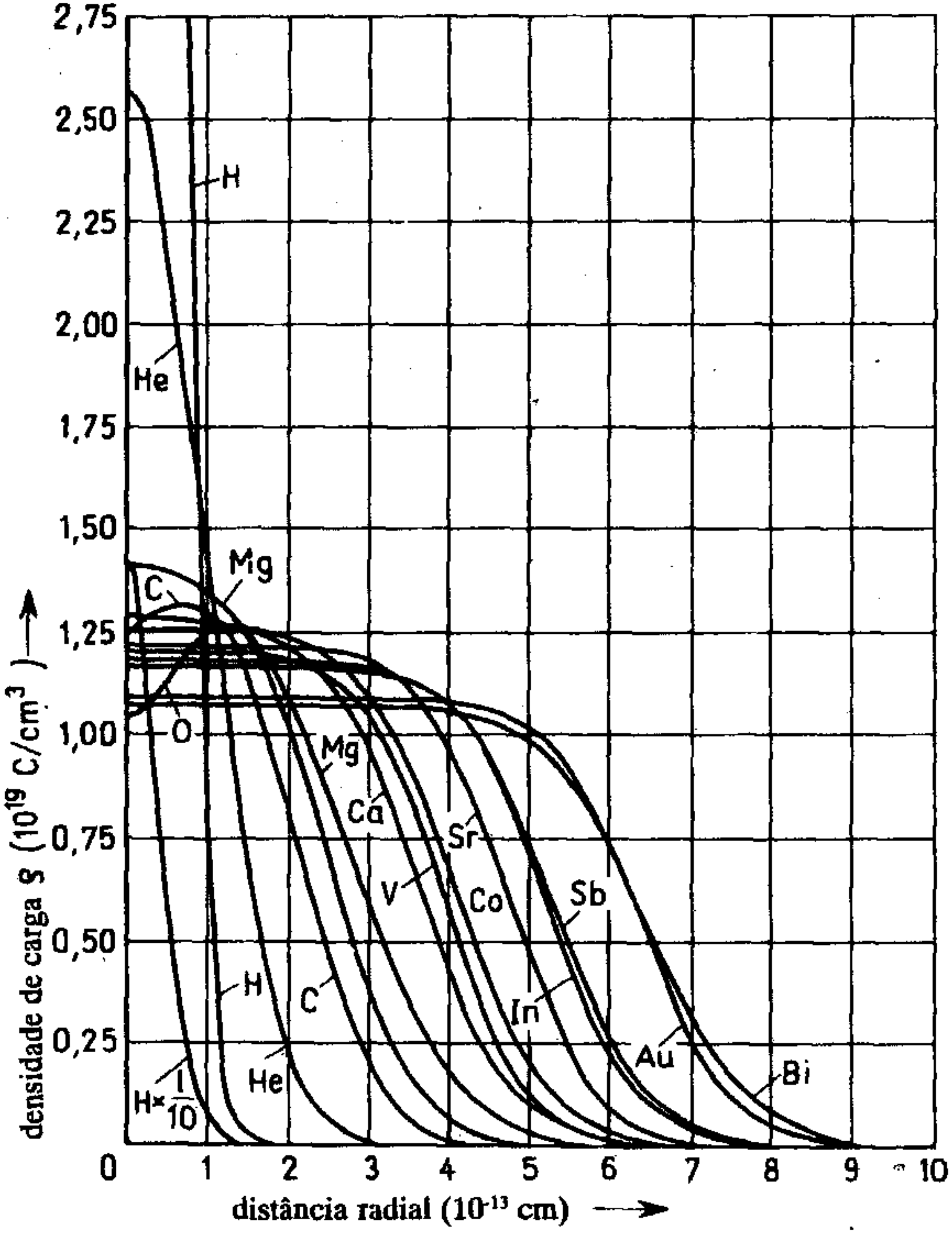
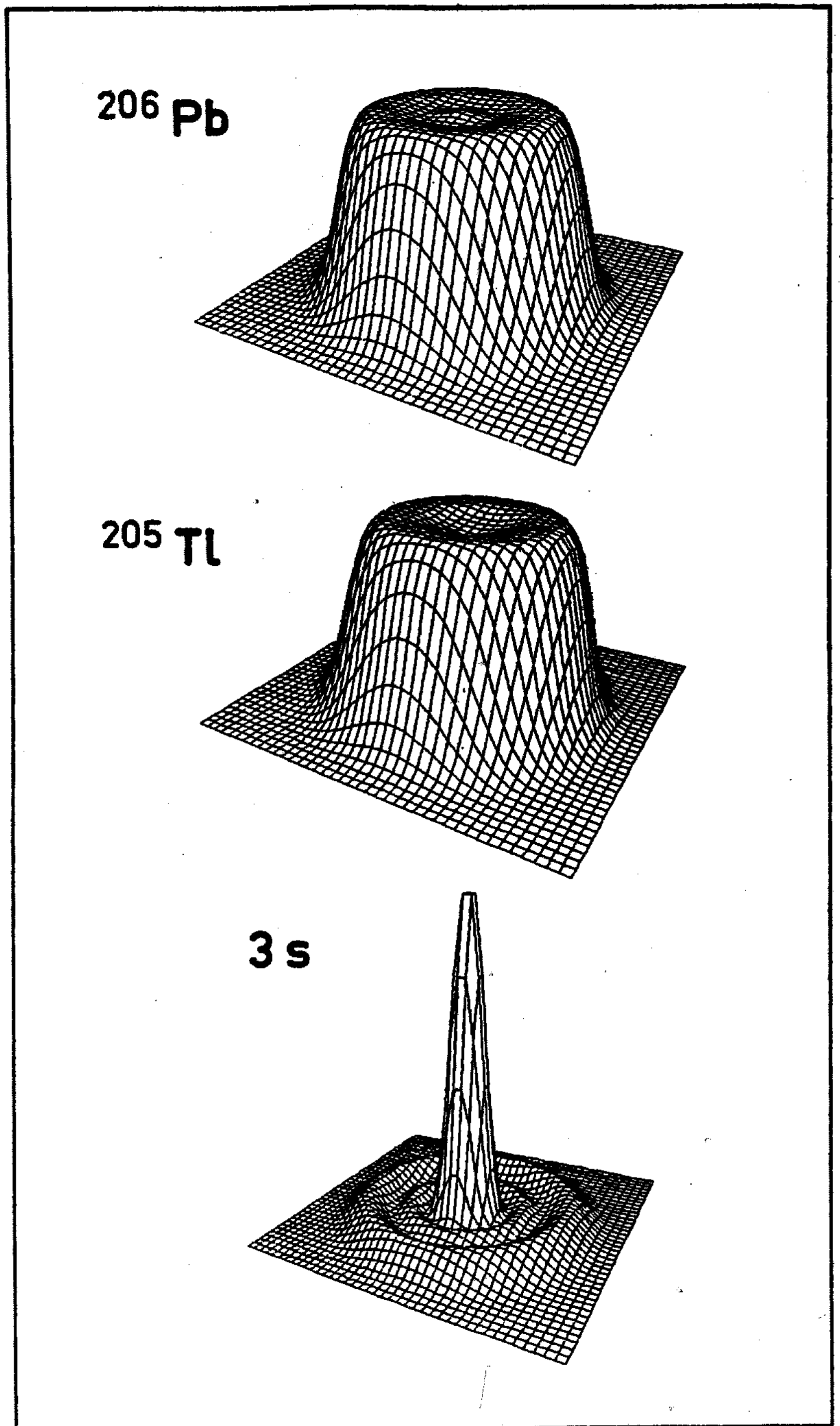


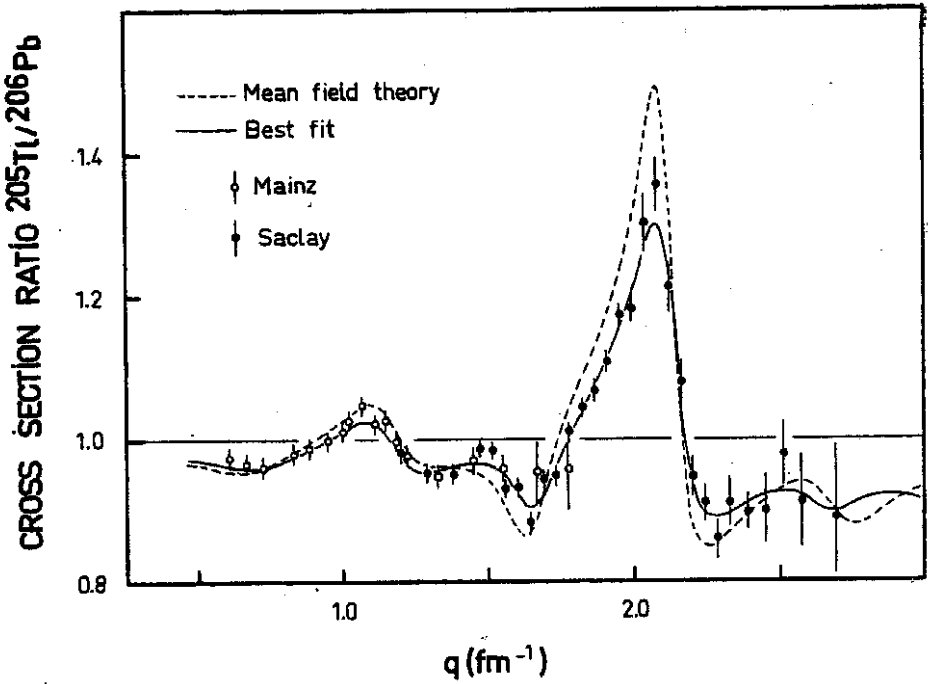
Figure 2.17 Dipole (M1) and octupole (M3) magnetic form factors for ^{11}B deduced from electron-scattering measurements. (After Rand *et al.*, 1966)



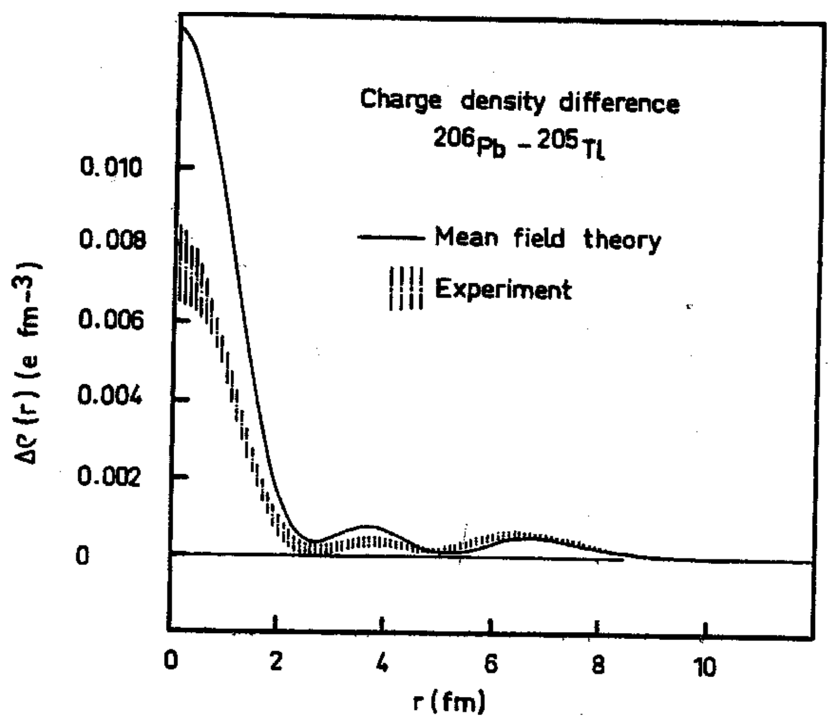
2,75
2,50
2,25
2,00
1,75
1,50
1,25
1,00
0,75
0,50
0,25
0
1
2
3
4
5
6
7
8
9
10
H
He
Mg
C
O
Mg
Ca
Sr
V
Co
Sb
H
C
In
He
Au
Bi
H× 1/10
densidade de carga ρ (10¹⁹ C/cm³) →
distância radial (10⁻¹³ cm) →



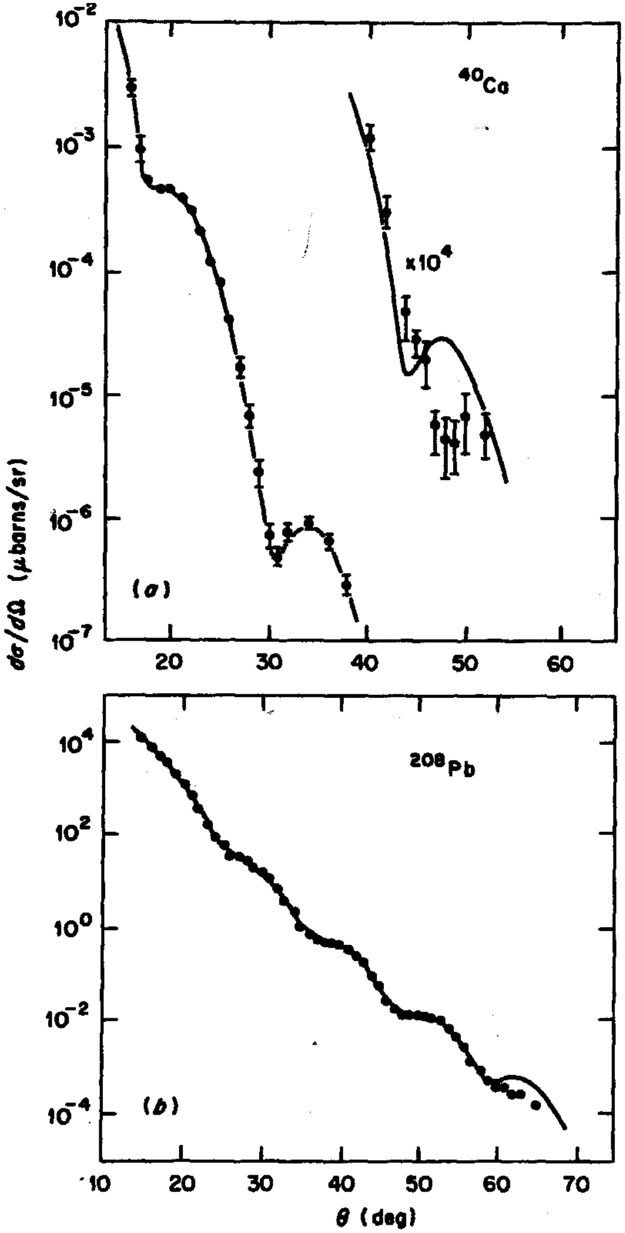
206 Pb
205 Tl
3 s



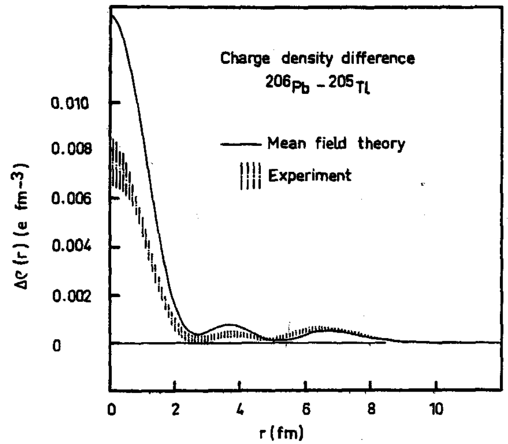
Mean field theory
Best fit
Mainz
Saclay
CROSS SECTION RATIO 205Tl/206Pb
1.4
1.2
1.0
0.8
1.0
2.0
q (fm⁻¹)



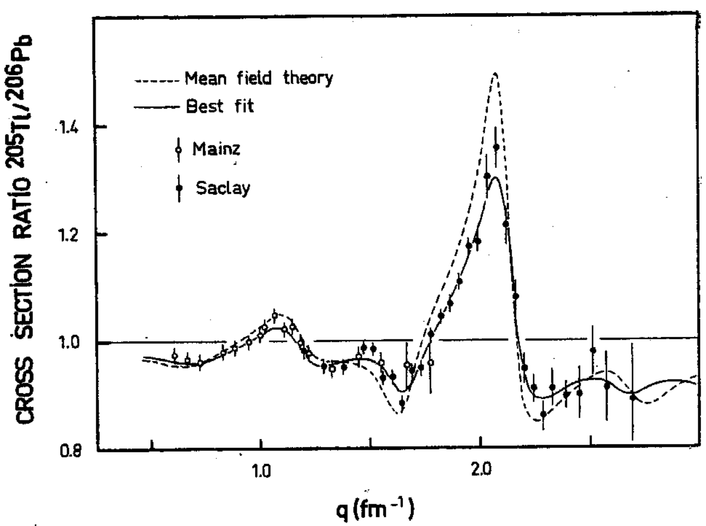
Charge density difference
206Pb - 205Tl
Mean field theory
Experiment
Δϱ (r) (e fm-3)
0.010
0.008
0.006
0.004
0.002
0
0
2
4
6
8
10
r (fm)



^{40}Ca
x10^4
(a)
$d\sigma/d\Omega$ (μbarns/sr)
^{208}Pb
(b)
θ (deg)



Charge density difference
$^{206}Pb - ^{205}Tl$
Mean field theory
Experiment
$\Delta\rho(r)$ (e fm-3)
r (fm)



Mean field theory
Best fit
Mainz
Saclay
CROSS SECTION RATIO $^{205}Tl/^{206}Pb$
q (fm^{-1})

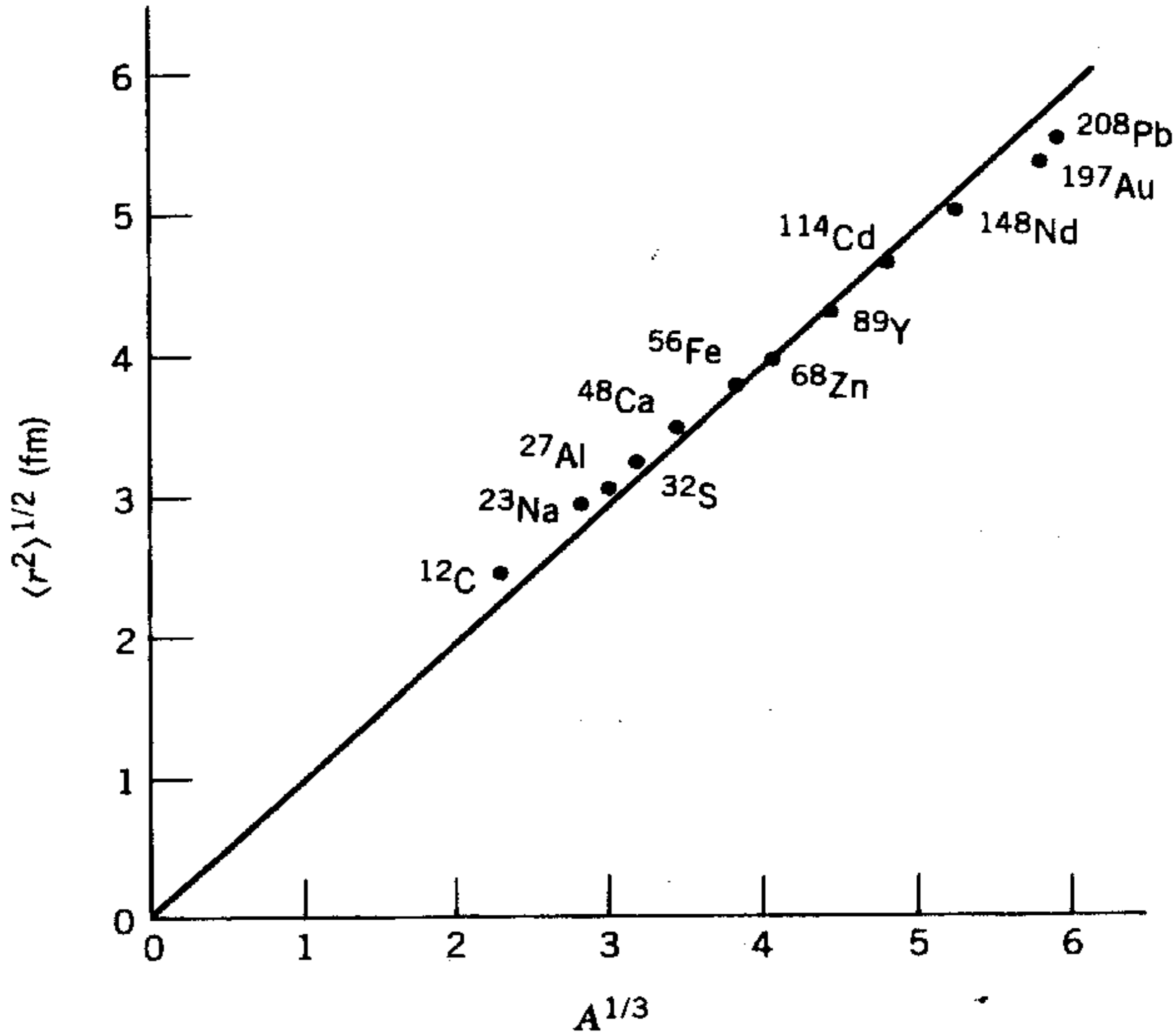
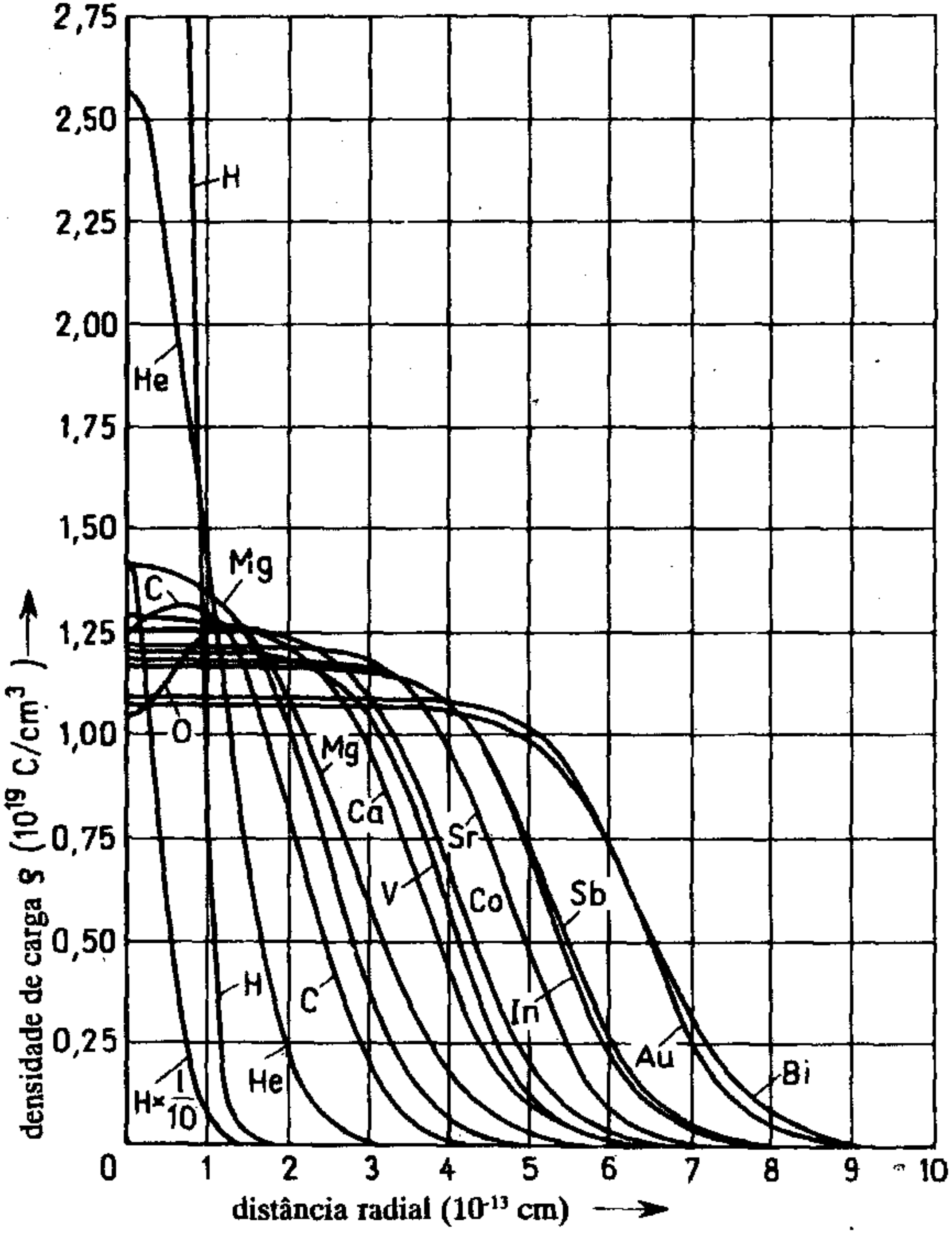
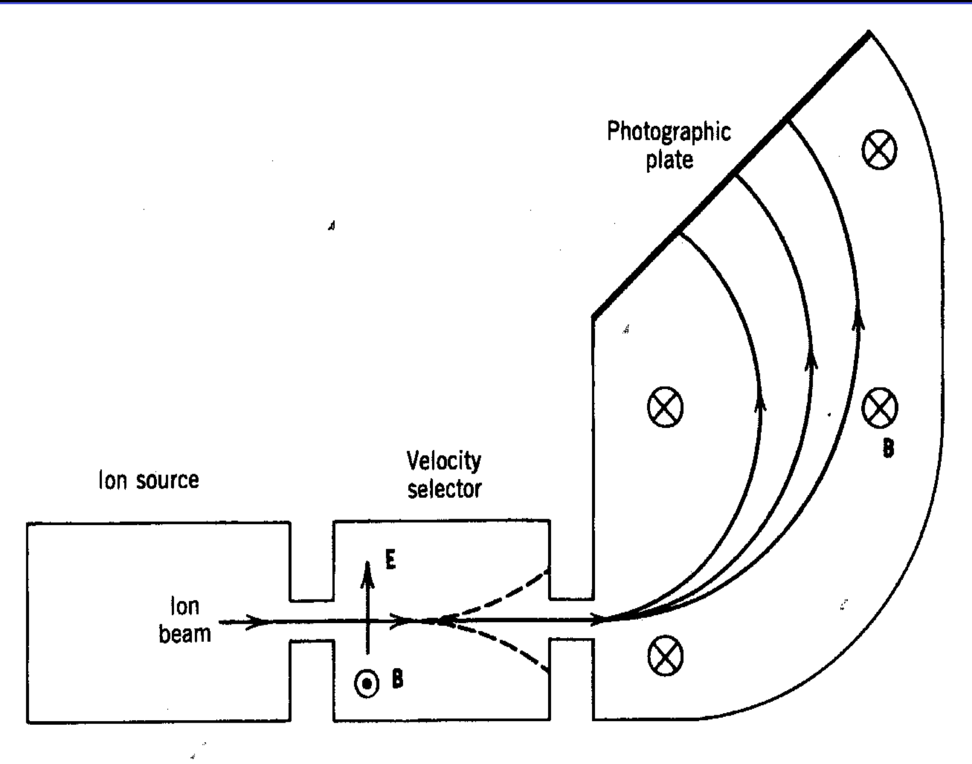


Figure 3.5 The rms nuclear radius determined from electron scattering experiments. The slope of the straight line gives $R_0 = 1.23$ fm. (The line is not a true fit to the data points, but is forced to go through the origin to satisfy the equation $R = R_0 A^{1/3}$.) The error bars are typically smaller than the size of the points (± 0.01 fm). More complete listings of data and references can be found in the review of C. W. de Jager et al., *Atomic Data and Nuclear Data Tables* **14**, 479 (1974).

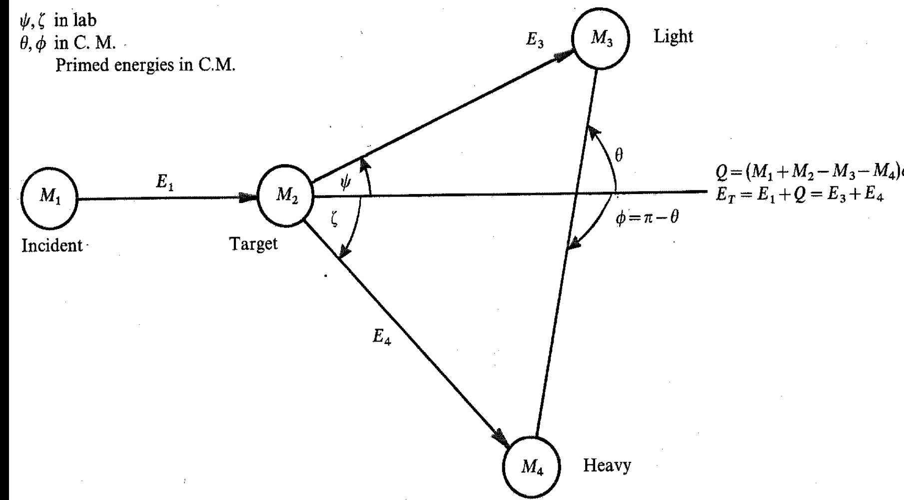


2,75
2,50
2,25
2,00
1,75
1,50
1,25
1,00
0,75
0,50
0,25
H
He
Mg
C
O
Mg
Ca
Sr
V
Co
Sb
In
Au
Bi
H
C
He
H× 1/10
0
1
2
3
4
5
6
7
8
9
10
densidade de carga ρ (10^19 C/cm^3) →
distância radial (10^-13 cm) →

reações nucleares (tipos)
cinematica de reação



Kinematics of nuclear reactions and scattering (continued)



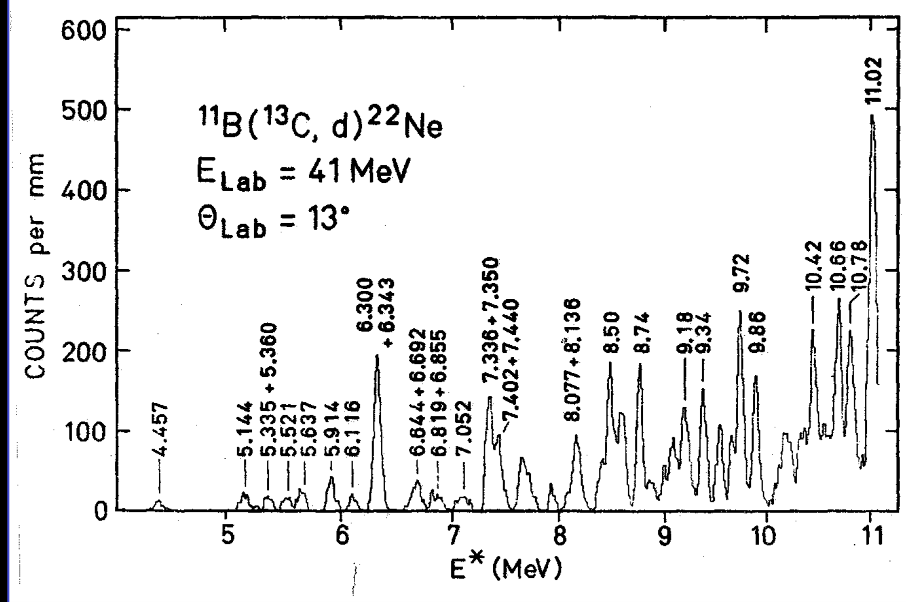
Define:

$$A = \frac{M_1 M_4 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad C = \frac{M_2 M_3}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1 Q}{M_2 E_T}\right) = \frac{E'_4}{E_T}$$

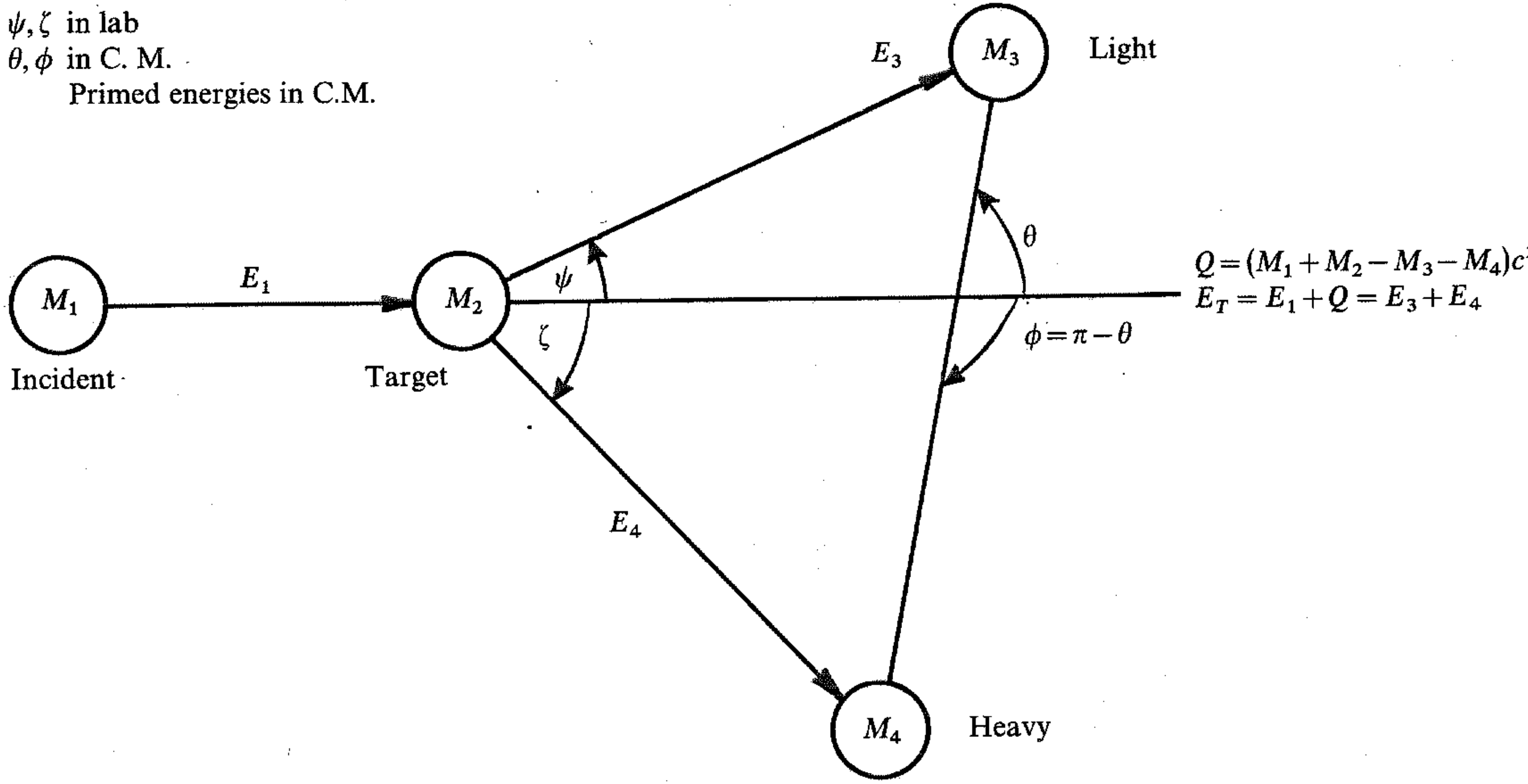
$$B = \frac{M_1 M_3 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad D = \frac{M_2 M_4}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1 Q}{M_2 E_T}\right) = \frac{E'_3}{E_T}$$

Note that $A+B+C+D=1$ and $AC=BD$

$$\frac{mv^2}{r} = qvB \;\Rightarrow\; m = \frac{qrB^2}{E}$$



Kinematics of nuclear reactions and scattering (continued)



Define:

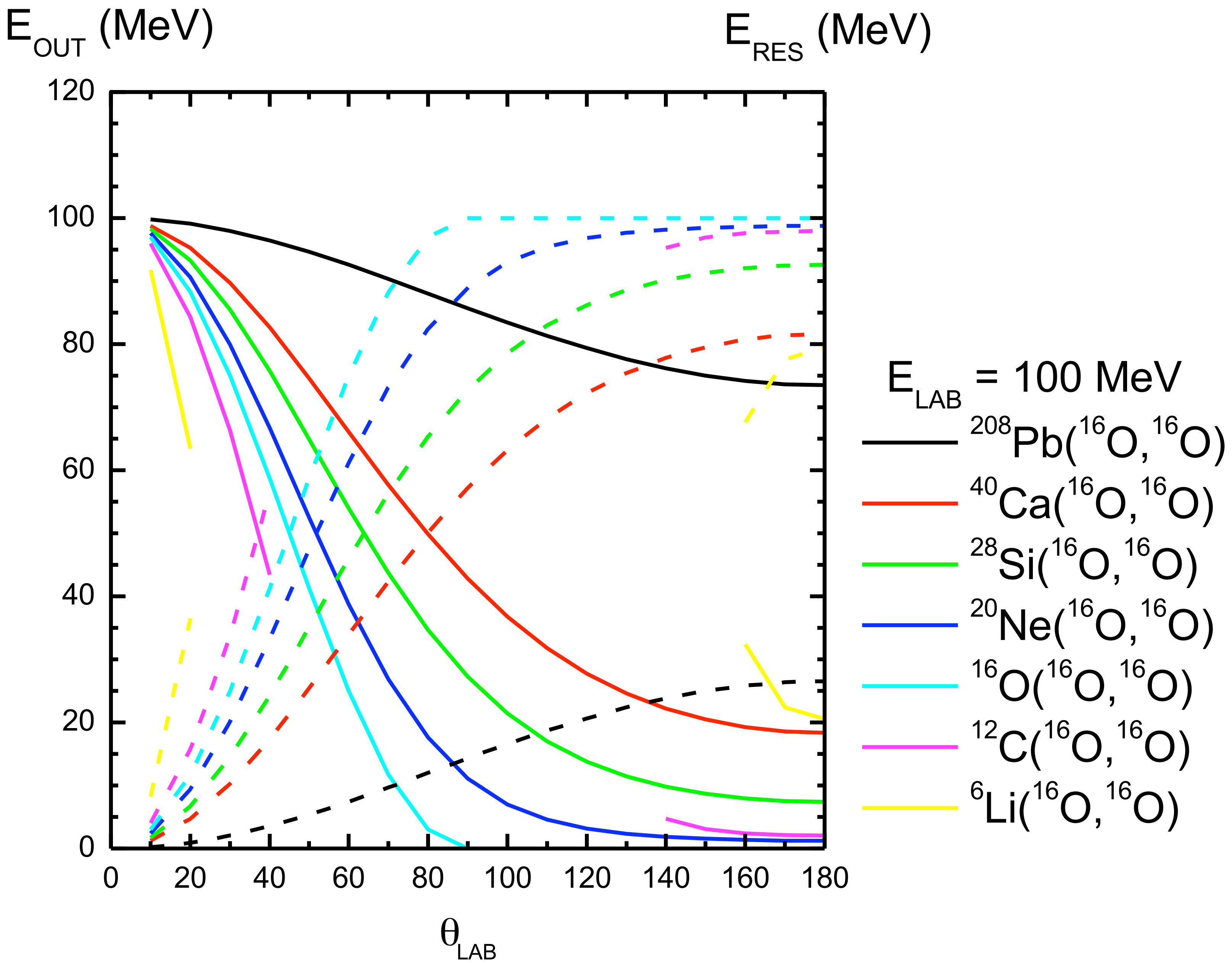
$$A = \frac{M_1 M_4 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad C = \frac{M_2 M_3}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1 Q}{M_2 E_T}\right) = \frac{E'_4}{E_T}$$

$$B = \frac{M_1 M_3 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad D = \frac{M_2 M_4}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1 Q}{M_2 E_T}\right) = \frac{E'_3}{E_T}$$

Note that $A+B+C+D=1$ and $AC=BD$

Note that $A+B+C+D=1$ and $AC=BD$

Lab energy of light product:	$\frac{E_3}{E_T}=B+D+2(AC)^{\frac{1}{2}}\cos\theta$ $=B[\cos\psi\pm(D/B-\sin^2\psi)^{\frac{1}{2}}]^2$		Use only plus sign unless $B>D$, in which case $\psi_{max}=\sin^{-1}(D/B)^{\frac{1}{2}}$
Lab energy of heavy product:	$\frac{E_4}{E_T}=A+C+2(AC)^{\frac{1}{2}}\cos\phi$ $=A[\cos\zeta\pm(C/A-\sin^2\zeta)^{\frac{1}{2}}]^2$		Use only plus sign unless $A>C$, in which case $\zeta_{max}=\sin^{-1}(C/A)^{\frac{1}{2}}$
Lab angle of heavy product:	$\sin\zeta=\left(\frac{M_3E_3}{M_4E_4}\right)^{\frac{1}{2}}\sin\psi$	C.M. angle of light product:	$\sin\theta=\left(\frac{E_3/E_T}{D}\right)\sin\psi$
Intensity or solid-angle ratio for light product:	$\frac{\sigma(\theta)}{\sigma(\psi)}=\frac{I(\theta)}{I(\psi)}=\frac{\sin\psi\,d\psi}{\sin\theta\,d\theta}=\frac{\sin^2\psi}{\sin^2\theta}\cos(\theta-\psi)=\frac{(AC)^{\frac{1}{2}}(D/B-\sin^2\psi)^{\frac{1}{2}}}{E_3/E_T}$		
Intensity or solid-angle ratio for heavy product:	$\frac{\sigma(\phi)}{\sigma(\zeta)}=\frac{I(\phi)}{I(\zeta)}=\frac{\sin\zeta\,d\zeta}{\sin\phi\,d\phi}=\frac{\sin^2\zeta}{\sin^2\phi}\cos(\phi-\zeta)=\frac{(AC)^{\frac{1}{2}}(C/A-\sin^2\zeta)^{\frac{1}{2}}}{E_4/E_T}$		
Intensity or solid-angle ratio for associated particles in the lab system:	$\frac{\sigma(\zeta)}{\sigma(\psi)}=\frac{I(\zeta)}{I(\psi)}=\frac{\sin\psi\,d\psi}{\sin\zeta\,d\zeta}=\frac{\sin^2\psi\cos(\theta-\psi)}{\sin^2\zeta\cos(\phi-\zeta)}$		



E_{OUT} (MeV)
E_{RES} (MeV)
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
160
180
θ_{LAB}
E_{LAB} = 100 MeV
$^{208}Pb(^{16}O,^{16}O)$
$^{40}Ca(^{16}O,^{16}O)$
$^{28}Si(^{16}O,^{16}O)$
$^{20}Ne(^{16}O,^{16}O)$
$^{16}O(^{16}O,^{16}O)$
$^{12}C(^{16}O,^{16}O)$
$^{6}Li(^{16}O,^{16}O)$

EXERCÍCIO

CINEMÁTICA DE REAÇÕES: **CALCULAR**

E_3 , E_4 vs θ ou E_{out} , E_{res} vs θ

E_3 , E_4 vs E_1

Ψ vs χ

θ vs φ

ESPALHAMENTO ELASTICO

REAÇÕES DIRETAS

ESPALHAMENTO INELASTICO

TRANSFERENCIA DE NUCLEONS
- "STRIPPING"
- "PICK-UP"

"KNOCK-OUT"

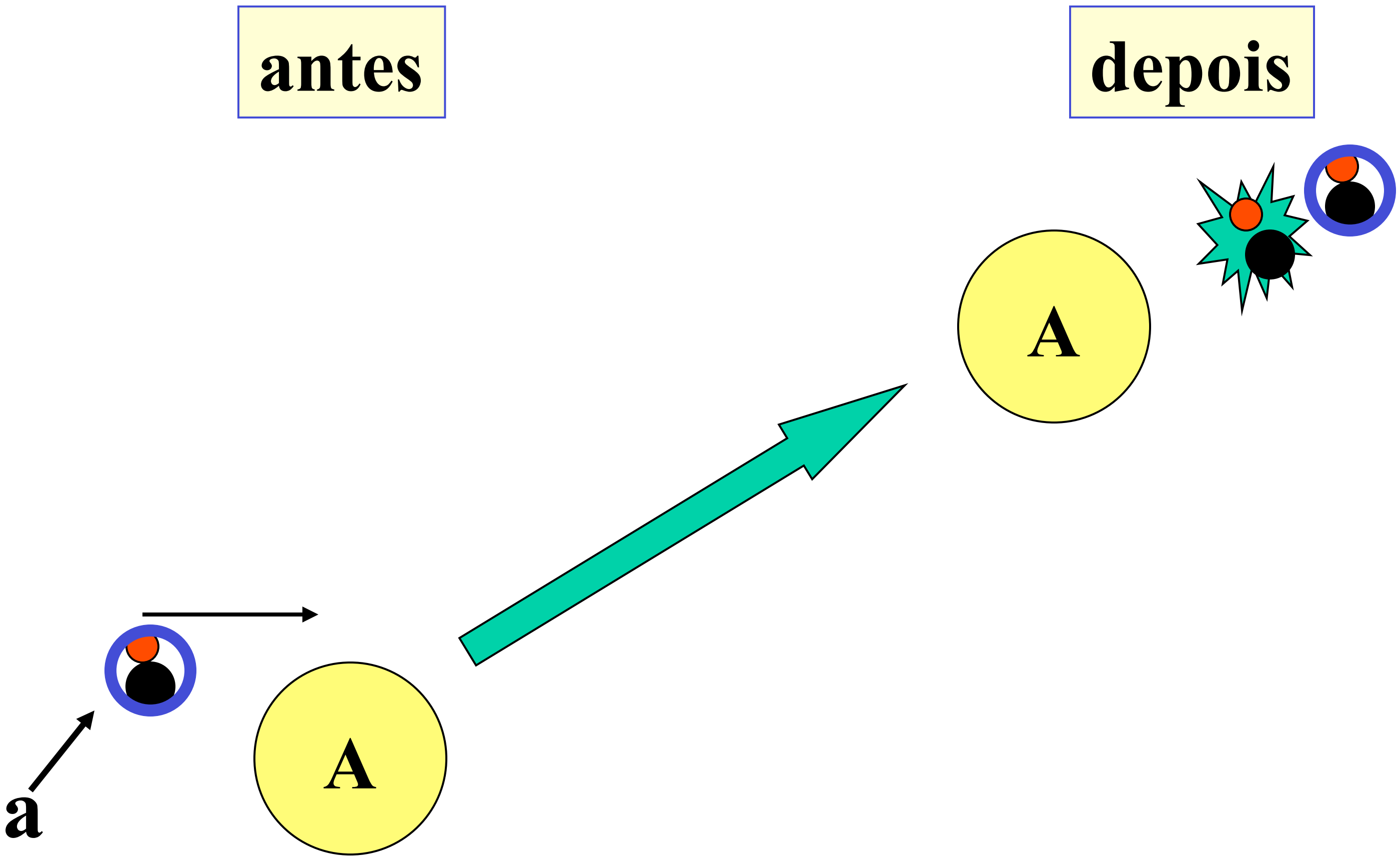
QUEBRA NUCLEAR ("BREAK-UP")

PRÉ-EQUILIBRIO

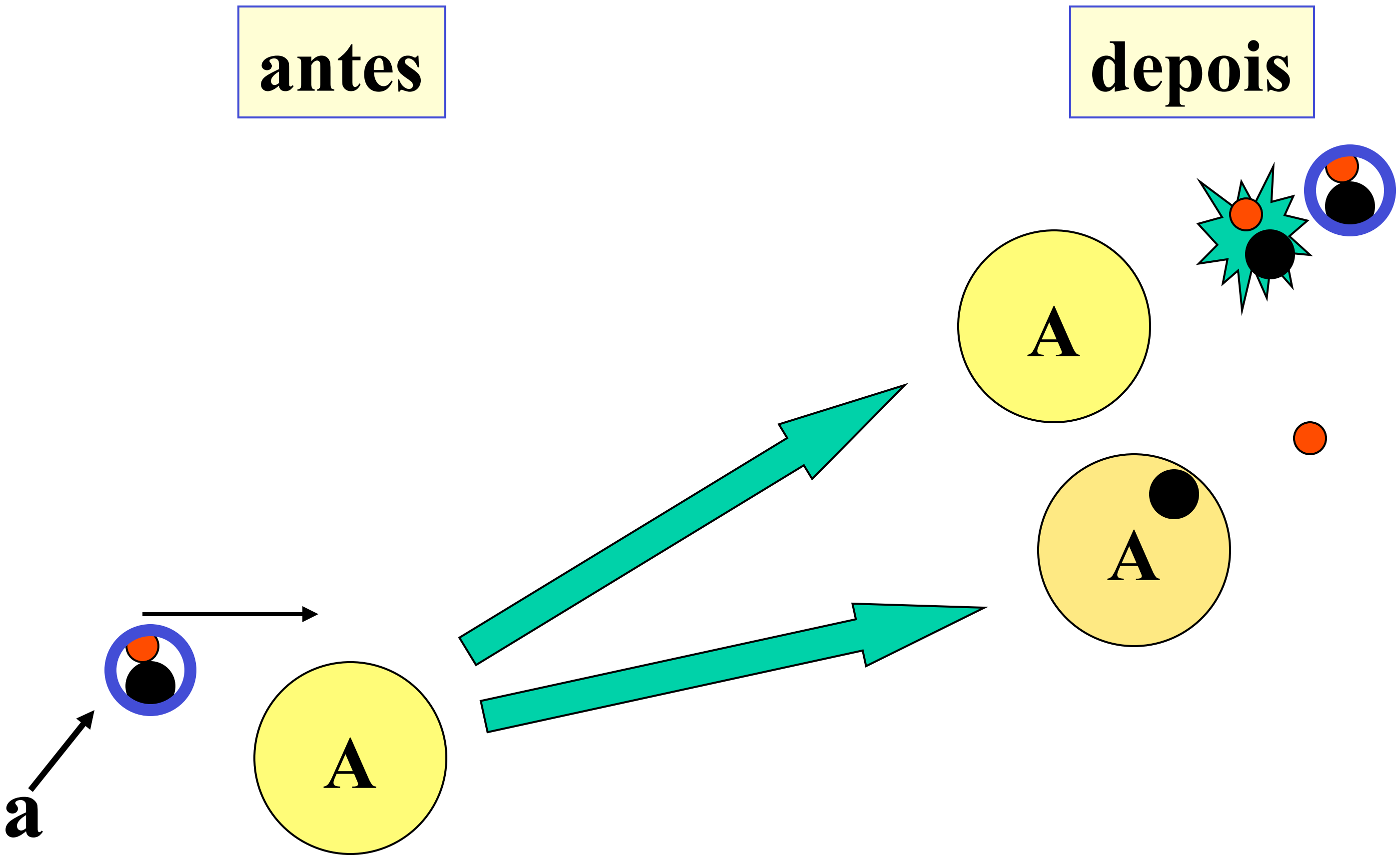
NUCLEO COMPOSTO

FUSÃO
- FUSÃO COMPLETA
- FUSÃO INCOMPLETA

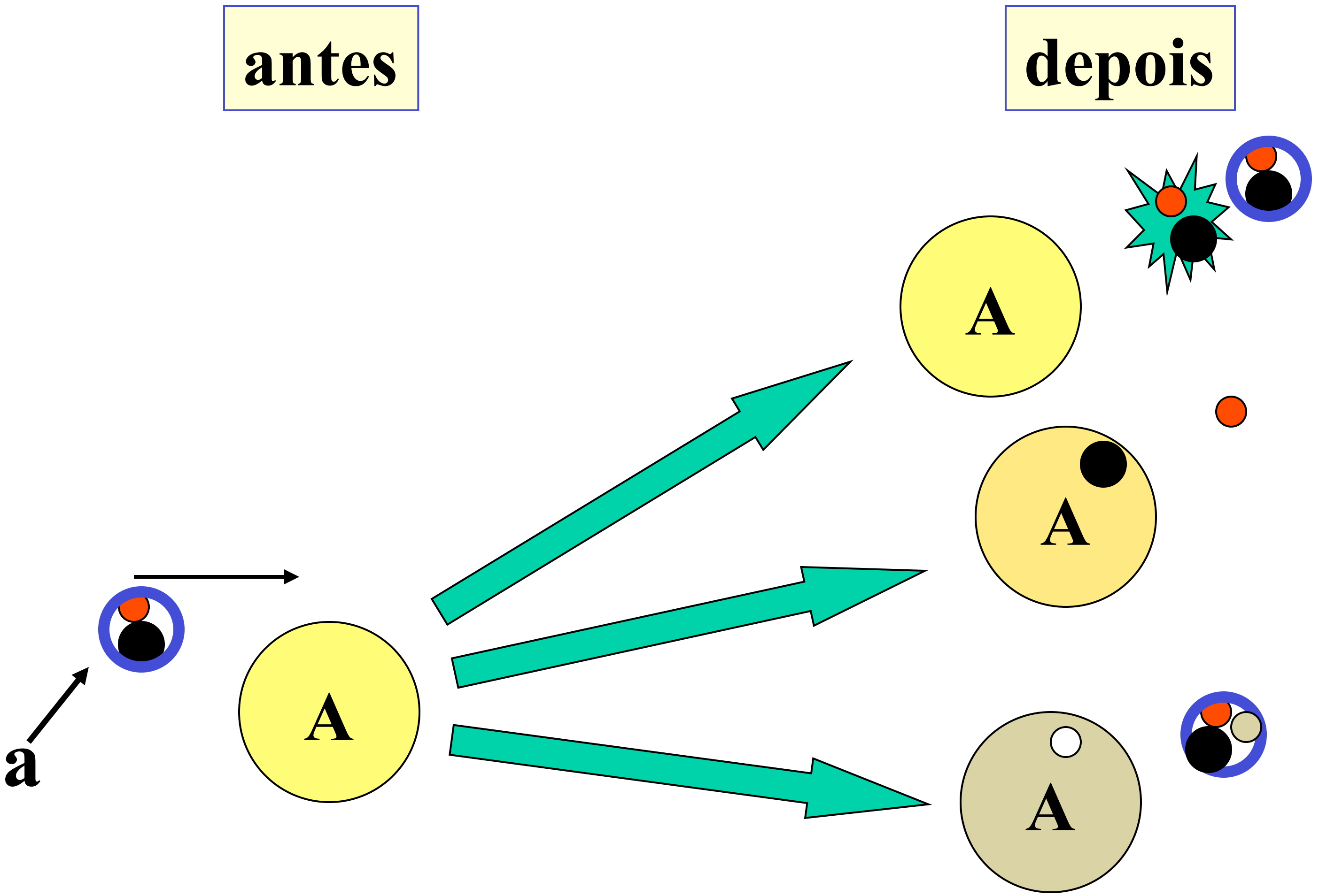
FISSÃO



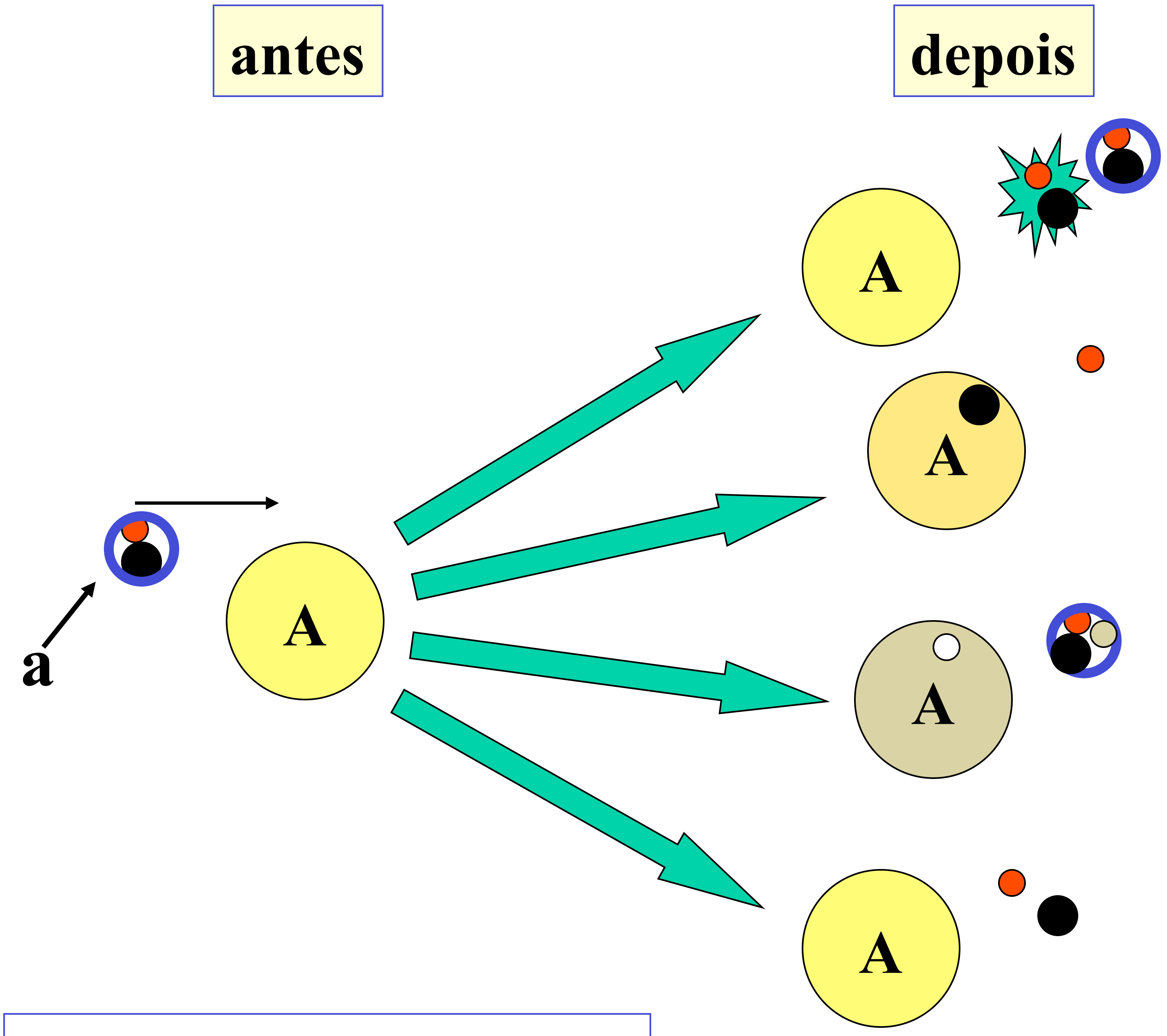
Reações diretas (rápidas)



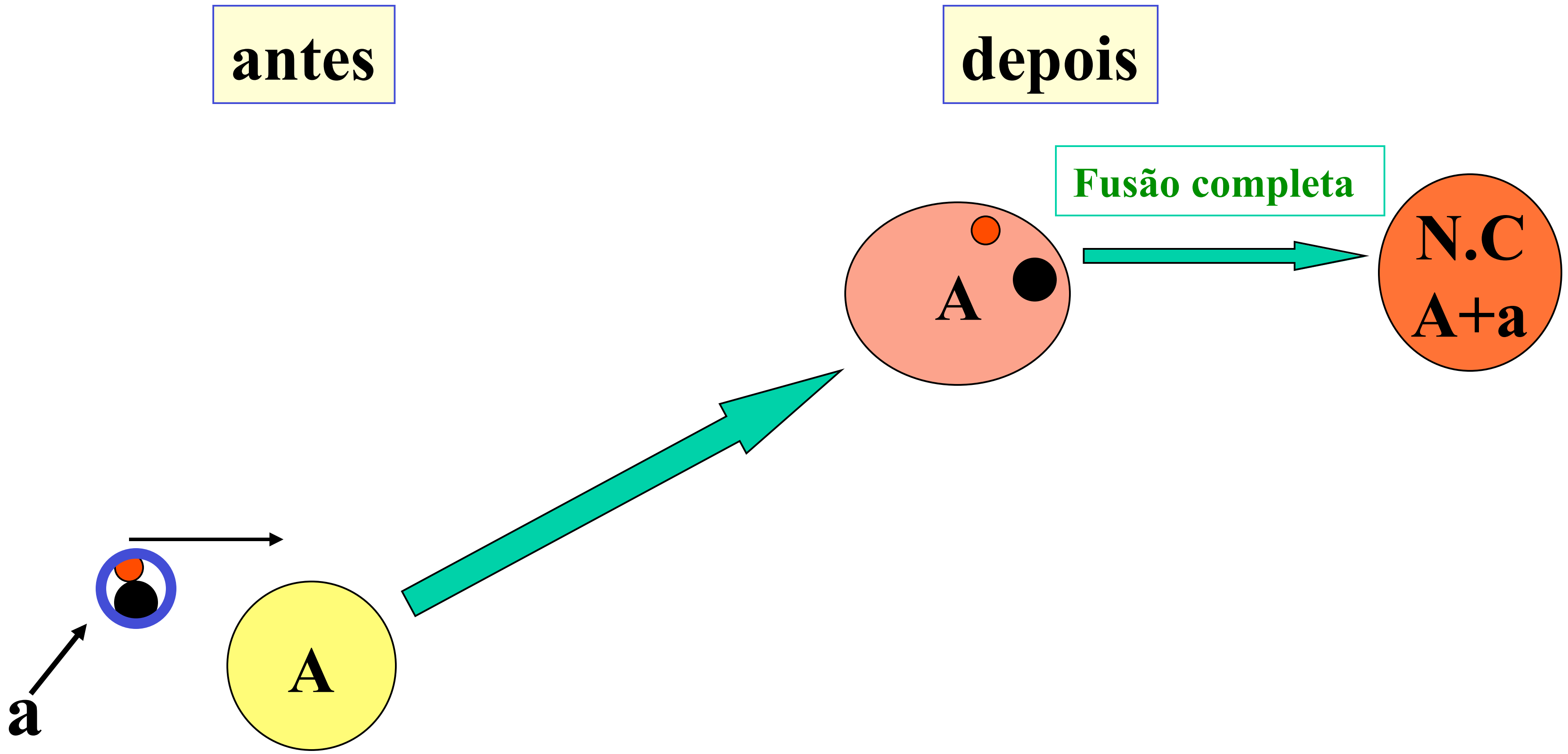
Reações diretas (rapidas)



Reações diretas (rapidas)

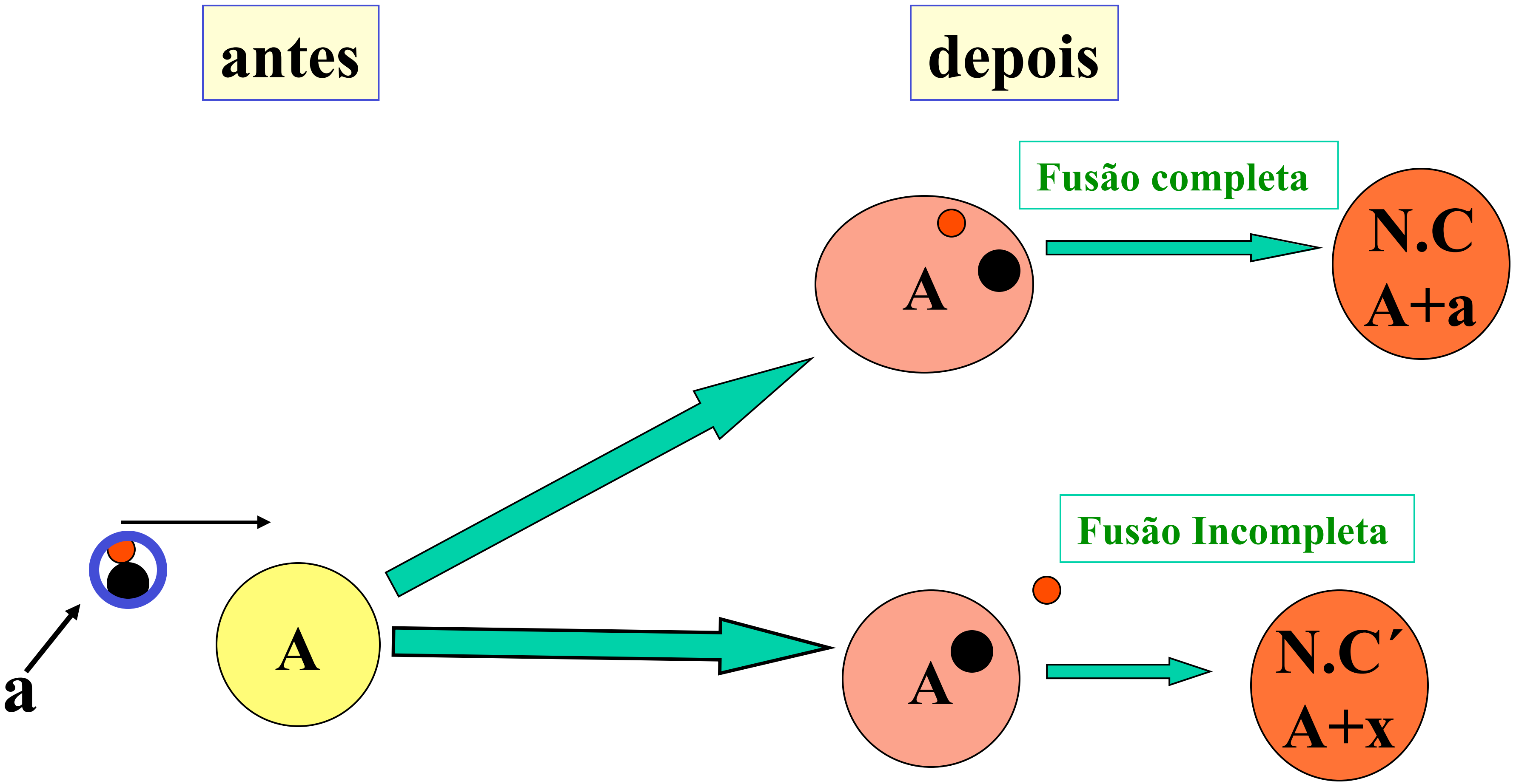


Reações diretas (rapidas)

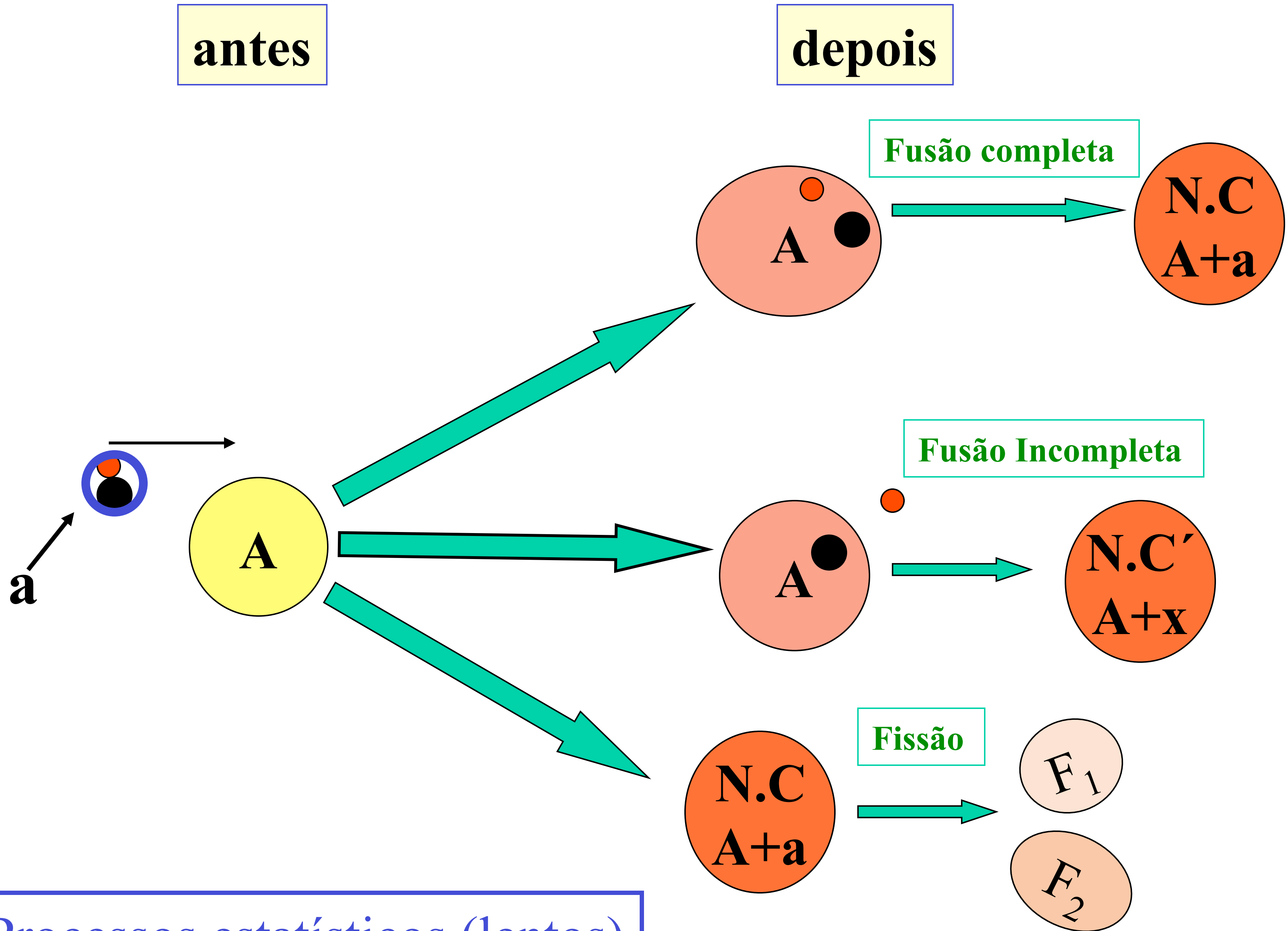


Processos estatísticos (lentos)

via **Núcleo Composto (N.C.)**



Processos estatísticos (lentos)



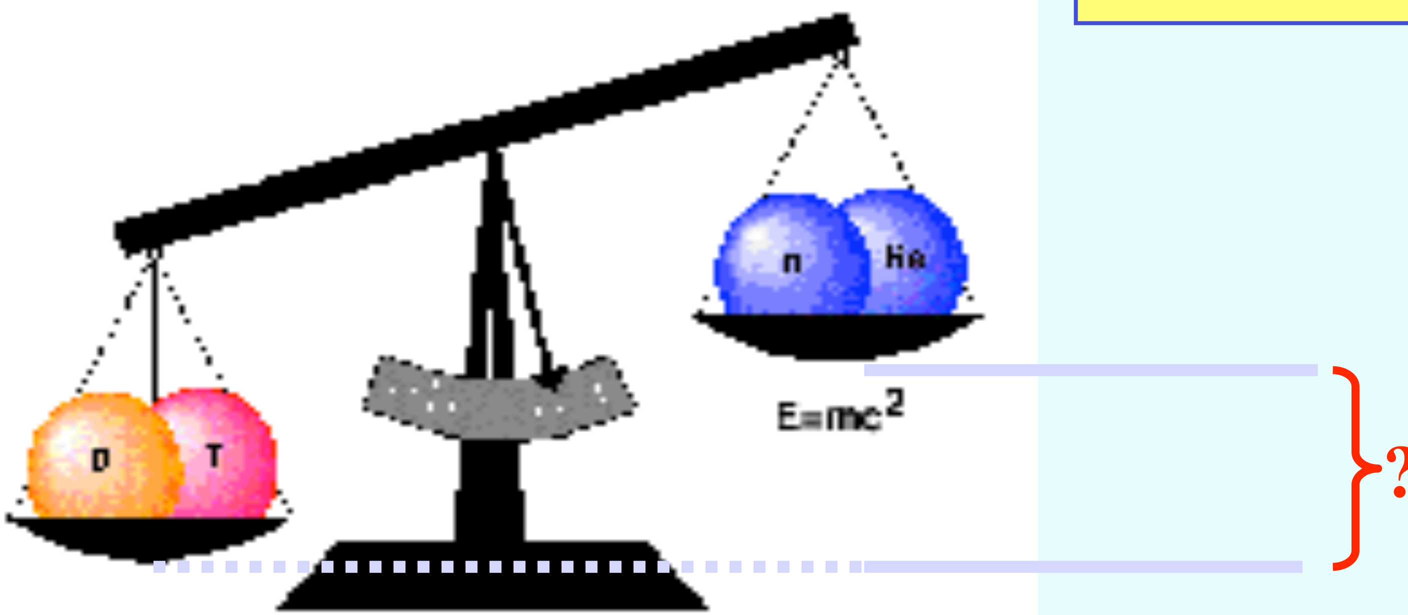
Processos estatísticos (lentos)

massa dos nucleos

d = p+n

t = p+n+n

4He = p+p+n+n



n =1.00866 u.m.a.

p = 1.0079 u.m.a.

d = 2.01410 u.m.a.

t = 3.01860 u.m.a.

^{4}He = 4. 00260 u.m.a.

^{6}Li = 6. 01512 u.m.a.

^{12}C = 0.00000 u.m.a.

$(m_{12C}) = 12.0000$ u → $u = m_u = (m_{12C})/12$

m_e	0,511 MeV
m_n	939,566 MeV
m_p	938,272 MeV
m_d	1875,613 MeV
$m\ (^3He)$	2808,350 MeV
m_α	3727,323 MeV
u	931,494 MeV

$1u = 1.66056 \times 10^{-24}$ g

$\Rightarrow m_u c^2 = 931.50\ MeV/c^2$

$Z+N = A$

$m(Z,N) = Zm_H + Nm_n - B(Z,N)/c^2$

$B(Z,N) = [\ Zm_H + Nm_n - m(Z,N)]\ c^2$

$B(Z,N) = [\ \Delta m\ ]\ c^2$

Excesso de massa (mass excess)
Δ (MeV)

Δ_A (MeV) = (m_A – A)u . c^2

Δ (^{12}C) = 0

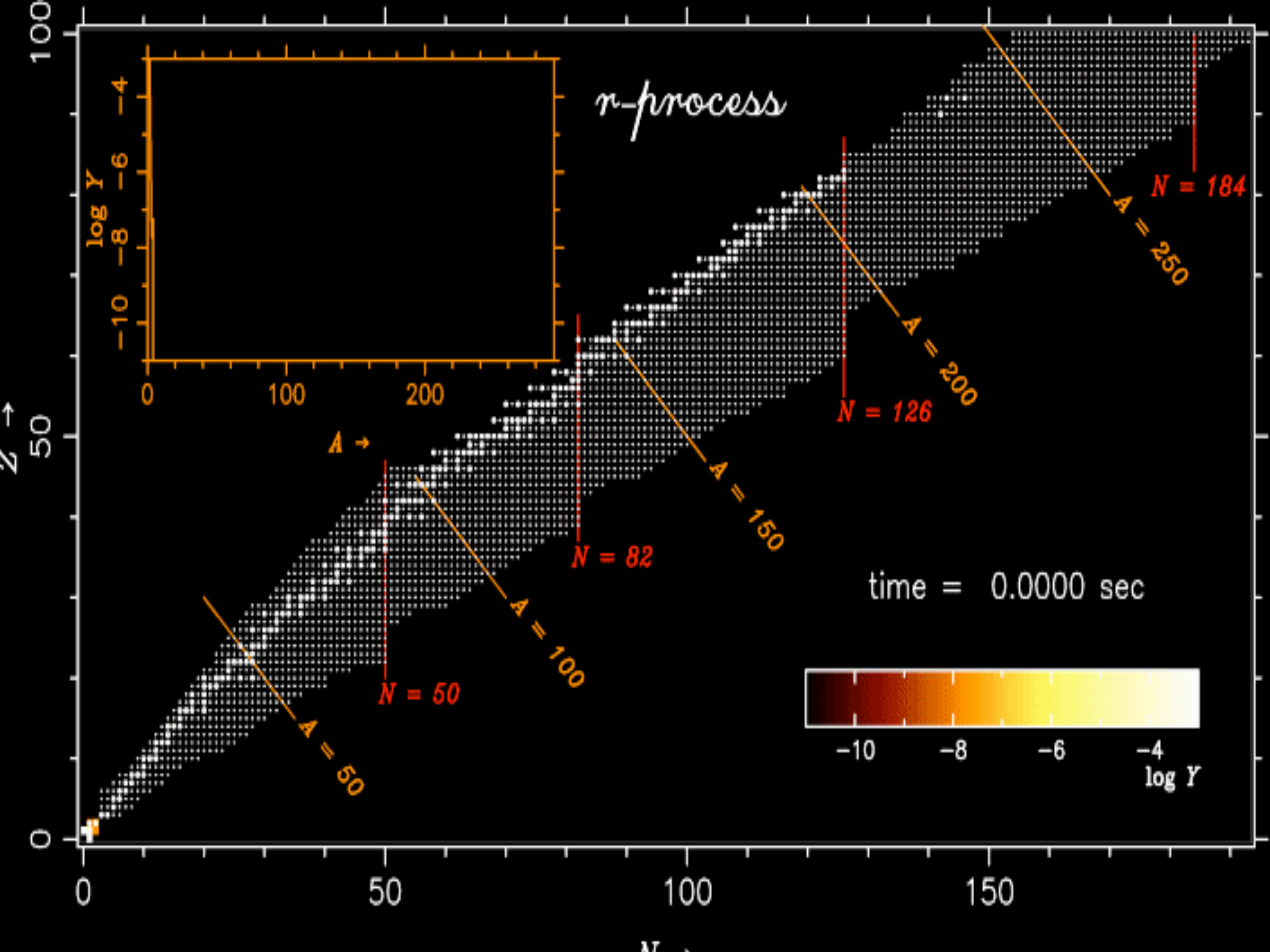
dada a reação: A(B,C)D
A (Z_A,N_A) + B (Z_B,N_B) $\rightarrow$ C (Z_C,N_C) + D (Z_D,N_D)

Definimos o valor de "Q" de uma reação
(M_A+ M_B) = (M_C+ M_D) + Q(A+B $\longrightarrow$C+D)

$Q_{(A+B \rightarrow C+D)} = (B_C + B_D) - (B_A + B_B)$

$Q_{(A+B \rightarrow C+D)} = (\Delta_C + \Delta_D) - (\Delta_A + \Delta_B)$

N	Z	A	EL	O	MASS EXCESS (KEV)		B/A	BINDING ENERGY (KEV)		ATOMIC MASS (U)	
1	0	1	N		8071.69	0.10		0.0	0.0	1.00866522	0.00000006
0	1		H		7289.22	0.09		0.0	0.0	1.00782522	0.00000004
1	1	2	H		13136.27	0.1	1.11	2224.64	0.04	2.01410222	0.00000007
2	1	3	H		14950.38	0.22	2.42	8482.22	0.15	3.01604972	0.00000016
1	2		HE		14931.73	0.22	2.57	7718.40	0.14	3.01602970	0.00000016
3	1	4	H	-N	25920	500		5580	500	4.02783	0.00054
2	2		HE		2424.94	0.2	7.07	28296.9	0.4	4.00260326	0.00000027
1	3		LI	+NN	25130	300		4810	300	4.02697	0.00032
4	1	5	H	+	33790	800		5790	800	5.03627	0.00086
3	2		HE	-N	11390	50		27410	50	5.01222	0.00005
2	3		LI	-P	11680	50		26330	50	5.01254	0.00005
4	2	6	HE		17597.3	3.6		29267.9	3.6	6.0188913	0.0000039
3	3		LI		14087.5	0.7	5.33	31995.2	0.8	6.0151234	0.0000008
2	4		BE	-	18375	5		26926	5	6.019726	0.000006
5	2	7	HE	+	26111	30		28826	30	7.028031	0.000032
4	3		LI		14908.6	0.8	5.60	39245.9	0.9	7.0160048	0.0000008
3	4		BE		15770.3	0.8		37601.6	0.9	7.0169299	0.0000008
2	5		B	-	27940	100		24650	100	7.02999	0.00011
6	2	8	HE	+	31650	120		31360	120	8.03397	0.00013
5	3		LI	-N	20947.5	1.0		41278.6	1.2	8.0224879	0.0000011
4	4		BE		4941.8	0.5	7.06	56501.9	0.8	8.0053052	0.0000005
3	5		B	-PP	22922.3	1.2		37738.8	1.3	8.0246079	0.0000013
6	3	9	LI	+	24966	5		45331	5	9.026802	0.000005
5	4		BE		11348.4	0.6	6.46	58167.0	0.9	9.0121828	0.0000006
4	5		B	-	12415.7	0.9		56317.1	1.1	9.0133287	0.0000010
3	6		C		28912	5		39038	5	9.031038	0.000006
7	3	10	LI	-N	35340	SYST		43030	SYST	10.03794	SYST
6	4		BE		12608.1	0.7		64978.9	1.0	10.0135352	0.0000008
5	5		B		12052.3	0.4	6.47	64752.3	0.9	10.0129385	0.0000004
4	6		C		15702.7	1.8		60319.4	2.0	10.0168573	0.0000020
8	3	11	LI	-N	43310	SYST		43130	SYST	11.04649	SYST
7	4		BE		20177	6		65482	6	11.021660	0.000007
6	5		B		8667.95	0.2	6.93	76208.3	1.0	11.00930533	0.00000030
5	6		C	-	10650.2	1.1		73443.6	1.4	11.0114333	0.0000011
4	7		N	-	25450	SYST		57860	SYST	11.02732	SYST
8	4	12	BE	-N	24950	SYST		68780	SYST	12.02678	SYST
[illegible]	[illegible]		B		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]
6	6		C		0.0	0.0	7.68	92165.5	1.1	12.00000000	0.0
5	7		N		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]

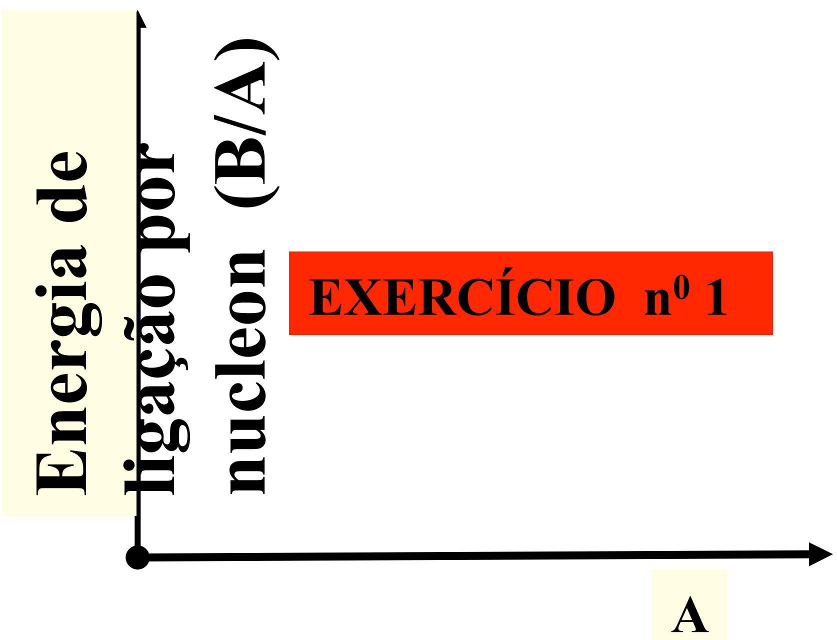


r-process
log Y
−4
−6
−8
−10
0
100
200
A →
N = 50
N = 82
N = 126
N = 184
A = 50
A = 100
A = 150
A = 200
A = 250
time = 0.0000 sec
−10
−8
−6
−4
log Y
Z →
0
50
100
0
50
100
150

http://ie.lbl.gov/toimass.html

http://ie.lbl.gov/mass/

http://www.phy.ornl.gov/hribf/calculators/mass-diff.shtml



EXERCÍCIO n^0 1

ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT

Nuclear Physics A 729 (2003) 337–676

NUCLEAR PHYSICS A

www.elsevier.com/locate/npe

The AME2003 atomic mass evaluation *

(II). Tables, graphs and references

G. Audi[a,§], A.H. Wapstra[b] and C. Thibault[a]

[a] *Centre de Spectrometrie Nucleaire et de Spectrometrie de Masse, CSNSM, IN2P3-CNRS&UPS, Batiment 108, F-91405 Orsay Campus, France*

[b] *National Institute of Nuclear Physics and High-Energy Physics, NIKHEF, PO Box 41882, 1009DB Amsterdam, The Netherlands*

Abstract

This paper is the second part of the new evaluation of atomic masses AME2003. From the results of a least-squares calculation described in Part I for all accepted experimental data, we derive here tables and graphs to replace those of 1993. The first table lists atomic masses. It is followed by a table of the influences of data on primary nuclides, a table of separation energies and reaction energies, and finally, a series of graphs of separation and decay energies. The last section in this paper lists all references to the input data used in Part I of this AME2003 and also to the data entering the NUBASE2003 evaluation (first paper in this volume).

AMDC: http://csnwww.in2p3.fr/AMDC/

1. Introduction

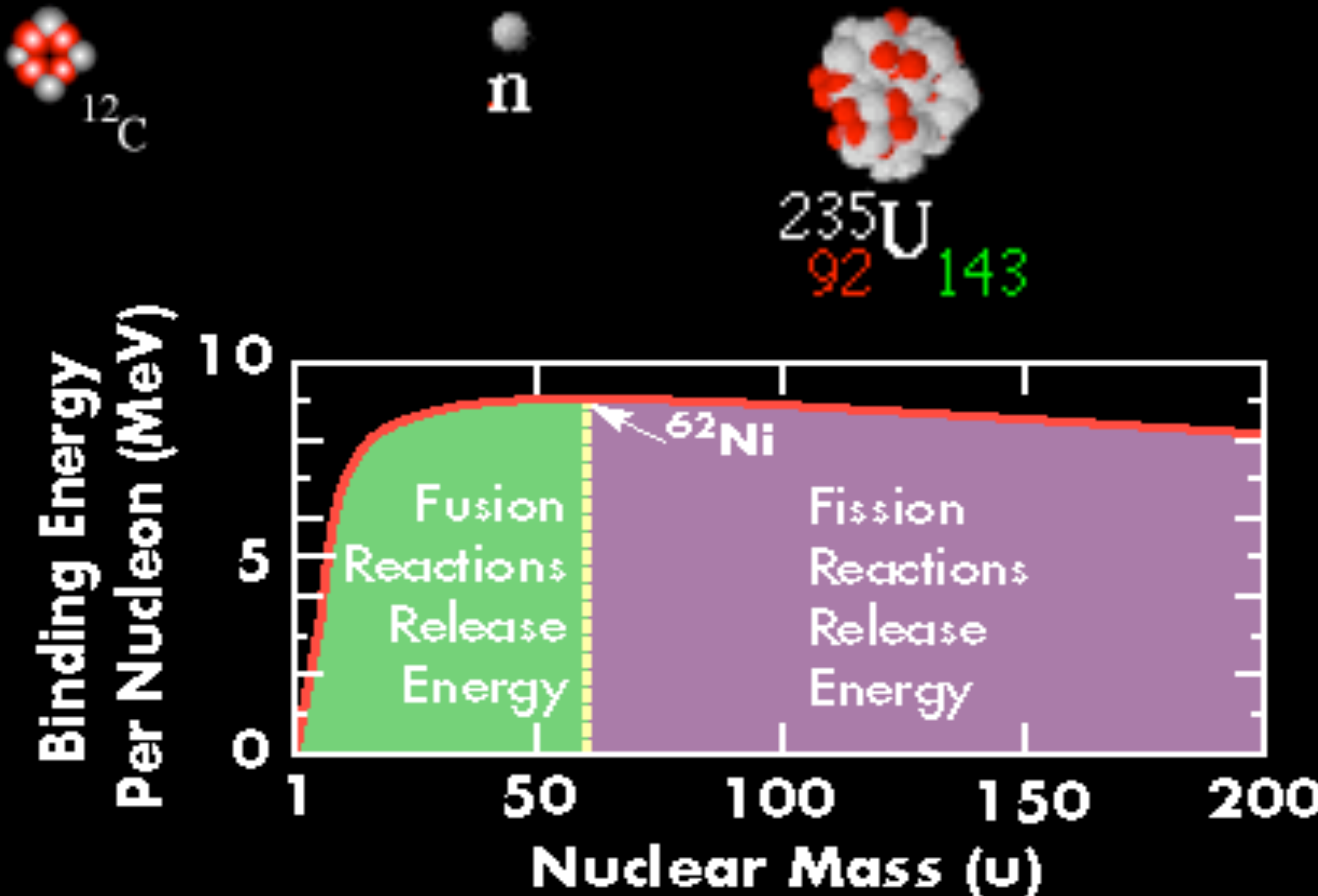
The description of the general procedures and policies are given in Part I of this series of two papers, where the input data used in the evaluation are presented. In this paper we give tables and graphs derived from the evaluation of the input data in Part I.

Firstly, we present the table of atomic masses (Table I) expressed as mass excesses in energy units, together with the binding energy per nucleon, the beta-decay energy and the full atomic mass in mass units.

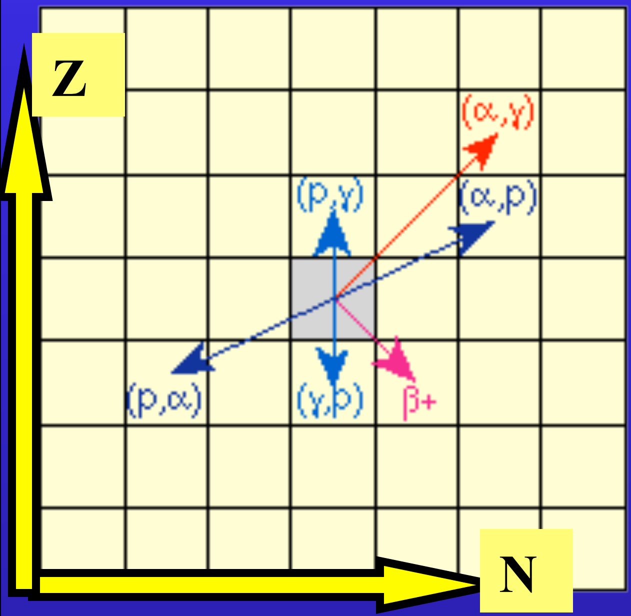
* This work has been undertaken with the encouragement of the IUPAP Commission on Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants (SUN-AMCO).

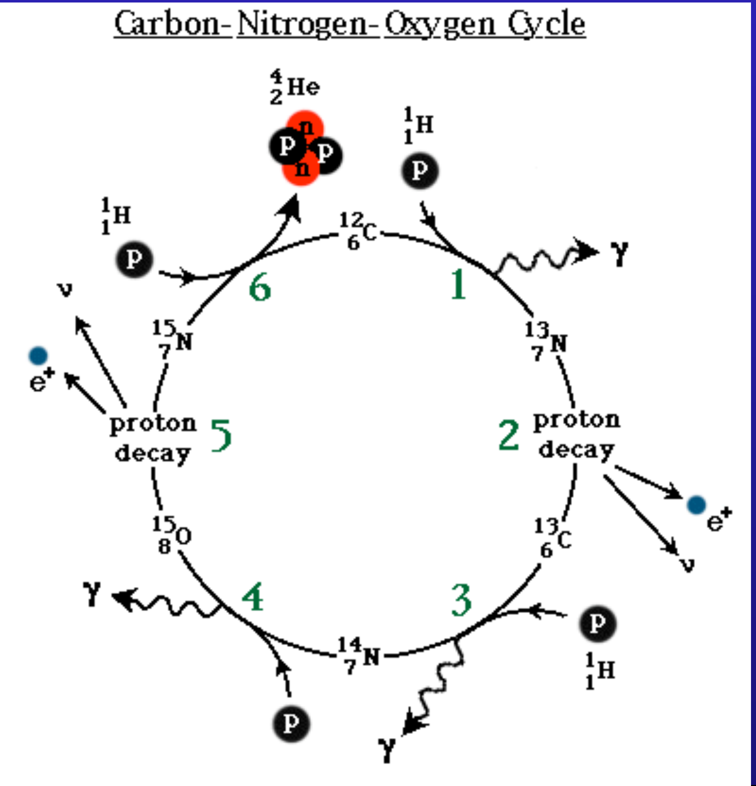
§ Corresponding author. *E-mail address:* audi@csnsm.in2p3.fr (G. Audi).

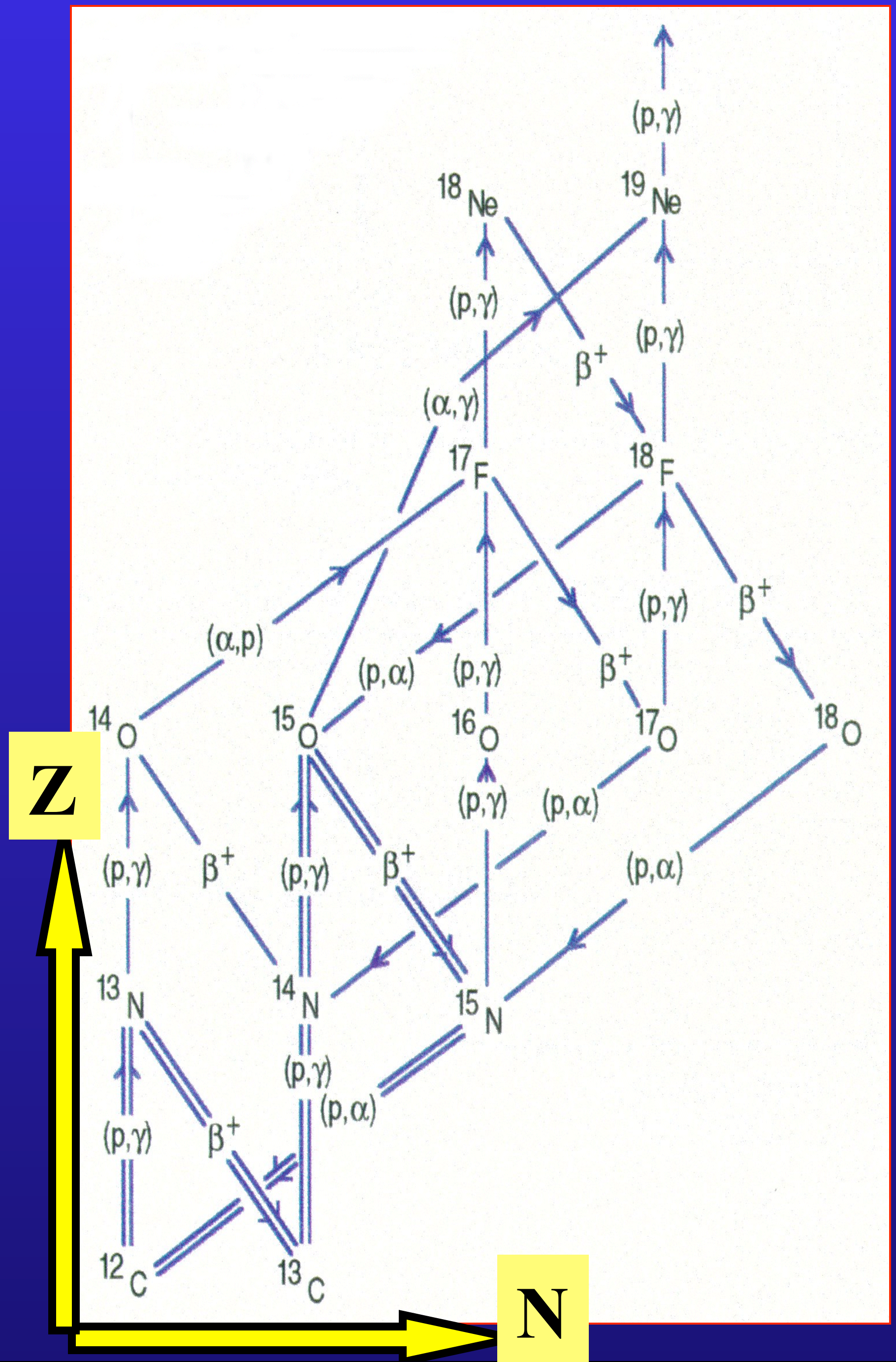
doi:10.1016/j.nuclphysa.2003.11.003

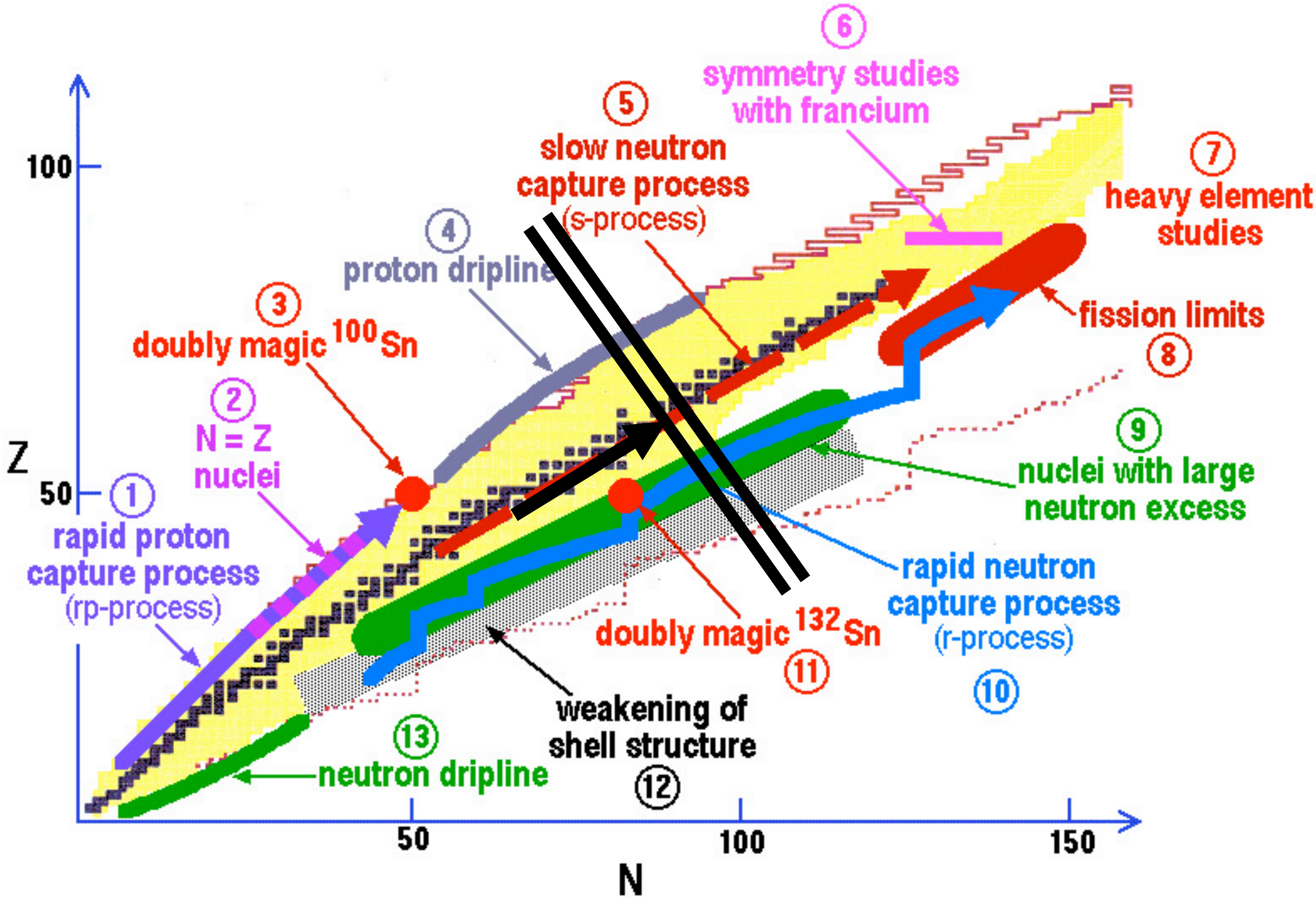


12C
n
235U
92
143
Binding Energy
Per Nucleon (MeV)
10
5
0
62Ni
Fusion
Reactions
Release
Energy
Fission
Reactions
Release
Energy
1
50
100
150
200
Nuclear Mass (u)

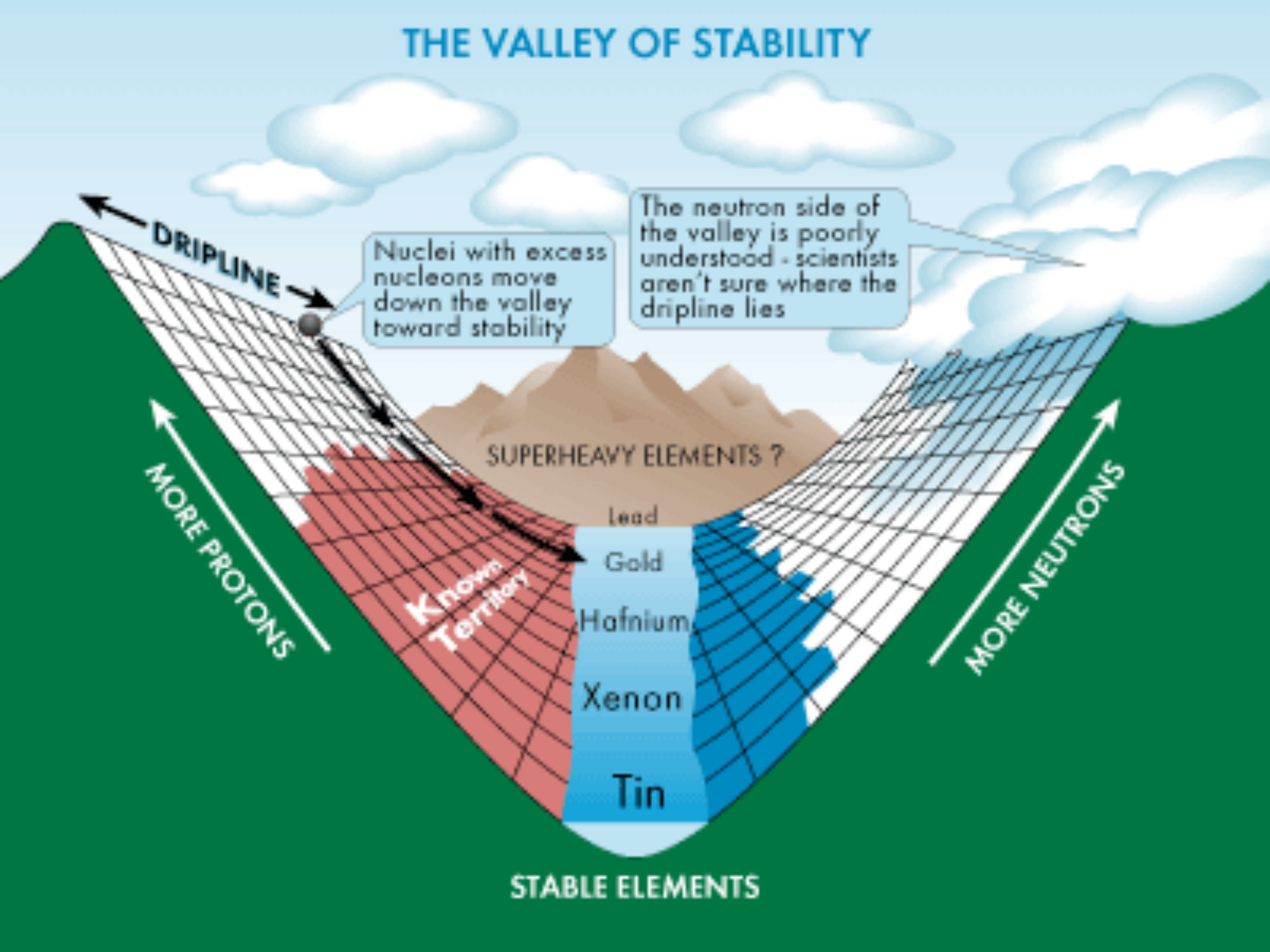

Z
N
(α,γ)
(p,γ)
(α,p)
(p,α)
(γ,p)
β+

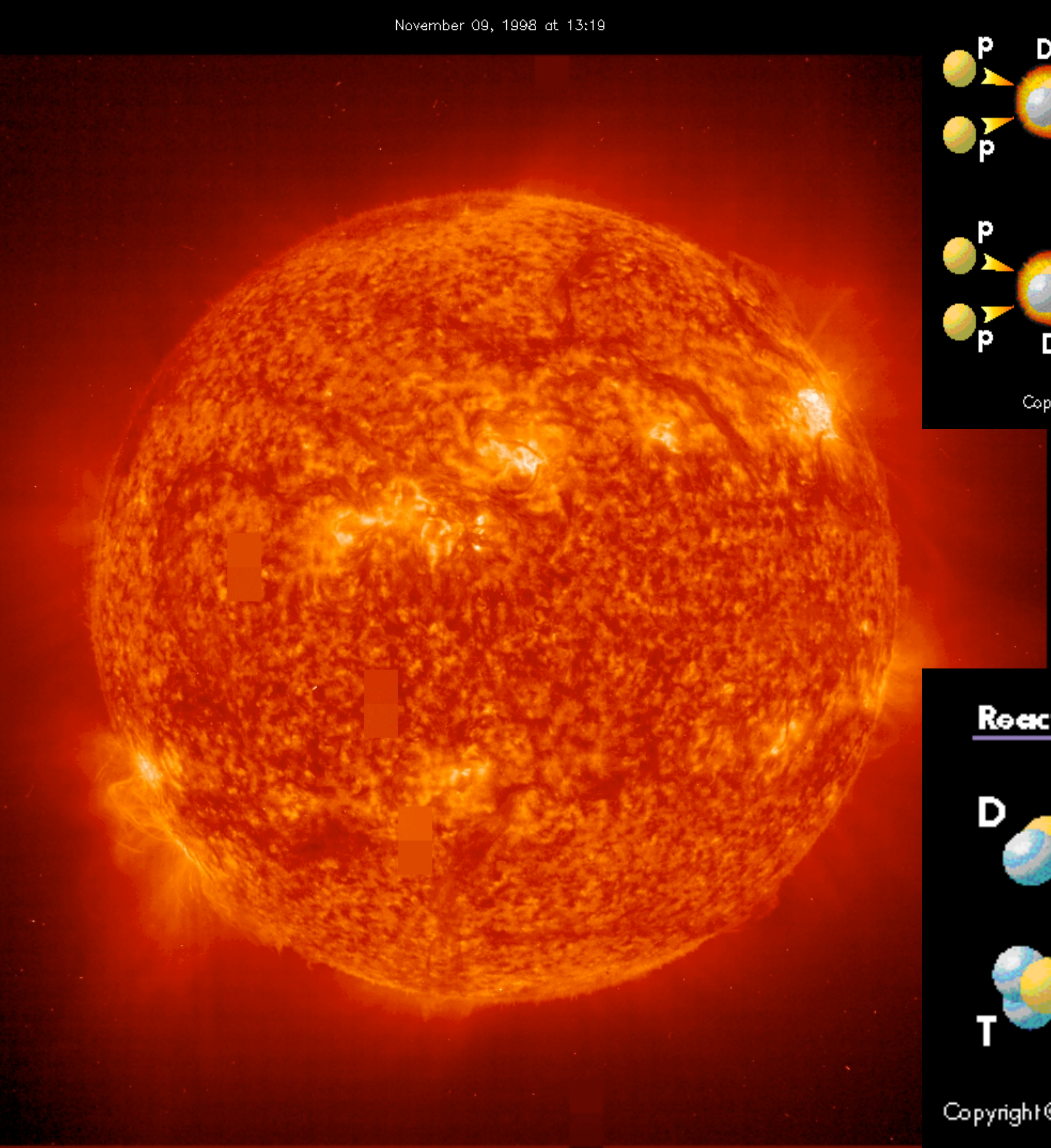

Carbon-Nitrogen-Oxygen Cycle
proton decay
proton decay
γ
ν
e+

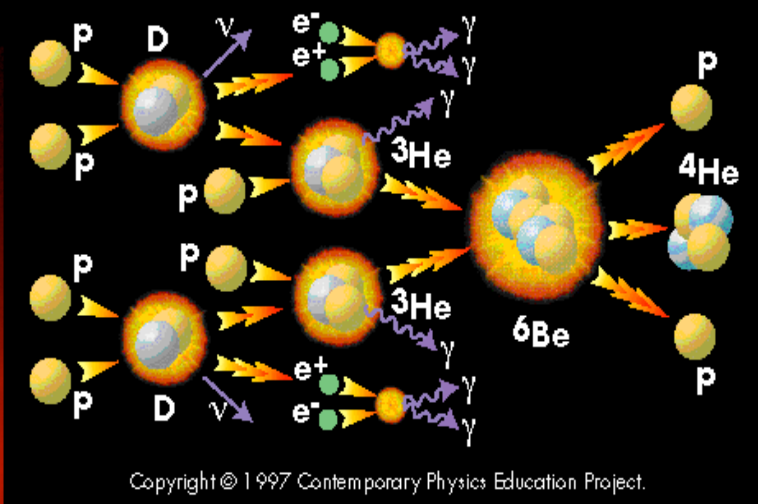

Z
N
(p,γ)
(α,γ)
(α,p)
(p,α)
β+

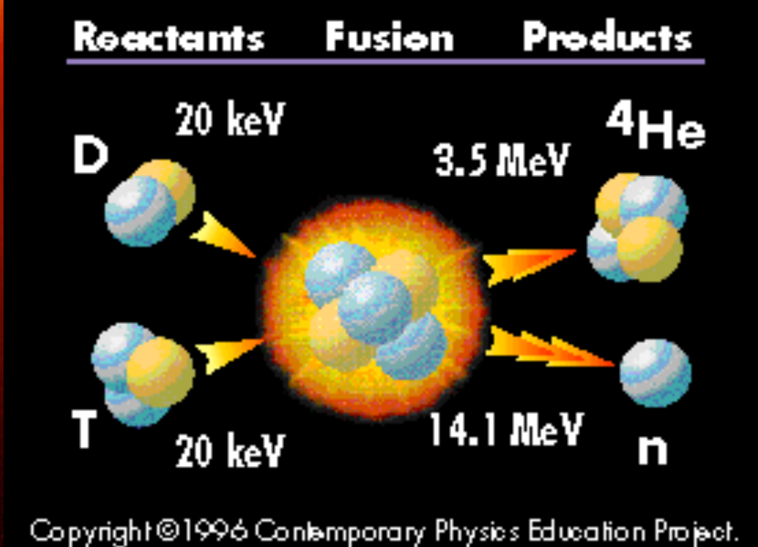


100
50
Z
N
50
100
150
1
rapid proton
capture process
(rp-process)
2
N = Z
nuclei
3
doubly magic 100Sn
4
proton dripline
5
slow neutron
capture process
(s-process)
6
symmetry studies
with francium
7
heavy element
studies
fission limits
8
9
nuclei with large
neutron excess
rapid neutron
capture process
(r-process)
10
doubly magic 132Sn
11
weakening of
shell structure
12
13
neutron dripline


THE VALLEY OF STABILITY
DRIPLINE
Nuclei with excess nucleons move down the valley toward stability
The neutron side of the valley is poorly understood - scientists aren't sure where the dripline lies
SUPERHEAVY ELEMENTS ?
Lead
Gold
Hafnium
Xenon
Tin
Known Territory
MORE PROTONS
MORE NEUTRONS
STABLE ELEMENTS

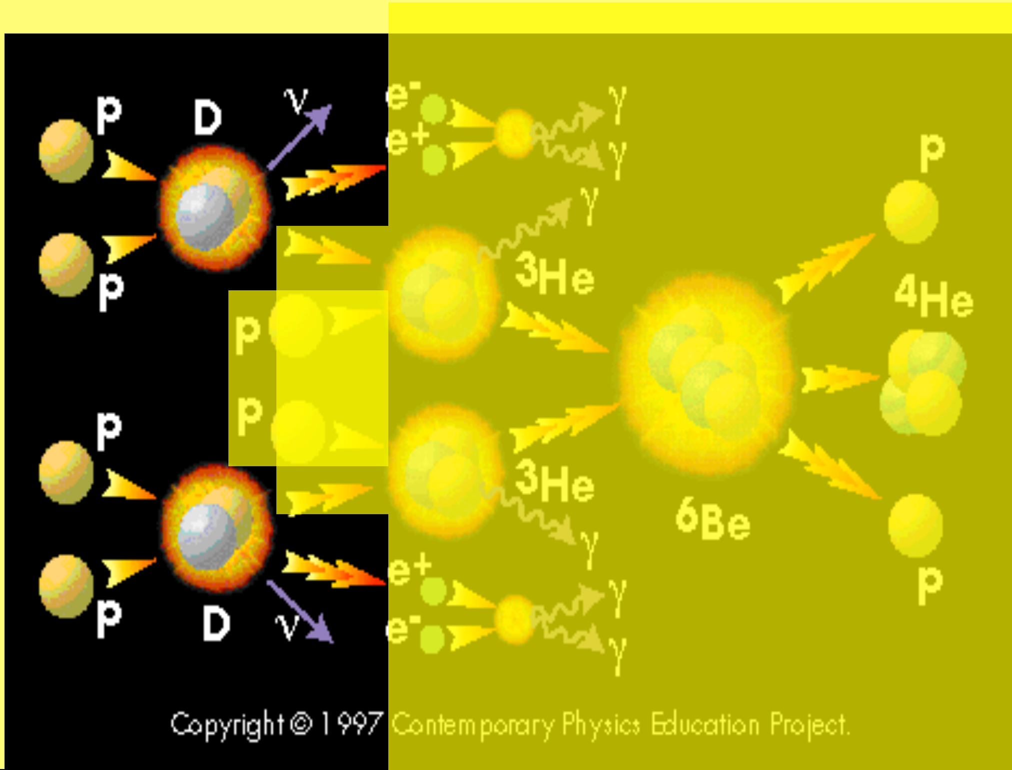

November 09, 1998 at 13:19


p
p
D
ν
e-
e+
γ
γ
γ
p
3He
p
4He
p
p
p
3He
6Be
γ
e+
e-
γ
γ
D
ν
p
p
Copyright © 1997 Contemporary Physics Education Project.


Reactants
Fusion
Products
20 keV
D
3.5 MeV
4He
T
20 keV
14.1 MeV
n
Copyright ©1996 Contemporary Physics Education Project.

$p + p \rightarrow d + e^{+} + \nu$

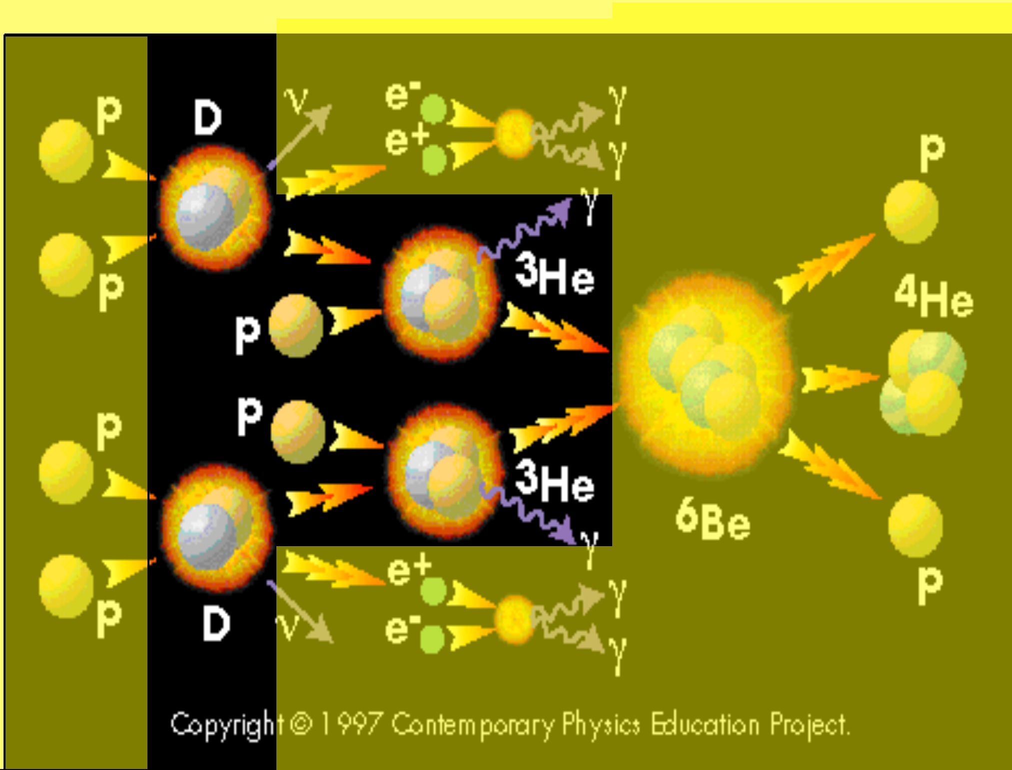
n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He =	14.931
^{4}He =	2.425
^{6}Be =	18.375



$$p + p \rightarrow d + e^{+} + \nu$$

$$p + d \rightarrow {}^{3}He + \gamma$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He = 14.931
^{4}He = 2.425
^{6}Be = 18.375

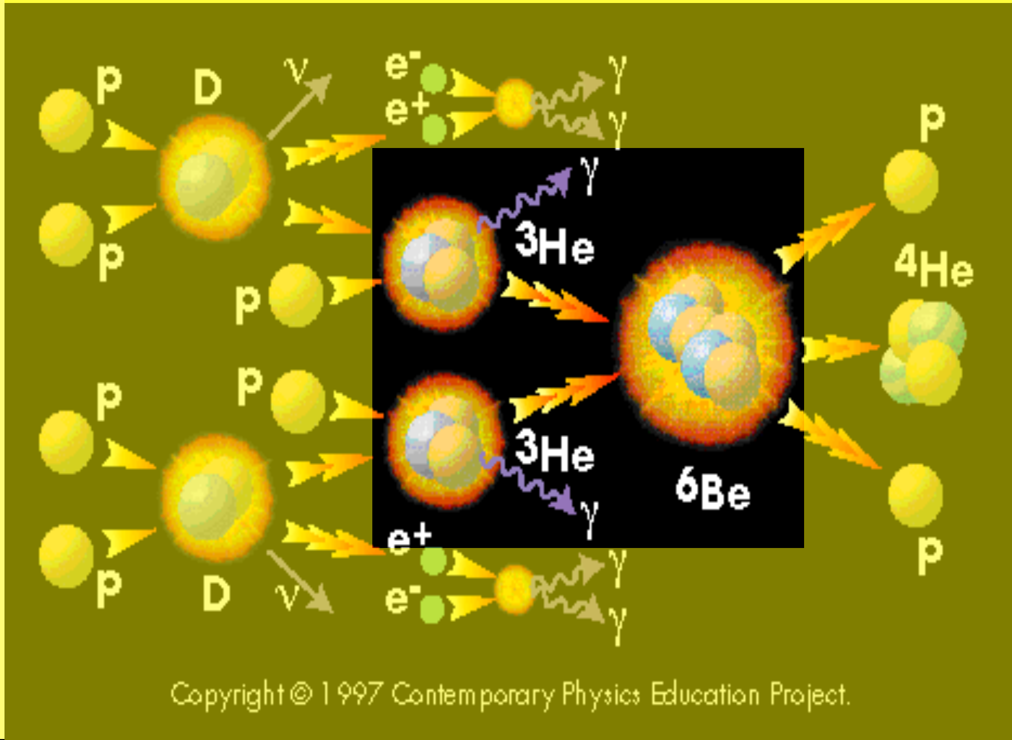


$$p + p \rightarrow d + e^{+} + \nu$$

$$p + d \rightarrow {}^{3}He + \gamma$$

$${}^{3}He + {}^{3}He \rightarrow {}^{6}Be + \gamma$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He= 14.931
^{4}He= 2.425
^{6}Be= 18.375



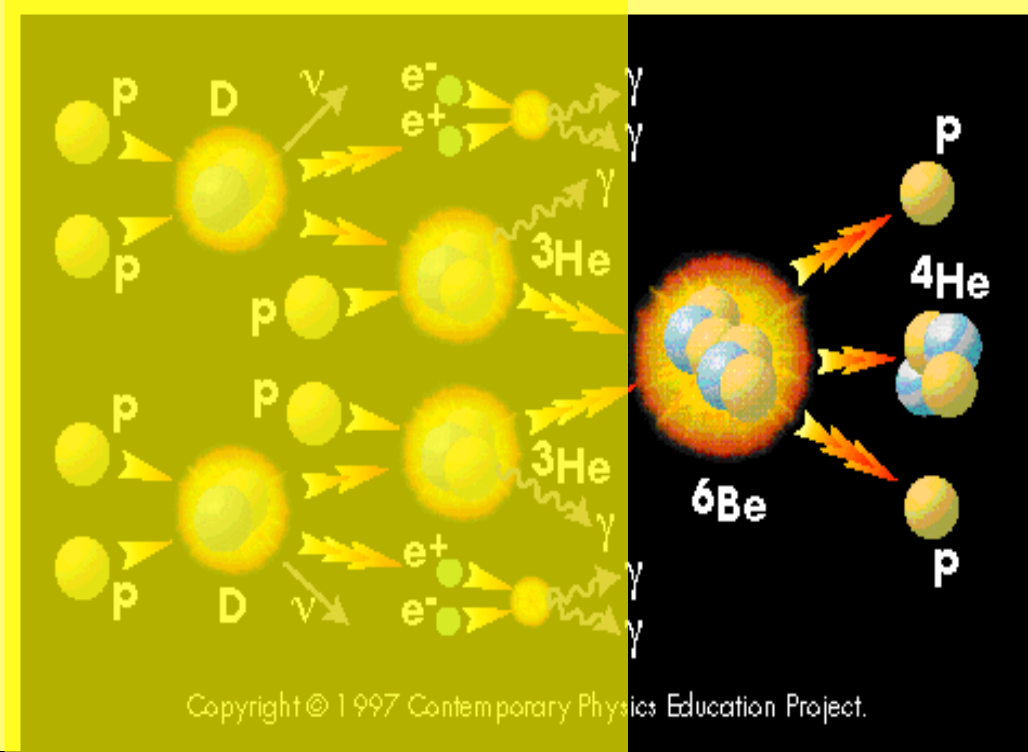
$$p + p \rightarrow d + e^{+} + \nu$$

$$p + d \rightarrow {}^{3}He + \gamma$$

$$^{3}He + {}^{3}He \rightarrow {}^{6}Be + \gamma$$

$$^{6}Be \rightarrow {}^{4}He + p + p$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He= 14.931
^{4}He= 2.425
^{6}Be= 18.375



$$p + p \rightarrow d + e^+ + \nu$$

$$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931\ \text{MeV}$$

$$p + d \rightarrow {}^3He + \gamma$$

$$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494\ \text{MeV}$$

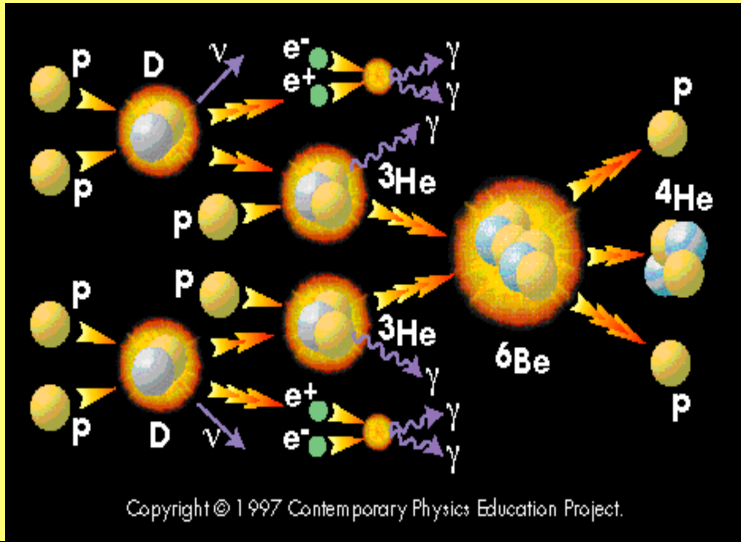
$$^3He + {}^3He \rightarrow {}^6Be + \gamma$$

$$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487\ \text{MeV}$$

$$^6Be \rightarrow {}^4He + p + p$$

$$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372\ \text{MeV}$$

n =	8.071
p =	7.289
d =	13.136
t =	14.950
^3He =	14.931
^4He =	2.425
^6Be =	18.375



$p + p \rightarrow d + e^+ + \nu$

$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931$ MeV

n =	8.071
p =	7.289
d =	13.136
t =	14.950
^3He =	14.931
^4He =	2.425
^6Be =	18.375

$p + d \rightarrow {}^3He + \gamma$

$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494$ MeV

${}^3He + {}^3He \rightarrow {}^6Be + \gamma$

$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487$ MeV

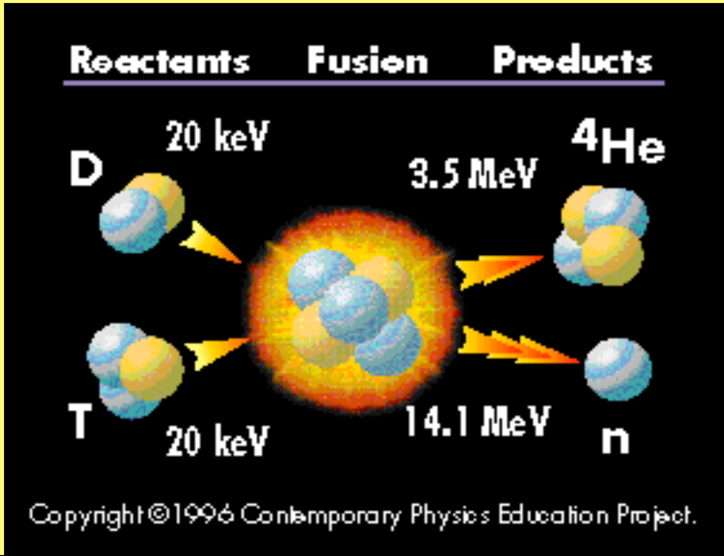
${}^6Be \rightarrow {}^4He + p + p$

$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372$ MeV

$d + t \rightarrow {}^4He + n$

$13.136 + 14.950 \rightarrow 2.425 + 8.071 + Q$

$Q = 17.59$ MeV



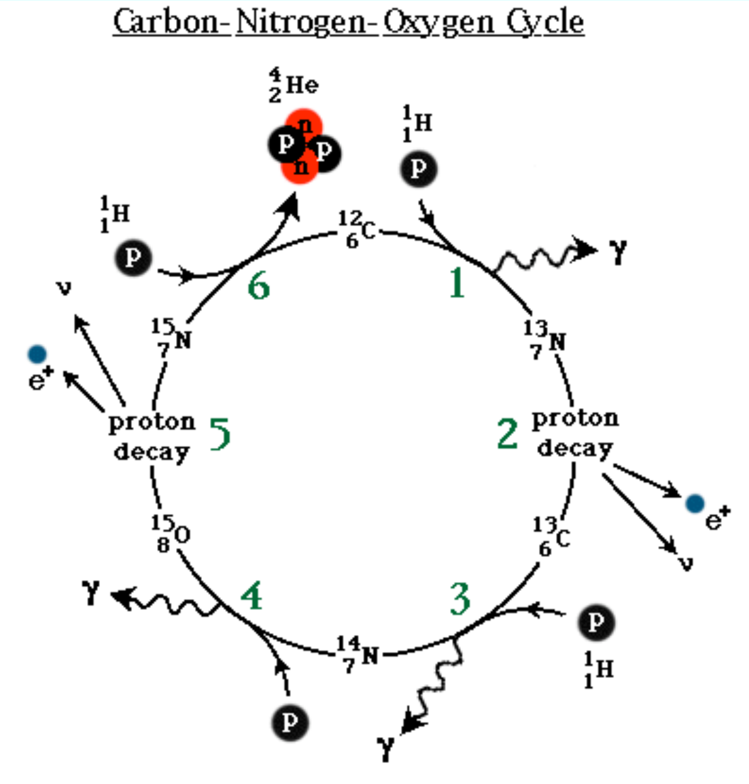
EXERCÍCIOS

CALCULAR O BALANÇO ENERGÉTICO NAS SEGUINTES REAÇÕES

$\Delta = (M-A)c^2$ (MeV)

Exemplo== $^{12}C + p \rightarrow {}^{13}N + \gamma$

n = 8.071	^{12}C = 0.00
p = 7.289	^{13}C = 3.125
d = 13.136	^{13}N = 5.345
t = 14.950	^{14}N = 2.863
^{3}He = 14.931	^{15}N = 0.011
^{4}He = 2.425	^{15}O = 2.855
^{6}Li = 14.086	^{16}O = -4.737
^{7}Li = 14.908	^{17}O = -0.809
^{6}Be = 18.375	^{18}O = -0.782



$^{12}C + p \rightarrow {}^{13}N + \gamma$

$0 + 7.289 \rightarrow 5.345 + Q \Rightarrow Q = 1.944$ MeV

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$

$5.345 \rightarrow 3.125 + 0.511 + Q \Rightarrow Q = 1.709$ MeV

$^{13}C + p \rightarrow {}^{14}N + \gamma$

$3.125 + 7.289 \rightarrow 2.863 + Q \Rightarrow Q = 7.551$ MeV

$^{14}N + p \rightarrow {}^{15}O + \gamma$

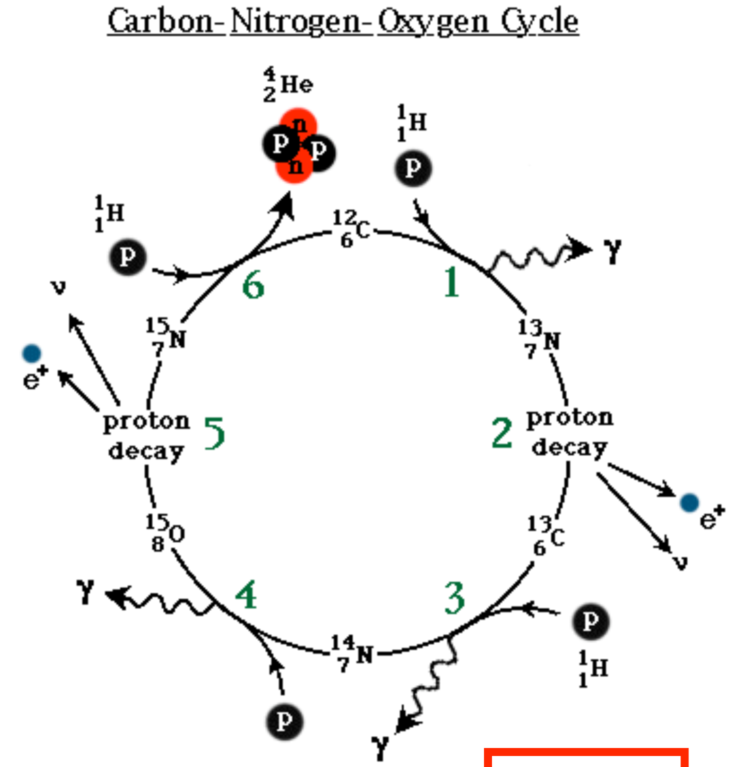
$2.863 + 7.289 \rightarrow 2.855 + Q \Rightarrow Q = 7.297$ MeV

$^{15}O \rightarrow {}^{15}N + e^+ + \nu$

$2.855 \rightarrow 0.101 + 0.511 + Q \Rightarrow Q = 2.243$ MeV

$^{15}N + p \rightarrow {}^{12}C + \alpha + \gamma$

$0.101 + 7.289 \rightarrow 0 + 2.425 + Q \Rightarrow Q = 4.965$ MeV



1.944
+1.709
+7.551
+7.297
+2.243
+4.965

25.709

$^{12}C + p + p + p + p \rightarrow {}^{12}C + {}^{4}He + 2e^+ + 2\nu$

$0 + 4x\,(7.289) \rightarrow 0 + 2.425 + 1.022 + Q$

Q = 25.709 MeV

BLOCO 2

GRANDEZAS MACROSCÓPICAS DO NUCLEO

ESPALHAMENTO RUTHERFORD
- PARÂMETRO DE IMPACTO
- ÂNGULO DE DEFLEXÃO
- BARREIRA COULOMBIANA

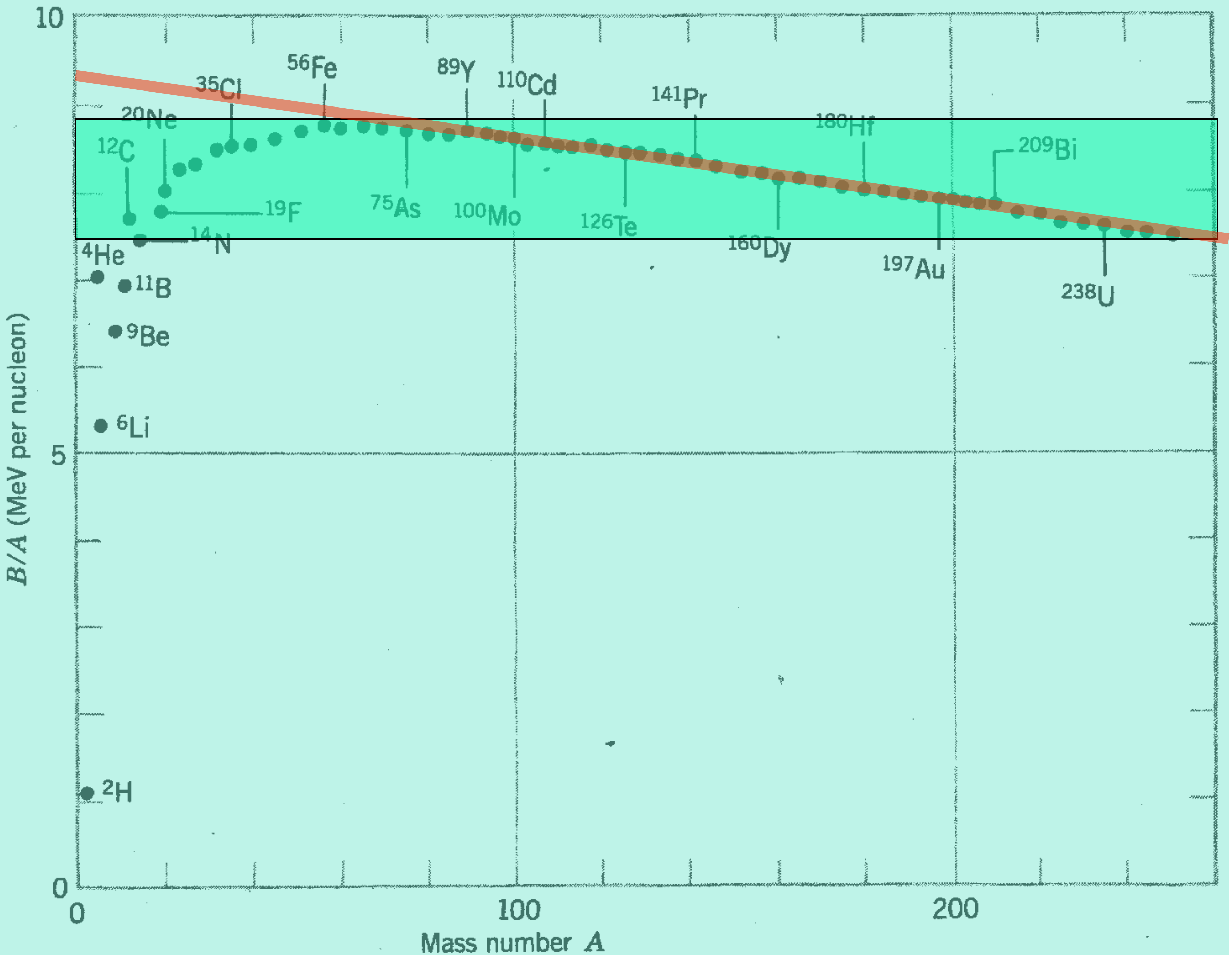
RAIO NUCLEAR
- ESPALHAMENTO DE ELETRONS
- DIFUSIVIDADE

MASSA NUCLAR:
- ENERGIA DE LIGAÇÃO
- EXCESSO DE MASSA
- ENERGIA DE SEPARAÇÃO
- VALOR "Q" de REAÇÃO
- MODELO DA GOTA LÍQUIDA

CINEMÁTICA DE REAÇÃO - CINEMÁTICA INVERSA

massa dos nucleos

N	Z	A	EL	O	MASS EXCESS (KEV)		B/A	BINDING ENERGY (KEV)		BETA-DECAY ENERGY (KEV)		ATOMIC MASS (U)	
1	0	1	N		8071.69	0.10		0.0	0.0	782.47	0.05	1.00866522	0.00000006
0	1		H		7289.22	0.09		0.0	0.0	*		1.00782522	0.00000004
1	1	2	H		13136.27	0.16	1.11	2224.64	0.04	*		2.01410222	0.00000007
2	1	3	H		14950.38	0.22	2.42	8482.22	0.15	18.65	0.04	3.01604972	0.00000016
1	2		HE		14931.73	0.22	2.57	7718.40	0.14	*		3.01602970	0.00000016
3	1	4	H	-N	25920	500		5580	500	23500	500	4.02783	0.00054
2	2		HE		2424.94	0.25	7.07	28296.9	0.4	-22700	300	4.00260326	0.0000027
1	3		LI	+NN	25130	300		4810	300	*		4.02697	0.00032
4	1	5	H	+	33790	800		5790	800	22400	800	5.03627	0.00086
3	2		HE	-N	11390	50		27410	50	-290	70	5.01222	0.00005
2	3		LI	-P	11680	50		26330	50	*		5.01254	0.00005
4	2	6	HE		17597.3	3.6		29267.9	3.6	3509.8	3.6	6.0188913	0.0000039
3	3		LI		14087.5	0.7	5.33	31995.2	0.8	-4287	5	6.0151234	0.0000008
2	4		BE	-	18375	5		26926	5	*		6.019726	0.000006
5	2	7	HE	+	26111	30		28826	30	11203	30	7.028031	0.000032
4	3		LI		14908.6	0.8	5.60	39245.9	0.9	-861.75	0.09	7.0160048	0.0000008
3	4		BE		15770.3	0.8		37601.6	0.9	-12170	100	7.0169299	0.0000008
2	5		B	-	27940	100		24650	100	*		7.02999	0.00011
6	2	8	HE	+	31650	120		31360	120	10700	120	8.03397	0.00013
5	3		LI	-N	20947.5	1.0		41278.6	1.2	16005.8	1.1	8.0224879	0.0000011
4	4		BE		4941.8	0.5	7.06	56501.9	0.8	-17980.5	1.3	8.0053052	0.0000005
3	5		B	-PP	22922.3	1.2		37738.8	1.3	*		8.0246079	0.0000013
6	3	9	LI	+	24966	5		45331	5	13618	5	9.026802	0.000005
5	4		BE		11348.4	0.6	6.46	58167.0	0.9	-1067.3	0.7	9.0121828	0.0000006
4	5		B	-	12415.7	0.9		56317.1	1.1	-16497	5	9.0133287	0.0000010
3	6		C		28912	5		39038	5	*		9.031038	0.000006
7	3	10	LI	-N	35340	SYST		43030	SYST	22730	SYST	10.03794	SYST
6	4		BE		12608.1	0.7		64978.9	1.0	555.9	0.8	10.0135352	0.0000008
5	5		B		12052.3	0.4	6.47	64752.3	0.9	-3650.4	1.8	10.0129385	0.0000004
4	6		C		15702.7	1.8		60319.4	2.0	*		10.0168573	0.0000020
8	3	11	LI	-N	43310	SYST		43130	SYST	23130	SYST	11.04649	SYST
7	4		BE		20177	6		65482	6	11509	6	11.021660	0.000007
6	5		B		8667.95	0.29	6.93	76208.3	1.0	-1982.2	1.0	11.00930533	0.00000030
5	6		C	-	10650.2	1.1		73443.6	1.4	-14800	SYST	11.0114333	0.0000011
4	7		N	-	25450	SYST		57860	SYST	*		11.02732	SYST
8	4	12	BE	-N	24950	SYST		68780	SYST	11580	SYST	12.02678	SYST
[illegible]	[illegible]		[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	6		C		0.0	0.0	7.68	92165.5	1.1	-17344	5	12.00000000	0.0
5	7		N		[illegible]	[illegible]		[illegible]	[illegible]			[illegible]	[illegible]



10
5
0
B/A (MeV per nucleon)
0
100
200
Mass number A
56Fe
89Y
110Cd
35Cl
141Pr
20Ne
180Hf
12C
209Bi
19F
75As
100Mo
126Te
14N
160Dy
4He
197Au
11B
238U
9Be
6Li
2H

2.4 Modelo da gota líquida e limites de estabilidade

Na secção anterior ficaram claros os limites da analogia entre o comportamento da matéria nuclear e o de um líquido. Nesta secção, pretende-se desenvolver um modelo nuclear simples, em que apenas se faz uso de propriedades do núcleo análogas às de um líquido. Este *modelo da gota líquida* permite compreender o comportamento das energias de ligação e, por meio delas, as massas nucleares, não conseguindo contudo explicar outros tipos de propriedades.

No que se segue, considera-se o núcleo como uma gota de um líquido incompressível que se mantém coesa sob a acção de forças de alcance curto. A energia de ligação do núcleo, B, obtém-se pela soma de várias parcelas

$$B = B_1 + B_2 + B_3 + B_4 + B_5 \tag{2.47}$$

correspondentes a outras tantas contribuições que se discutem de seguida, nas alíneas 1) a 5), sendo a energia B expressa como função de Z e de A. Interessa apenas obter, para cada contribuição, a relação funcional com aquelas grandezas e determinar depois, empiricamente, os valores das constantes necessárias.

1) A principal contribuição para a energia de ligação é a "energia de condensação", libertada no momento em que os nucleões se reunem para formar o núcleo. Ela deve ser proporcional ao número de partículas ligadas, de acordo com o valor aproximadamente constante de B/A (Fig.10). Se a_V for a constante de proporcionalidade, tem-se, portanto,

$$B_1 = a_V A \tag{2.48}$$

Como A é proporcional ao volume do núcleo chama-se a este termo *energia de volume.*

2) Os nucleões que se encontram à superfície do núcleo têm menor número de ligações com os vizinhos do que os que estão no interior, ficando por isso menos ligados e contribuindo menos para uma energia de ligação. Introduz-se, portanto, um termo negativo, B_2, proporcional à superfície $4\pi R^2 = 4\pi r_o^2 A^{2/3}$ e, como importa apenas a dependência funcional em A, vem

$$B_2 = -a_S A^{\frac{2}{3}} \quad \text{(Energia de superfície)} \tag{2.49}$$

3) A energia de ligação é ainda mais reduzida devido à repulsão entre os protões. A energia de Coulomb duma esfera de raio R e carga q, carregada uniformemente, é $(3/5).(q^2/R)$. Para o núcleo de carga Ze e raio $R = r_o A^{1/3}$, a dependência funcional em Z e A conduz a um termo da forma

$$B_3 = -a_C Z^2 A^{-\frac{1}{3}} \quad \text{(Energia de Colomb)} \tag{2.50}$$

4) Ao considerar a dependência de B em A e Z, deve-se também considerar que o excesso de neutrões é acompanhado por uma diminuição da energia de ligação em relação à situação simétrica (N = Z). De acordo com (2.46), esta diferença de energia depende do excesso de neutrões, sendo o termo correspondente dado por

$$B_4 = -a_A \frac{T_z^2}{A} = -a_A \frac{(Z - A/2)^2}{A} \quad \text{(Energia de assimetria)} \tag{2.51}$$

5) Sabe-se, com base na sistemática das energias de separação, que os nucleões do mesmo tipo produzem uma ligação particularmente forte quando surgem aos pares. A *energia de emparelhamento* não pode ser explicada com base na analogia com a gota líquida, sendo necessário, neste contexto, introduzi-la como correcção empírica. Se tanto Z como N são números pares (núcleos par-par) esta energia é particularmente elevada, sendo pelo contrário particularmente baixa para núcleos em que Z e N são ímpares (núcleos ímpar-ímpar). Introduz-se pois da seguinte maneira a contribuição B_5:

$$B_5 = \begin{cases} +\delta & \text{núcleos par-par} \\ 0 & \text{núcleos par-ímpar ou ímpar-par} \\ -\delta & \text{núcleos impar-ímpar} \end{cases} \tag{2.52}$$

Uma fórmula empírica, válida em boa aproximação, é

$$\delta \approx a_p A^{-\frac{1}{2}} \tag{2.53}$$

A energia de emparelhamento não pode ser explicada facilmente. Isso torna-se evidente se se pensar que um par de nucleões idênticos não está ligado. De facto nem o "diprotão" nem o "dineutrão" existem como sistemas ligados. Se esses núcleos existissem os seus nucleões apresentariam spins

desemparelhados no estado fundamental, de acordo com o princípio de Pauli. Pelo contrário, o spin do deuterão no estado fundamental é 1, ou seja, o protão e o neutrão têm spins paralelos. Isto significa que a estrutura das forças nucleares é tal que a energia de ligação é maior no caso de spins paralelos. Não se pode portanto compreender a energia de emparelhamento a partir do potencial da ligação entre pares de nucleões. Trata-se efectivamente dum fenómeno que surge somente nos sistemas de muitas partículas e cuja origem será discutida no secção 6.5.

Veja-se agora de que modo as contribuições 1) a 5) se adicionam para dar a energia de ligação. De acordo com (2.10) a massa nuclear m(Z,A) exprime-se por

$$m(Z, A) = Zm_H + (A - Z)m_n - B/c^2 .$$

Introduzindo aqui a expressão de B dada em (2.47) e fazendo uso das relações (2.48) a (2.52), resulta

$$\begin{aligned} m(Z, A) = {} & Zm_H + (A - Z)m_n - a_V A + a_S A^{\frac{2}{3}} \\ & + a_C Z^2 A^{-\frac{1}{3}} + a_A (Z - A/2)^2 A^{-1} \pm \delta \end{aligned} \qquad (2.54)$$

onde as constantes a_V até a_P contêm agora um factor $1/c^2$. Esta expressão é conhecida por *fórmula de Weizsaecker* (1935). Para determinar os valores das constantes serão precisas em princípio cinco massas nucleares. Contudo, o ajuste é muito melhor se forem consideradas tantas mais massas quantas for possível, uma vez que a fórmula apenas descreve um comportamento médio. Um conjunto de valores para aquelas constantes é o seguinte [Wap 58]:

$$a_V = 17{,}011 \text{ mu} = 15{,}85 \text{ MeV}/c^2$$

$$a_S = 19{,}691 \text{ mu} = 18{,}34 \text{ MeV}/c^2$$

$$a_C = 0{,}767 \text{ mu} = 0{,}71 \text{ MeV}/c^2$$

$$a_A = 99{,}692 \text{ mu} = 92{,}86 \text{ MeV}/c^2$$

$$a_P = \pm\, 12{,}3 \text{ mu} = 11{,}46 \text{ MeV}/c^2$$

A contribuição dos termos individuais da expressão (2.54) para a energia de ligação por nucleão está representada na Fig.19. A figura mostra como o decréscimo da energia de superfície e do crescimento da energia de Coulomb, conduz a um máximo de B/A para $A \approx 60$. É claro que a fórmula de Weizsaecker exprime apenas o comportamento médio dos núcleos, não podendo decerto reproduzir quaisquer efeitos da estrutura em camadas. A expressão só é aplicável para $A > 30$ (ver Fig.10), produzindo para $A > 40$ valores de B/A correctos dentro de $\approx 1\%$. É, de facto, notável que um modelo tão simples seja capaz de descrever tão bem a energia de ligação. Para aplicações práticas existem fórmulas de massa que foram refinadas à custa da inclusão de hipóteses suplementares, e que produzem resultados ainda melhores que a expressão (2.54) (ver, por exemplo, [See 61, Mye 66, Gar 69]).

A constante a_A do termo de assimetria pode calcular-se a partir do modelo do gás de Fermi (v. (2.45)), mas o valor assim determinado representa apenas cerca de metade do valor determinado empiricamente a partir das massas nucleares. Existe porém uma outra contribuição para o termo de assimetria, que tem a ver com a já referida dependência que apresentam as forças nucleares relativamente ao spin. A ligação entre um neutrão e um protão que estejam alinhados paralelamente é maior que entre dois neutrões, os quais, devido ao princípio de Pauli, só podem ter orientação anti-paralela. Os núcleos com excesso de neutrões apresentam por isso uma energia de ligação menor. Verifica-se que esta contribuição é proporcional a T_Z/A.

A fórmula de Weizsaecker permite deduzir um certo número de regularidades importantes. Repare-se na variação da massa nuclear ao longo duma série de isóbaros, i.e., tome-se A = const. e faça-

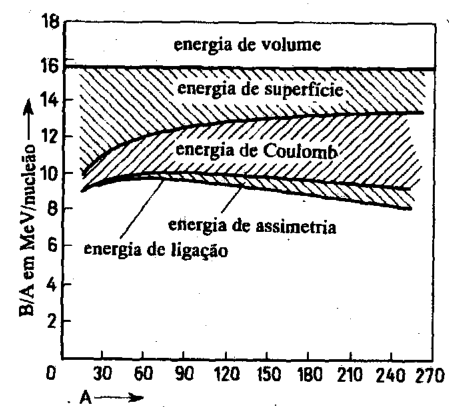


Fig.19
Contribuição dos diferentes termos da fórmula das massas nucleares para a energia de ligação média por nucleão [Eva 55].

-se variar Z em (2.54). Olhando para a expressão vê-se que ela é quadrática em Z. Para A ímpar obtém-se pois uma parábola como a representada na Fig.20a. Nos casos de A par surgem duas parábolas diferentes, devido à energia de emparelhamento $\pm\delta$. O núcleo está numa ou noutra parábola conforme seja do tipo par-par ou ímpar-ímpar (Fig.20b). Como se vê na Fig.20, nuclidos de Z vizinho podem transformar-se uns nos outros por emissão duma partícula β^+ ou β^-. Na Fig.20 lê-se também a regra segundo a qual para A ímpar apenas existe um isóbaro estável, enquanto para A par se têm vários isóbaros estáveis possíveis.

O número de protões, Z_0, para a qual a massa nuclear duma série de isóbaros é mínima ocorre para

$$\left(\frac{\partial m(Z,A)}{\partial Z}\right)_{A=\text{const}} = 0$$

Introduzindo aqui (2.54), resulta

$$-m_n + 2Z_0 a_C A^{-1/3} + 2a_A(Z_0 - A/2)A^{-1} = 0$$

e, resolvendo esta equação em ordem a Z_0, tem-se

$$Z_0 = \frac{A}{2}\left[\frac{m_n - m_H + a_A}{a_C A^{2/3} + a_A}\right] = \frac{A}{1{,}98 + 0{,}015\,A^{2/3}} \qquad (2.55)$$

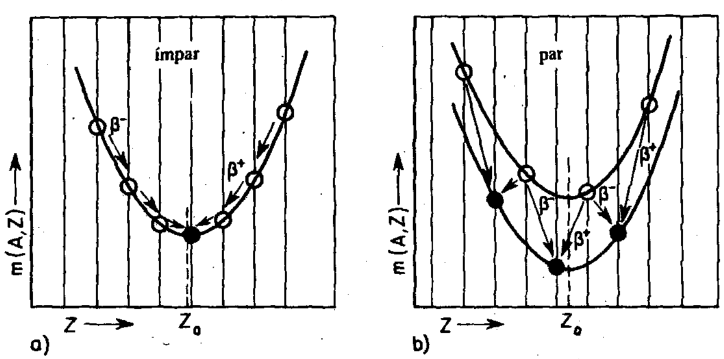


Fig.20 As energias dos núcleos com um mesmo A. Os núcleos estáveis são indicados pelos círculos a cheio.

Representando estes valores num diagrama de N em função de Z, obtém-se a Fig.21. Se, além disso, as massas nucleares forem representadas segundo o eixo perpendicular ao plano NZ, a linha a cheio na Fig.21 corresponde à localização aproximada dos núcleos estáveis, i.e., que não estão sujeitos ao decaimento β. Esses núcleos são os que se encontram no fundo do "vale" das massas nucleares.

Nos processos de transmutação por decaimento β o número de massa não é alterado. Pode igualmente usar-se a fórmula de Weizsaecker para saber se um dado processo de separação de nucleões pode libertar energia. É de esperar que, frequentemente, se ganhe energia na separação duma partícula α, em particular, devido à sua elevada energia de ligação. É realmente o que acontece sempre que a soma das massas da partícula, m_α, e do núcleo resultante, m(Z-2, A-4), seja inferior à massa do núcleo original. A energia cinética libertada será

$$E_\alpha = [m(Z,A) - m(Z-2, A-4) - m_\alpha]c^2 \qquad (2.56)$$

A comparação com (2.19) mostra que isto apenas significa uma energia de separação negativa. Em princípio, ganha-se energia pela separação duma partícula α sempre que $E_\alpha > 0$. Com a ajuda da fórmula de Weizsaecker é possível determinar as regiões do plano NZ que correspondem a $E_\alpha > 0, > 2, > 4, > 6$ MeV, etc. Na Fig.22 representam-se as fronteiras dessas regiões para diferentes valores de E_α. Para maior clareza, não se representa o plano NZ, mas sim N/Z em função de A. Representam-se igualmente na figura os limites das regiões de instabilidade para a separação de neutrões e de protões. Como se vê, esses limites afastam-se bastante da linha dos núcleos estáveis, que aliás nunca cruzam. Resulta daí que a emissão de neutrões ou de

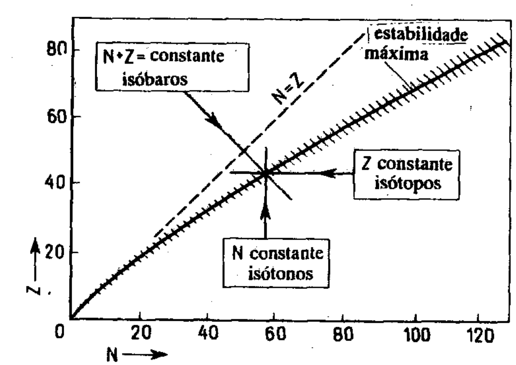


Fig.21
Localização dos núcleos estáveis no plano NZ.

actually observed, it must take this effect into account. (Otherwise it would allow stable isotopes of hydrogen with hundreds of neutrons!) This term is very important for light nuclei, for which $Z \simeq A/2$ is more strictly observed. For heavy nuclei, this term becomes less important, because the rapid increase in the Coulomb repulsion term requires additional neutrons for nuclear stability. A possible form for this term, called the symmetry term because it tends to make the nucleus symmetric in protons and neutrons, is $-a_{sym}(A-2Z)^2/A$ which has the correct form of favoring nuclei with $Z = A/2$ and reducing in importance for large A.

Finally, we must include another term that accounts for the tendency of like nucleons to couple pairwise to especially stable configurations. When we have an odd number of nucleons (odd Z and even N, or even Z and odd N), this term does not contribute. However, when both Z and N are odd, we gain binding energy by converting one of the odd protons into a neutron (or vice versa) so that it can now form a pair with its formerly odd partner. We find evidence for this *pairing force* simply by looking at the stable nuclei found in nature—there are only four nuclei with odd N and Z (^{2}H, ^{6}Li, ^{10}B, ^{14}N), but 167 with even N and Z. This pairing energy δ is usually expressed as $+a_p A^{-3/4}$ for Z and N even, $-a_p A^{-3/4}$ for Z and N odd, and zero for A odd.

Combining these five terms we get the complete binding energy:

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sym}\frac{(A-2Z)^2}{A} + \delta \tag{3.28}$$

and using this expression for B we have the *semiempirical mass formula*:

$$M(Z,A) = Zm(^1\text{H}) + Nm_n - B(Z,A)/c^2 \tag{3.29}$$

The constants must be adjusted to give the best agreement with the experimental curve of Figure 3.16. A particular choice of $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV, $a_{sym} = 23$ MeV, $a_p = 34$ MeV, gives the result shown in Figure 3.17, which reproduces the observed behavior of B rather well.

The importance of the semiempirical mass formula is not that it allows us to predict any new or exotic phenomena of nuclear physics. Rather, it should be regarded as a first attempt to apply nuclear models to understand the systematic behavior of a nuclear property, in this case the binding energy. It includes several different varieties of nuclear models: the *liquid-drop model*, which treats some of the gross collective features of nuclei in a way similar to the calculation of the properties of a droplet of liquid (indeed, the first three terms of Equation 3.28 would also appear in a calculation of the energy of a charged liquid droplet), and the *shell model*, which deals more with individual nucleons and is responsible for the last two terms of Equation 3.28.

For constant A, Equation 3.29 represents a parabola of M vs. Z. The parabola will be centered about the point where Equation 3.29 reaches a minimum. To compare this result with the behavior of actual nuclei, we must find the minimum, where $\partial M/\partial Z = 0$:

$$Z_{min} = \frac{[m_n - m(^1\text{H})] + a_c A^{-1/3} + 4a_{sym}}{2a_c A^{-1/3} + 8a_{sym}A^{-1}} \tag{3.30}$$

With $a_c = 0.72$ MeV and $a_{sym} = 23$ MeV, it follows that the first two terms in the numerator are negligible, and so

$$Z_{min} \simeq \frac{A}{2}\,\frac{1}{1+\frac{1}{4}A^{2/3}a_c/a_{sym}} \tag{3.31}$$

For small A, $Z_{min} \simeq A/2$ as expected, but for large A, $Z_{min} < A/2$. For heavy nuclei, Equation 3.31 gives $Z/A \simeq 0.41$, consistent with observed values for heavy stable nuclei.

Figure 3.18 shows a typical odd-A decay chain for $A = 125$, leading to the stable nucleus at $Z = 52$. The unstable nuclei approach stability by converting a neutron into a proton or a proton into a neutron by radioactive β decay. Notice how the decay energy (that is, the mass difference between neighboring isobars) increases as we go further from stability. For even A, the pairing term gives two parabolas, displaced by 2δ. This permits two unusual effects, not seen in odd-A decays: (1) some odd-Z, odd-N nuclei can decay in either direction, converting a neutron to a proton or a proton to a neutron; (2) certain *double* β *decays* can become energetically possible, in which the decay may change 2 protons to 2 neutrons. Both of these effects are discussed in Chapter 9.

3.4 NUCLEAR ANGULAR MOMENTUM AND PARITY

In Section 2.5 we discussed the coupling of orbital angular momentum ℓ and spin s to give total angular momentum j. To the extent that the nuclear potential is central, ℓ and s (and therefore j) will be constants of the motion. In the quantum mechanical sense, we can therefore label every nucleon with the corresponding quantum numbers ℓ, s, and j. The total angular momentum of a nucleus containing A nucleons would then be the vector sum of the angular momenta of all the nucleons. This total angular momentum is usually called the *nuclear spin* and is represented by the symbol I. The angular momentum I has all of the usual properties of quantum mechanical angular momentum vectors: $I^2 = \hbar^2 I(I+1)$ and $I_z = m\hbar$ $(m = -I, \ldots, +I)$. For many applications involving angular momentum, the nucleus behaves as if it were a single entity with an intrinsic angular momentum of I. In ordinary magnetic fields, for example, we can observe the nuclear Zeeman effect, as the state I splits up into its $2I+1$ individual substates $m = -I, -I+1, \ldots, I-1, I$. These substates are equally spaced, as in the atomic normal Zeeman effect. If we could apply an incredibly strong magnetic field, so strong that the coupling between the nucleons were broken, we would see each individual j splitting into its $2j+1$ substates. Atomic physics also has an analogy here: when we apply large magnetic fields we can break the coupling between the electronic ℓ and s and separate the $2\ell+1$ components of ℓ and the $2s+1$ components of s. No fields of sufficient strength to break the coupling of the nucleons can be produced. We therefore observe the behavior of I as if the nucleus were only a single "spinning" particle. For this reason, the spin (total angular momentum) I and the corresponding spin quantum number I are used to describe nuclear states.

To avoid confusion, we will always use I to denote the nuclear spin; we will use j to represent the total angular momentum of a single nucleon. It will often

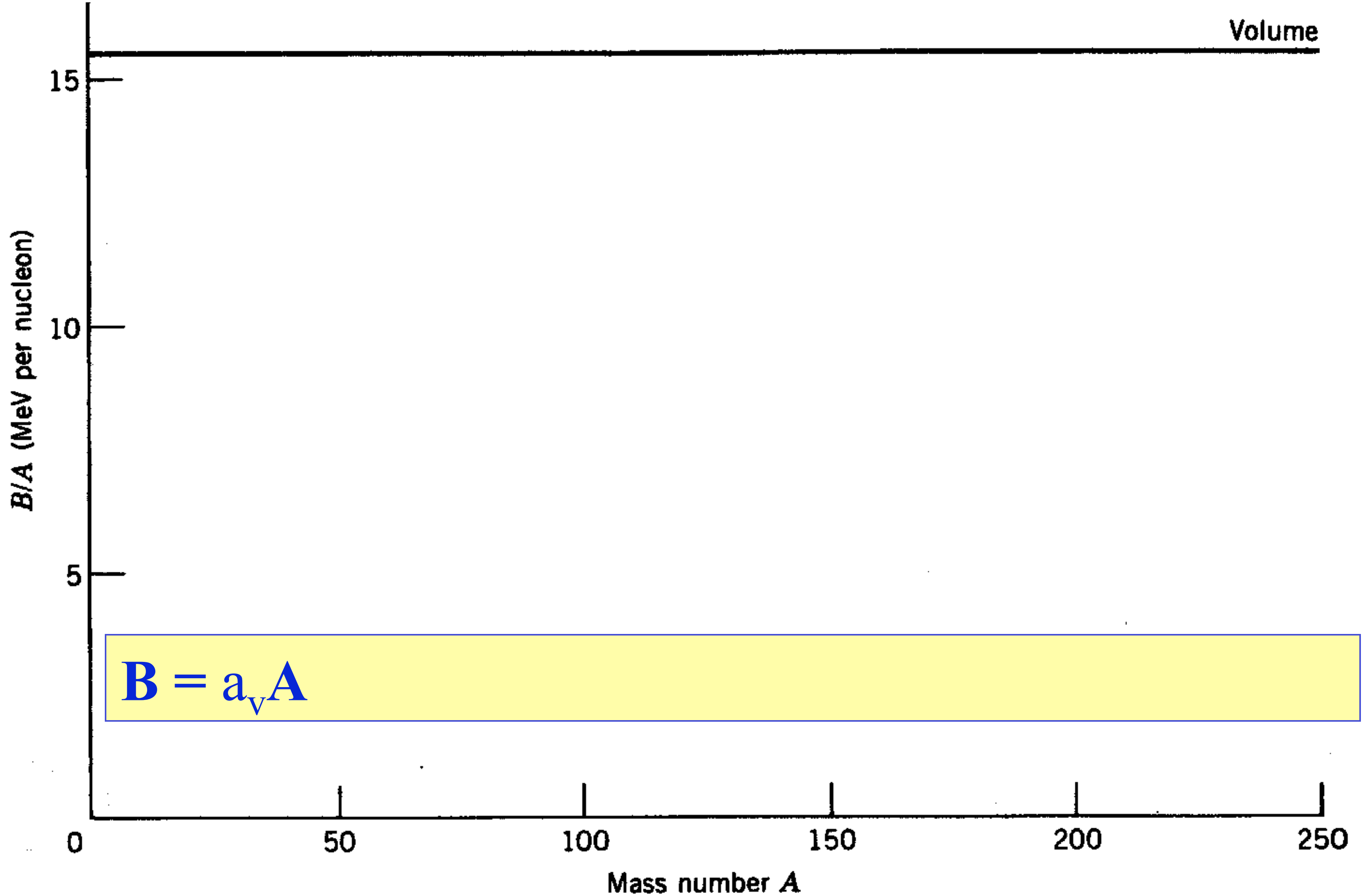
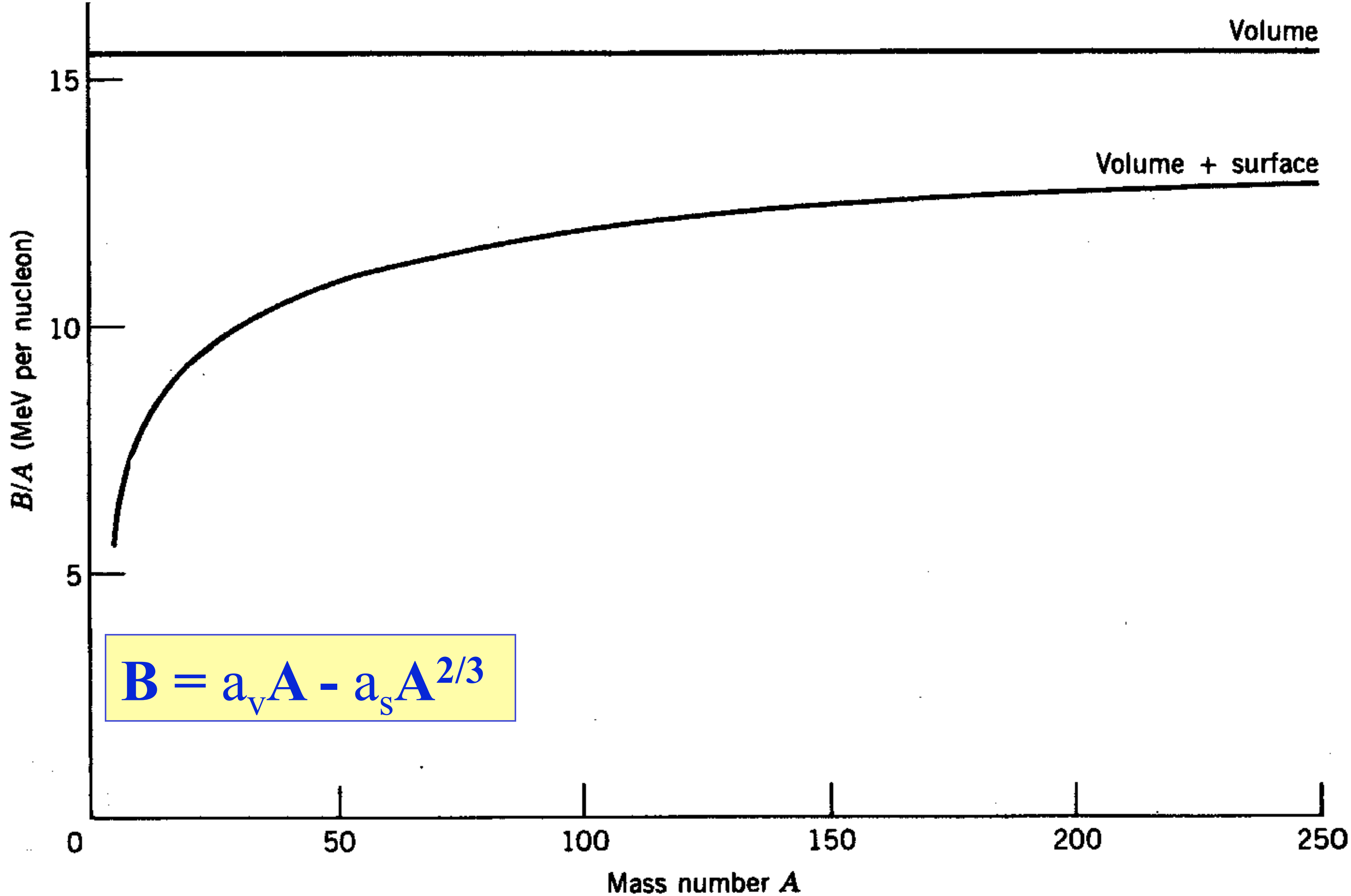
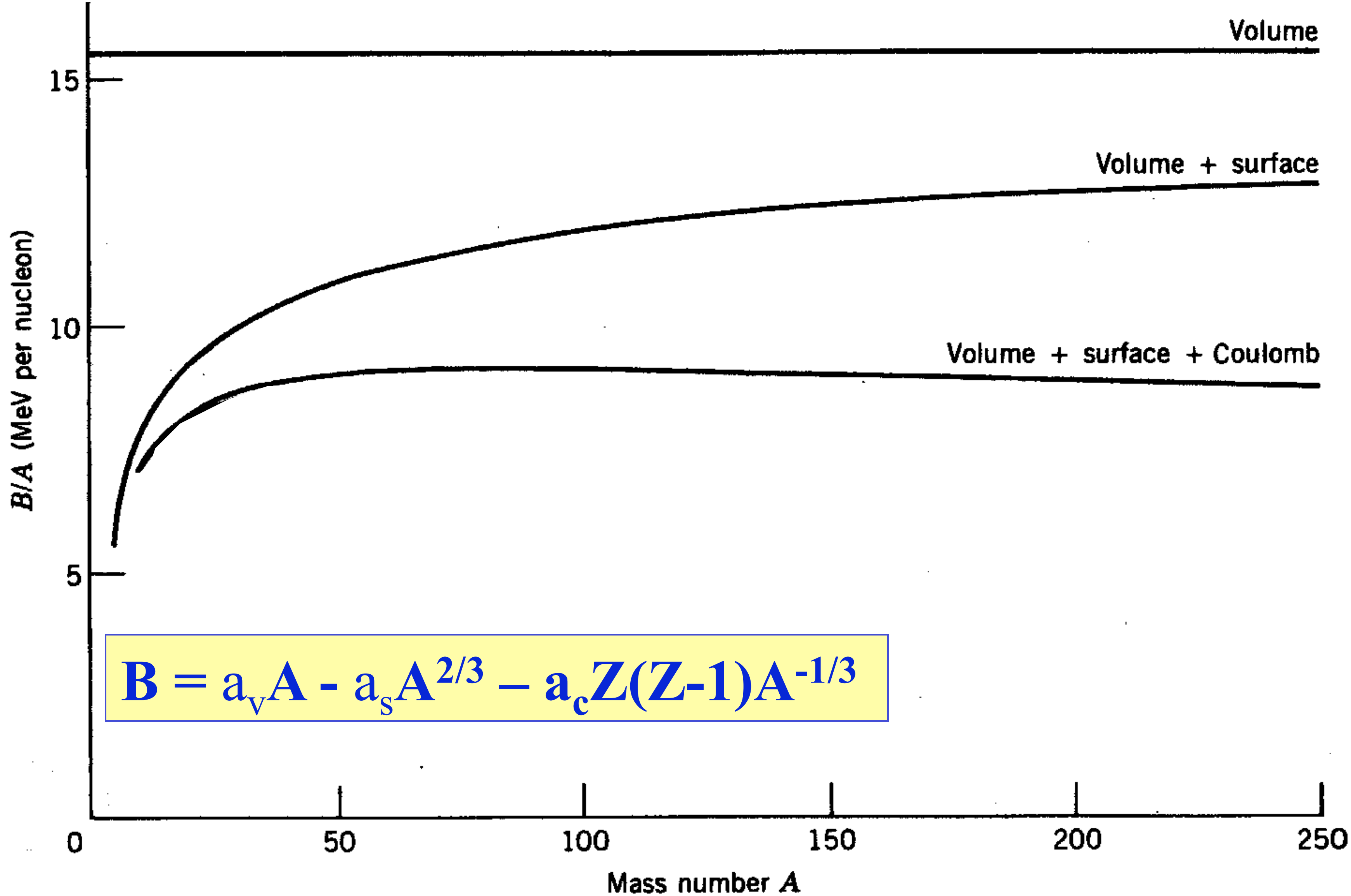


Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.



$$B = a_v A - a_s A^{2/3}$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

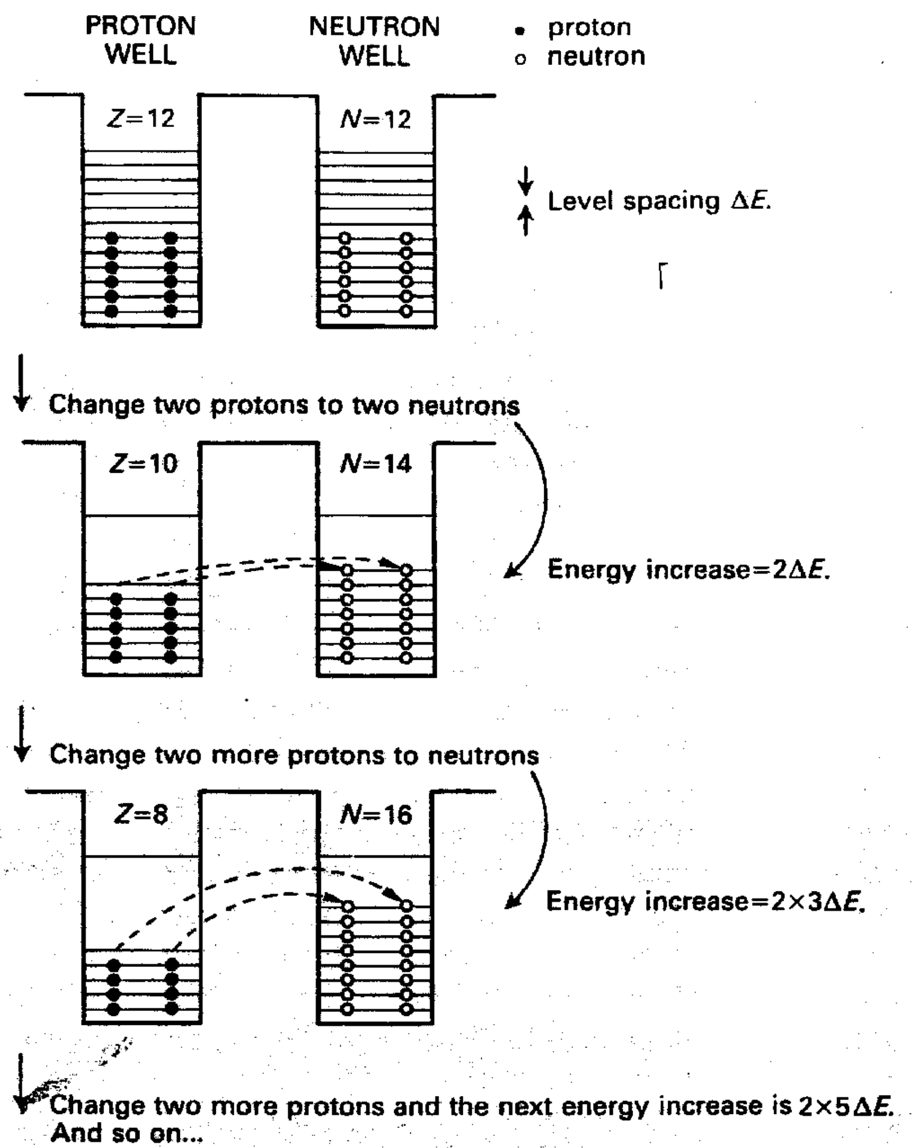


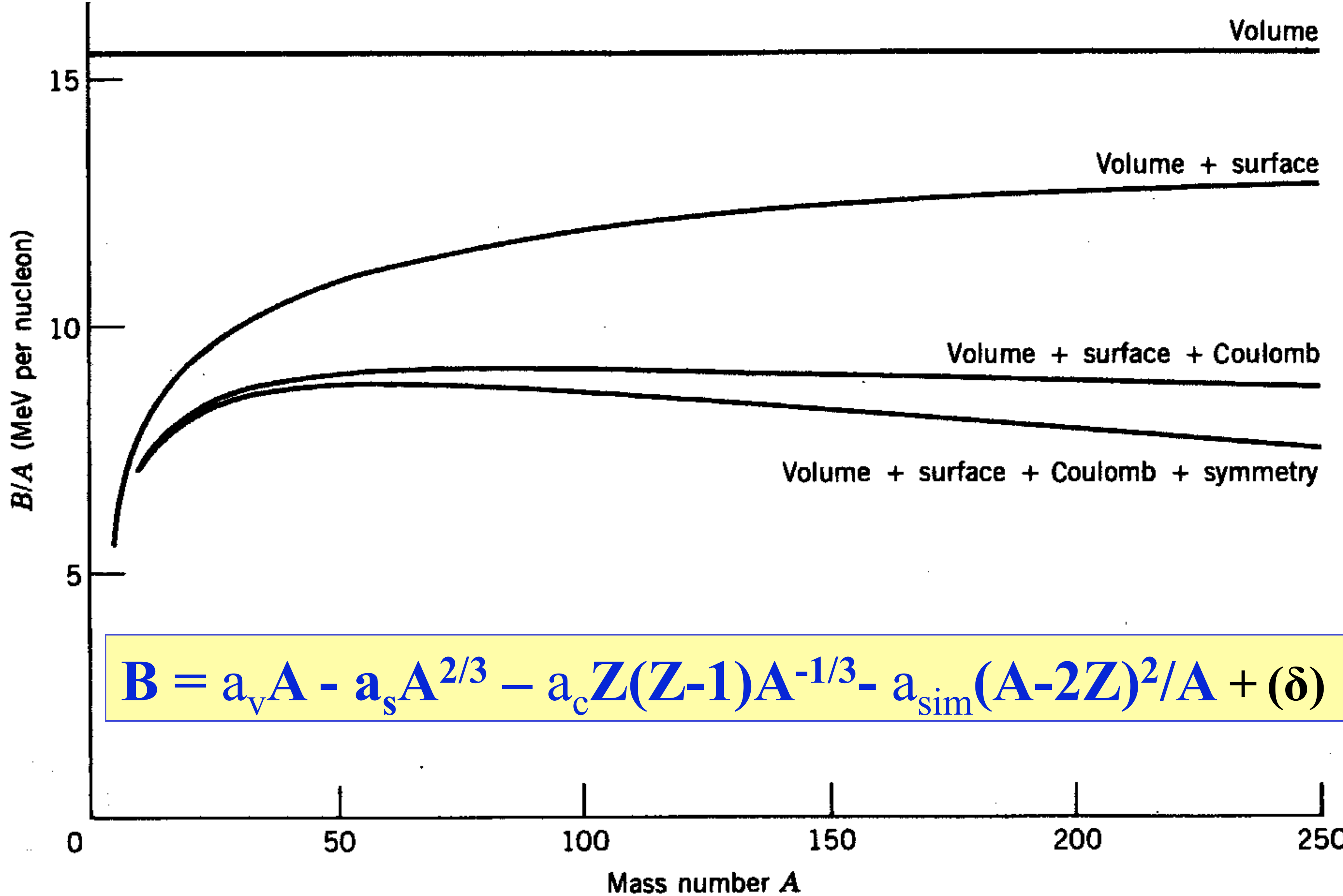
$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3}$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

TERMO DE SIMETRIA

Fig. 4.3 The occupation of energy levels of a nucleus by protons (●) and by neutrons (○) according to the Pauli exclusion principle in a nucleus which is changing from $Z=N$ to $N>Z$, while $A=Z+N$ remains constant. The cost in energy of making the change of two protons into two neutrons and placing the latter in unoccupied neutron levels increases at every change. (The cost or gain in energy due to the neutron–proton mass difference is not included.)

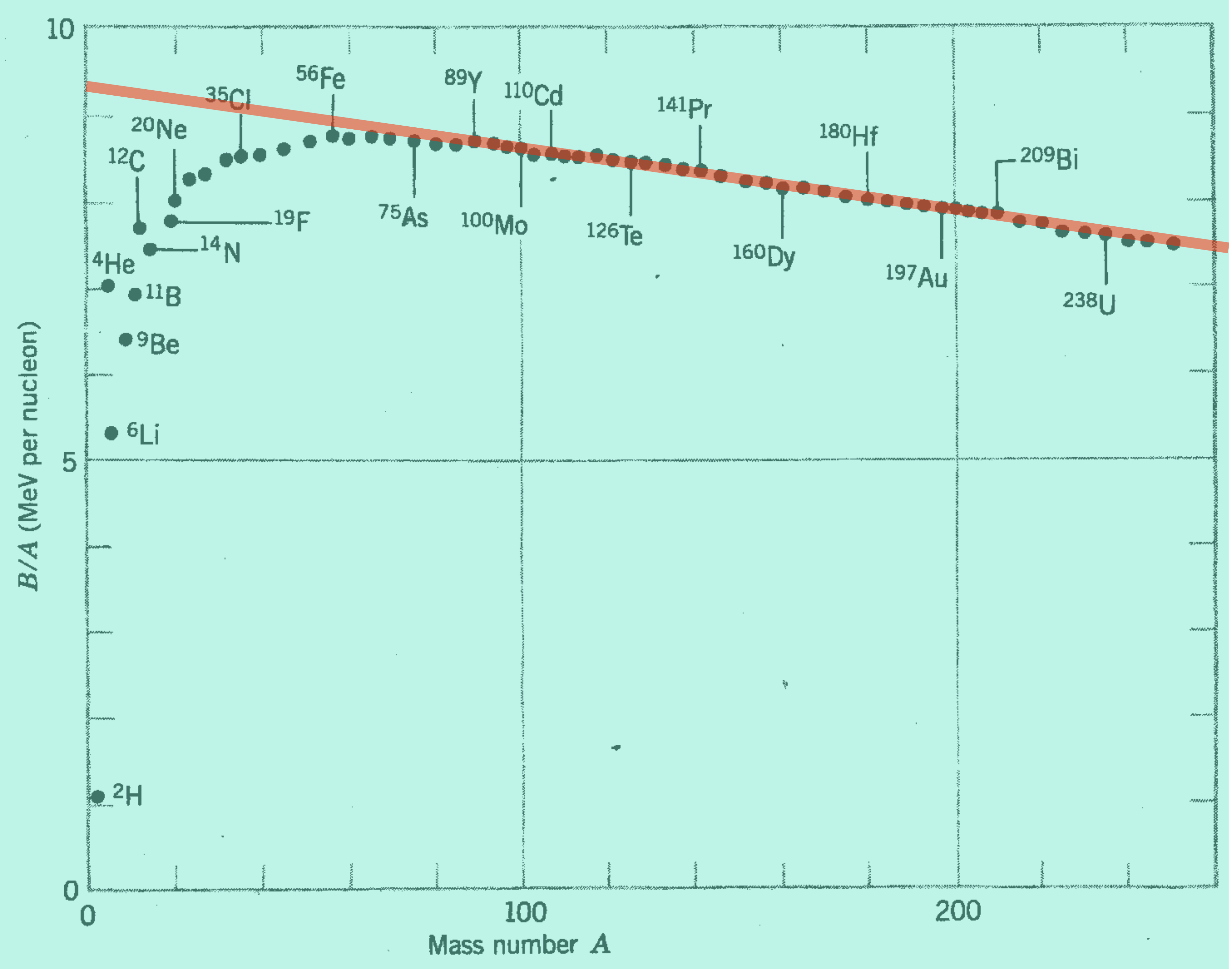




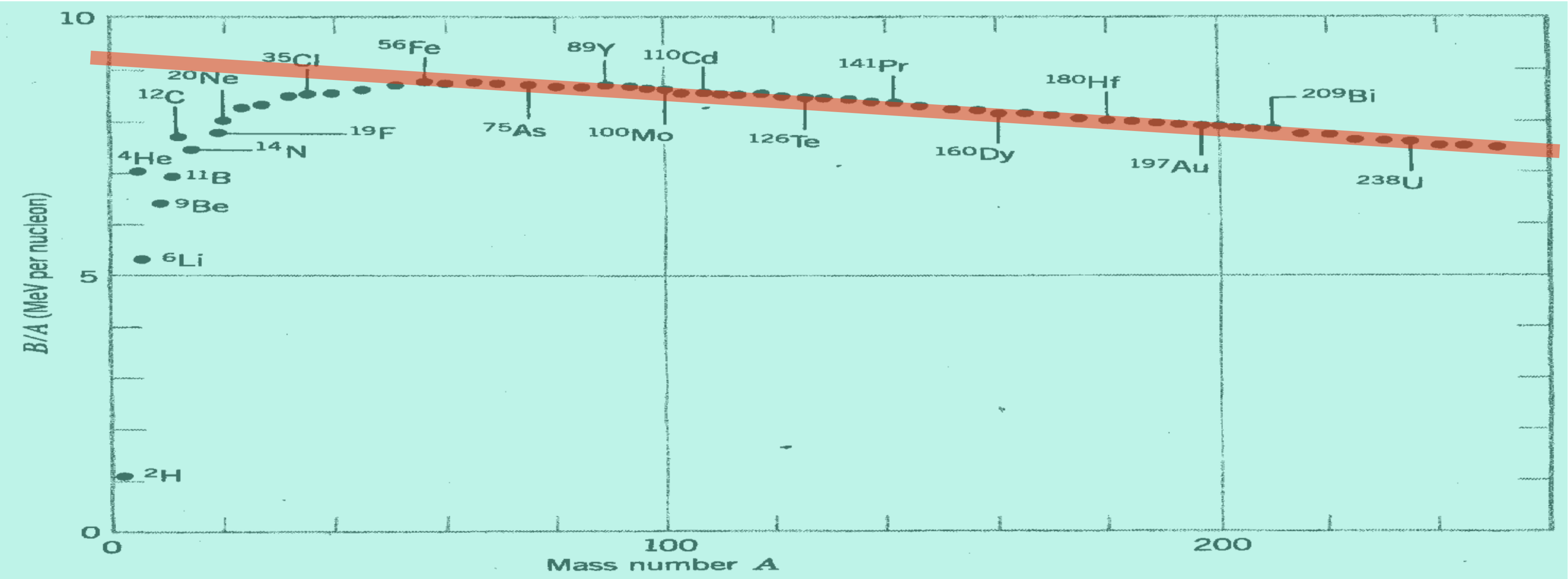
$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim}(A-2Z)^2/A + (\delta)$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

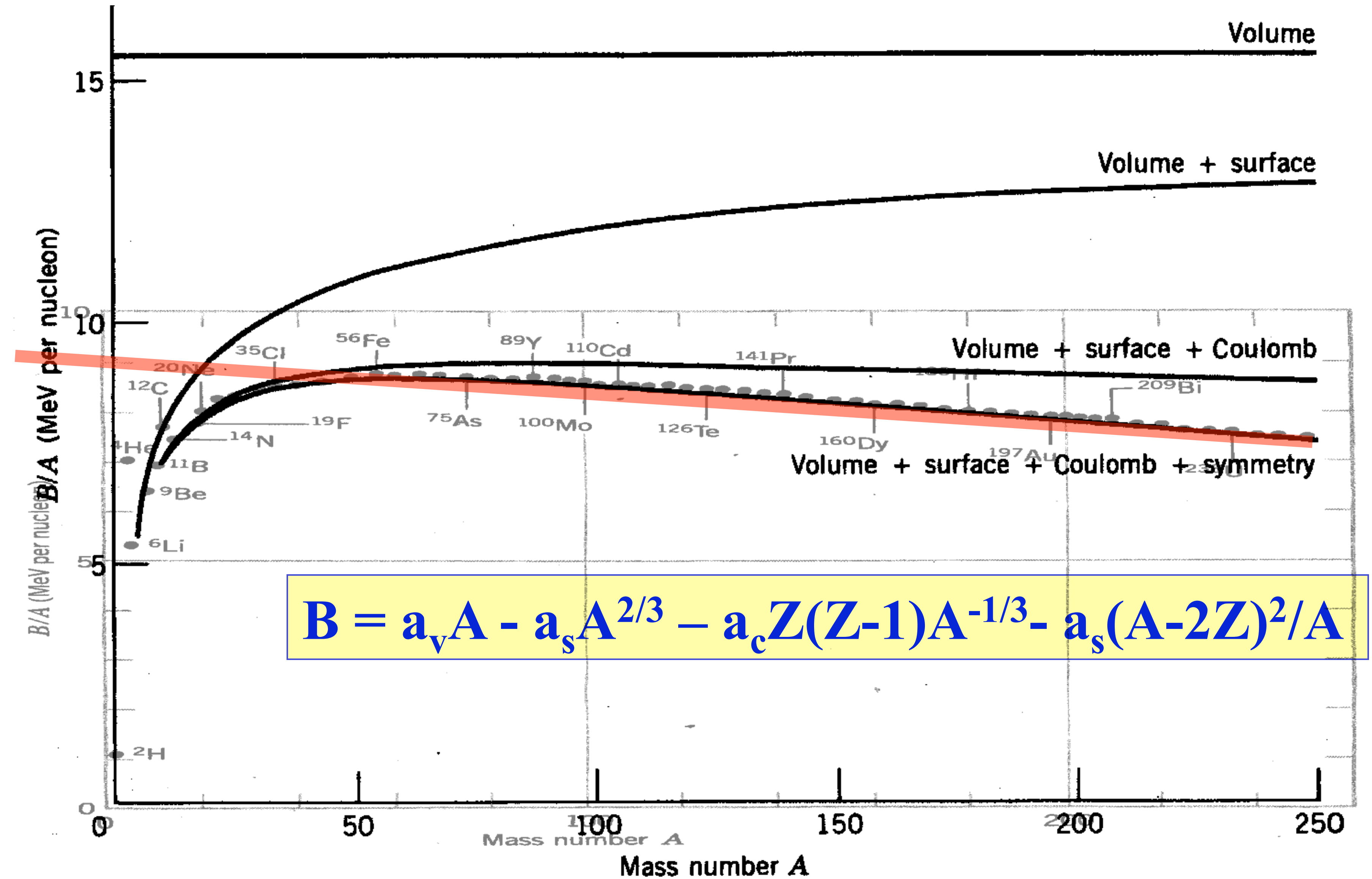
$$T_z = \frac{1}{2}(N-Z)$$



10
5
0
B/A (MeV per nucleon)
0
100
200
Mass number A
56Fe
35Cl
89Y
110Cd
141Pr
180Hf
209Bi
20Ne
12C
19F
14N
4He
11B
9Be
6Li
2H
75As
100Mo
126Te
160Dy
197Au
238U



10
5
0
0
100
200
Mass number A
B/A (MeV per nucleon)
²H
⁶Li
⁹Be
¹¹B
⁴He
¹⁴N
¹²C
¹⁹F
²⁰Ne
³⁵Cl
⁵⁶Fe
⁷⁵As
⁸⁹Y
¹⁰⁰Mo
¹¹⁰Cd
¹²⁶Te
¹⁴¹Pr
¹⁶⁰Dy
¹⁸⁰Hf
¹⁹⁷Au
²⁰⁹Bi
²³⁸U



$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_s (A-2Z)^2/A$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

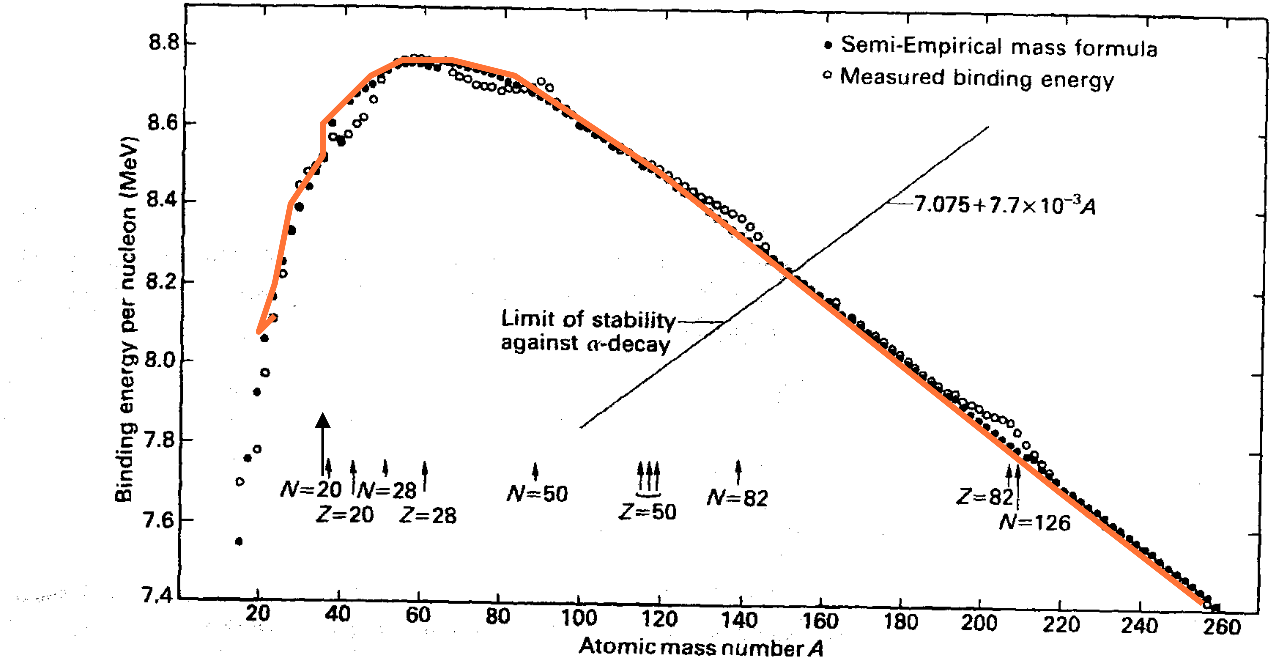


Fig. 4.6 The binding energy as a function of A for the odd-A nuclei from $A = 15$–259. The solid points are the prediction of the semi-empirical mass formula as given in Table 4.1. The open points are the measured values. The points for the formula do not lie on a smooth curve because Z for these nuclei is not a smooth function of A (see Fig. 4.1). Note that the zero of the ordinate is suppressed and its scale is much enlarged. Thus, in spite of the deviations from the formula, it is clear that the formula predicts the binding energy per nucleon for $A > 20$ with a precision which is, for the majority of cases, better than 0.1 MeV. The straight line crossing the curve at $A = 151$ gives the limit of stability of nuclei to α-decay (see Section 5.4).

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim}(A-2Z)^2/A + \delta(Z,A)$$

$$\delta(Z,A) = \begin{cases} -34A^{-3/4}\ \text{MeV} & \textbf{impar-impar} \\ 0 & \textbf{impar-par ou par-impar} \\ +34A^{-3/4}\ \text{MeV} & \textbf{par-par} \end{cases}$$

VALORES DOS COEFICIENTES DA EQUAÇÃO DE Bethe-Weizaker

$$\delta(Z,A) = \begin{cases} -34A^{-3/4}\ \text{MeV} & \text{impar-impar} \\ 0 & \text{impar-par ou par-impar} \\ +34A^{-3/4}\ \text{MeV} & \text{par-par} \end{cases}$$

$$B(Z,A) = [(Zm_p + Nm_n) - M(Z,A)]c^2$$

ou

$$M(Z,A) = Zm_p + Nm_n - B(Z,A)/c^2$$

$$M(Z,A) = \mathbf{Z}m_p + (A-\mathbf{Z})m_n - \mathbf{(}a_v A - a_s A^{2/3} - a_c \mathbf{Z(Z-1)}A^{-1/3} - a_{sim}(A-2\mathbf{Z})^{\mathbf{2}}/A\mathbf{)}$$

Para isobaros ($A=C^{te}$) → M vs Z é uma parabola apontando para

Z_{mim} tal que $(dM/dZ)_{Zmin}=0$

$$\mathbf{Z_{mim}} = \frac{(\mathbf{m_n}-\mathbf{m_p}) + a_c\mathbf{A^{-1/3}} + 4a_{sim}}{2a_c\mathbf{A^{-1/3}} + 8a_{sym}\mathbf{A^{-1}}}$$

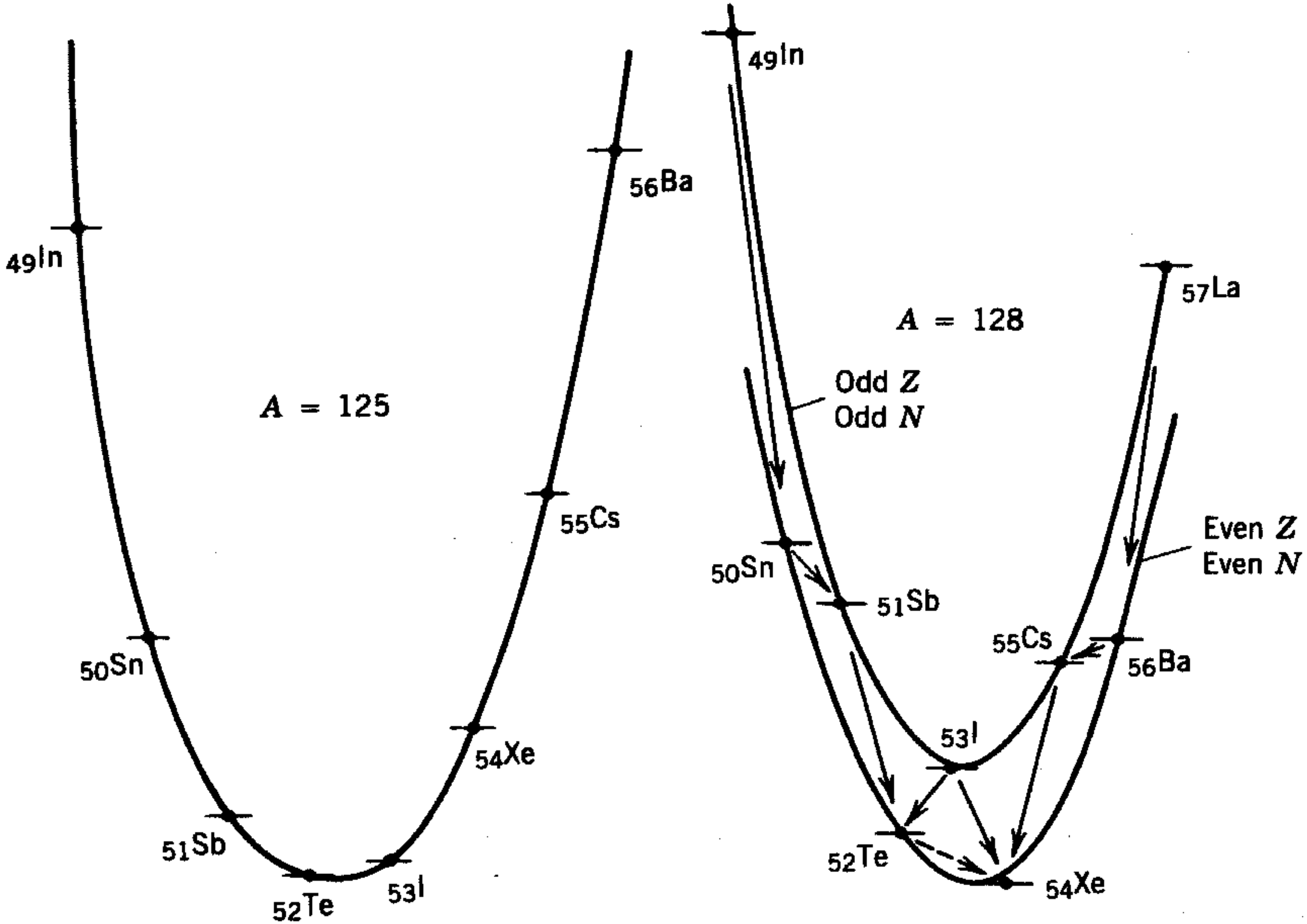
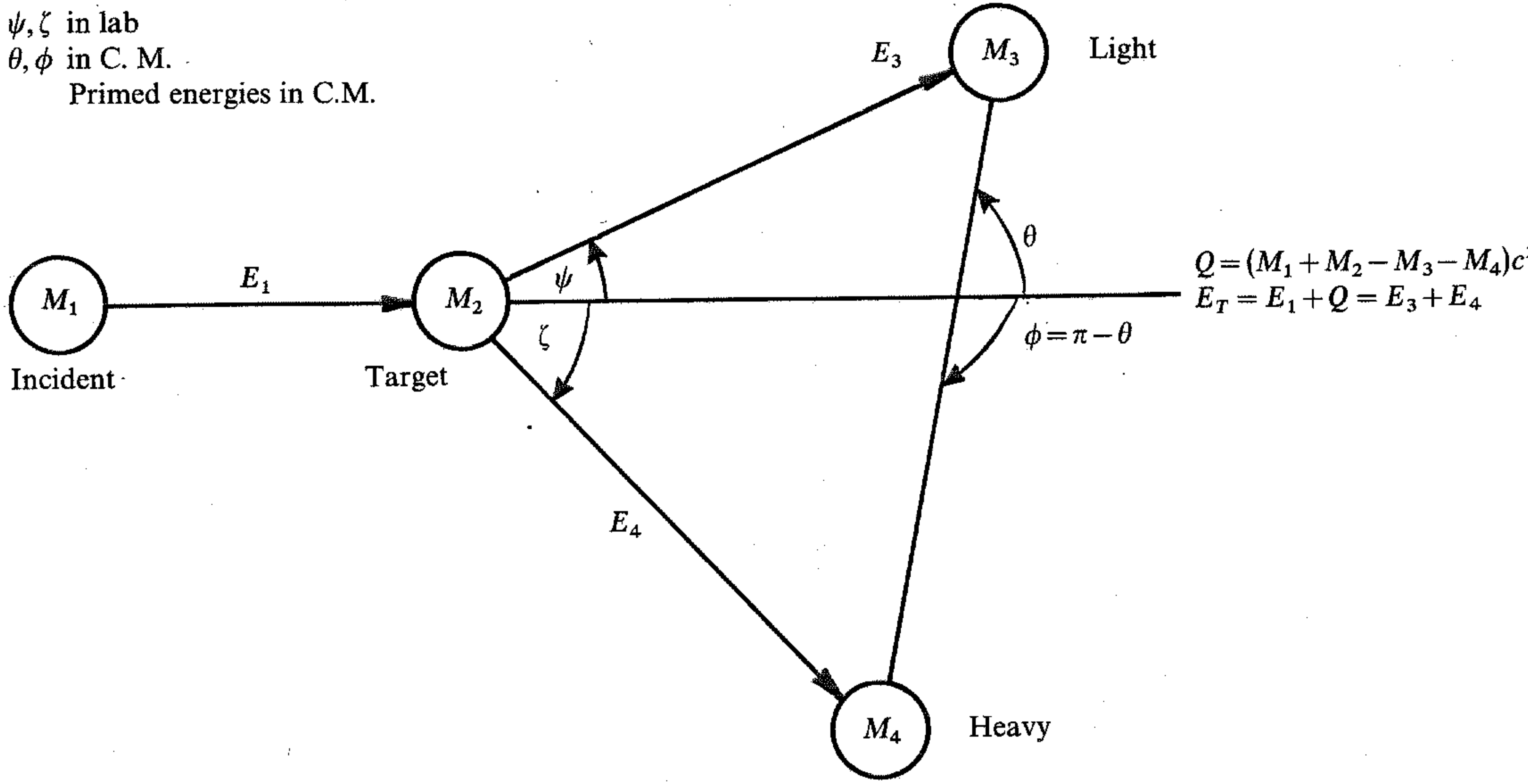


Figure 3.18 Mass chains for $A = 125$ and $A = 128$. For $A = 125$, note how the energy differences between neighboring isotopes increase as we go further from the stable member at the energy minimum. For $A = 128$, note the effect of the pairing term; in particular, ^{128}I can decay in either direction, and it is energetically possible for ^{128}Te to decay directly to ^{128}Xe by the process known as double β decay.

reações nucleares (tipos)
cinematica de reação

Kinematics of nuclear reactions and scattering (continued)



Define:

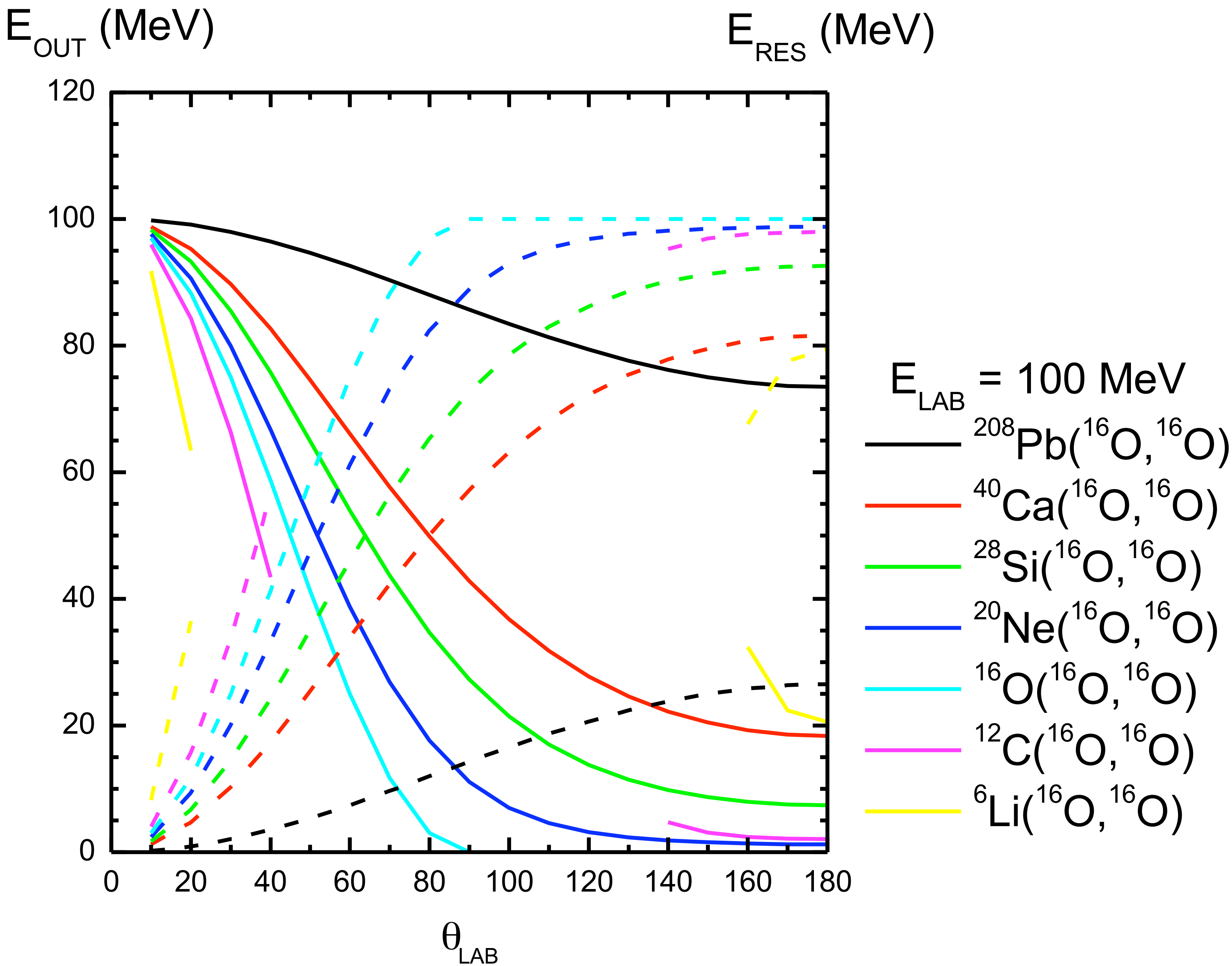
$$A = \frac{M_1 M_4 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad C = \frac{M_2 M_3}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1 Q}{M_2 E_T}\right) = \frac{E_4'}{E_T}$$

$$B = \frac{M_1 M_3 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad D = \frac{M_2 M_4}{(M_1+M_2)(M_3+M_4)}\left(1+\frac{M_1 Q}{M_2 E_T}\right) = \frac{E_3'}{E_T}$$

Note that $A+B+C+D=1$ and $AC=BD$

Note that $A+B+C+D=1$ and $AC=BD$

Lab energy of light product:	$\dfrac{E_3}{E_T}=B+D+2(AC)^{\frac{1}{2}}\cos\theta$ $=B[\cos\psi\pm(D/B-\sin^2\psi)^{\frac{1}{2}}]^2$	Use only plus sign unless $B>D$, in which case $\psi_{\max}=\sin^{-1}(D/B)^{\frac{1}{2}}$	
Lab energy of heavy product:	$\dfrac{E_4}{E_T}=A+C+2(AC)^{\frac{1}{2}}\cos\phi$ $=A[\cos\zeta\pm(C/A-\sin^2\zeta)^{\frac{1}{2}}]^2$	Use only plus sign unless $A>C$, in which case $\zeta_{\max}=\sin^{-1}(C/A)^{\frac{1}{2}}$	
Lab angle of heavy product:	$\sin\zeta=\left(\dfrac{M_3E_3}{M_4E_4}\right)^{\frac{1}{2}}\sin\psi$	C.M. angle of light product:	$\sin\theta=\left(\dfrac{E_3/E_T}{D}\right)\sin\psi$
Intensity or solid-angle ratio for light product:	$\dfrac{\sigma(\theta)}{\sigma(\psi)}=\dfrac{I(\theta)}{I(\psi)}=\dfrac{\sin\psi\,d\psi}{\sin\theta\,d\theta}=\dfrac{\sin^2\psi}{\sin^2\theta}\cos(\theta-\psi)=\dfrac{(AC)^{\frac{1}{2}}(D/B-\sin^2\psi)^{\frac{1}{2}}}{E_3/E_T}$		
Intensity or solid-angle ratio for heavy product:	$\dfrac{\sigma(\phi)}{\sigma(\zeta)}=\dfrac{I(\phi)}{I(\zeta)}=\dfrac{\sin\zeta\,d\zeta}{\sin\phi\,d\phi}=\dfrac{\sin^2\zeta}{\sin^2\phi}\cos(\phi-\zeta)=\dfrac{(AC)^{\frac{1}{2}}(C/A-\sin^2\zeta)^{\frac{1}{2}}}{E_4/E_T}$		
Intensity or solid-angle ratio for associated particles in the lab system:	$\dfrac{\sigma(\zeta)}{\sigma(\psi)}=\dfrac{I(\zeta)}{I(\psi)}=\dfrac{\sin\psi\,d\psi}{\sin\zeta\,d\zeta}=\dfrac{\sin^2\psi\cos(\theta-\psi)}{\sin^2\zeta\cos(\phi-\zeta)}$		



E_OUT (MeV)
E_RES (MeV)
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
160
180
θ_LAB
E_LAB = 100 MeV
208Pb(16O,16O)
40Ca(16O,16O)
28Si(16O,16O)
20Ne(16O,16O)
16O(16O,16O)
12C(16O,16O)
6Li(16O,16O)

EXERCÍCIO

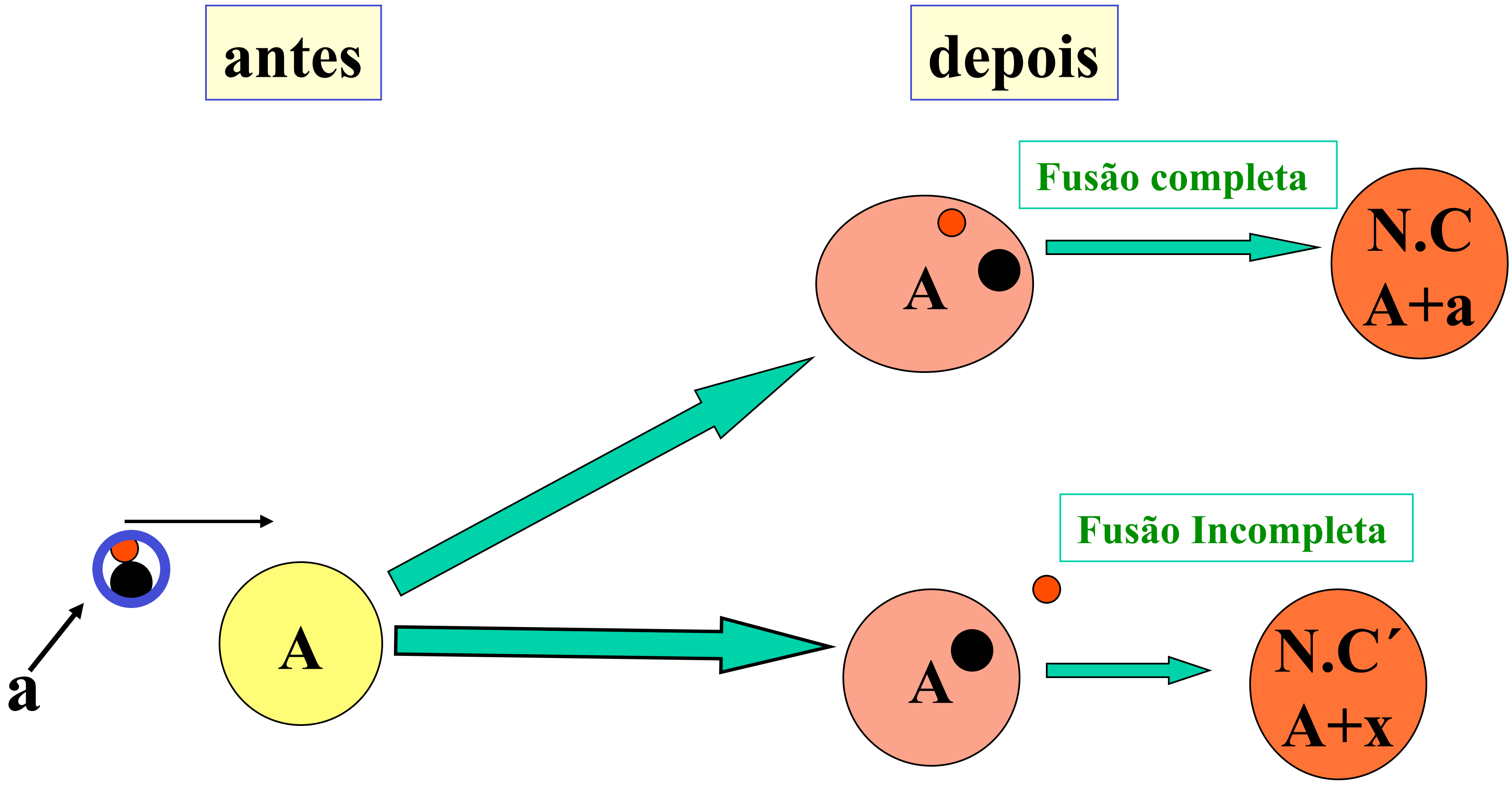
CINEMÁTICA DE REAÇÕES: **CALCULAR**

E_3 , E_4 vs θ ou E_{out} , E_{res} vs θ

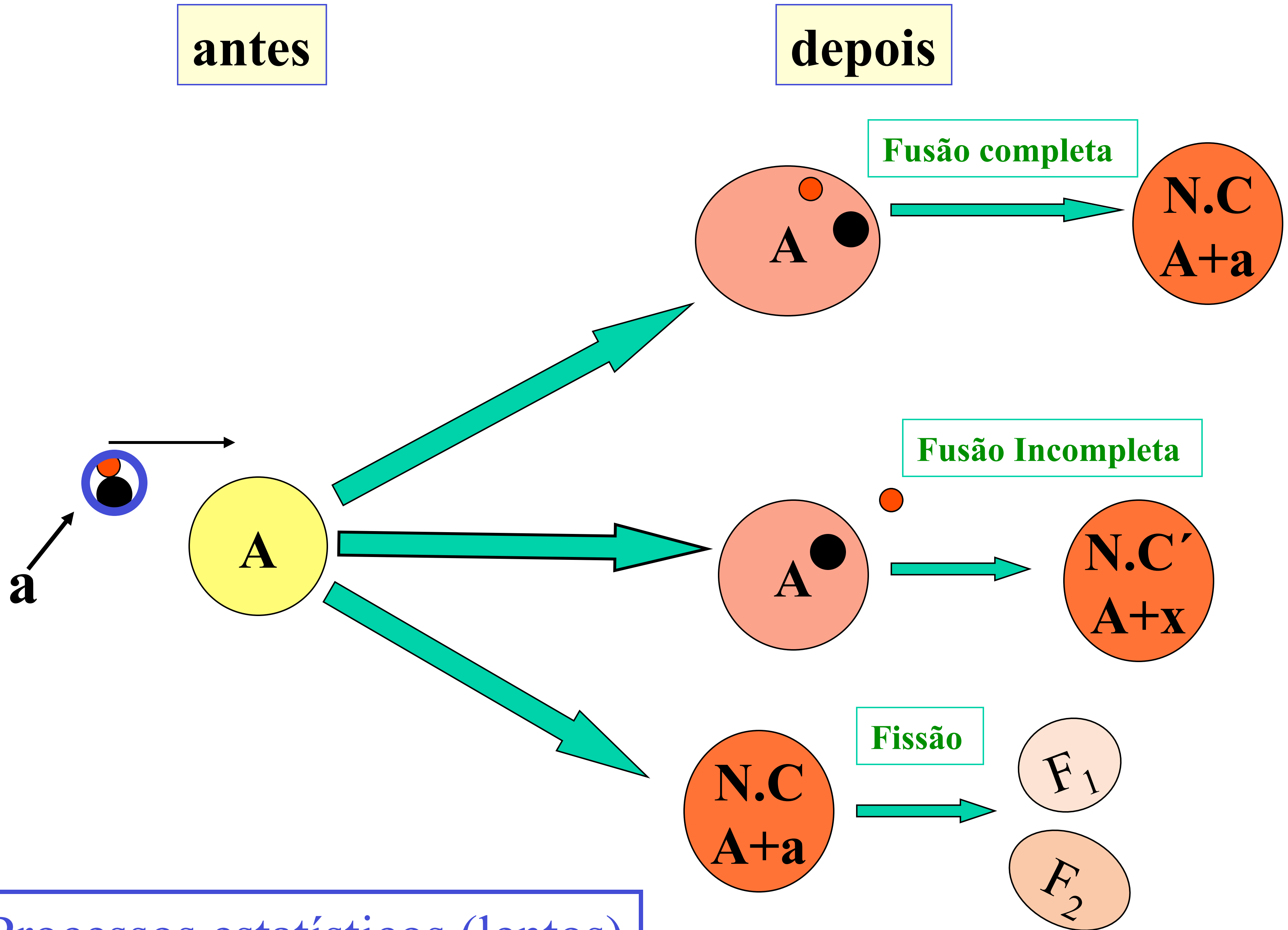
E_3 , E_4 vs E_1

Ψ vs χ

θ vs φ

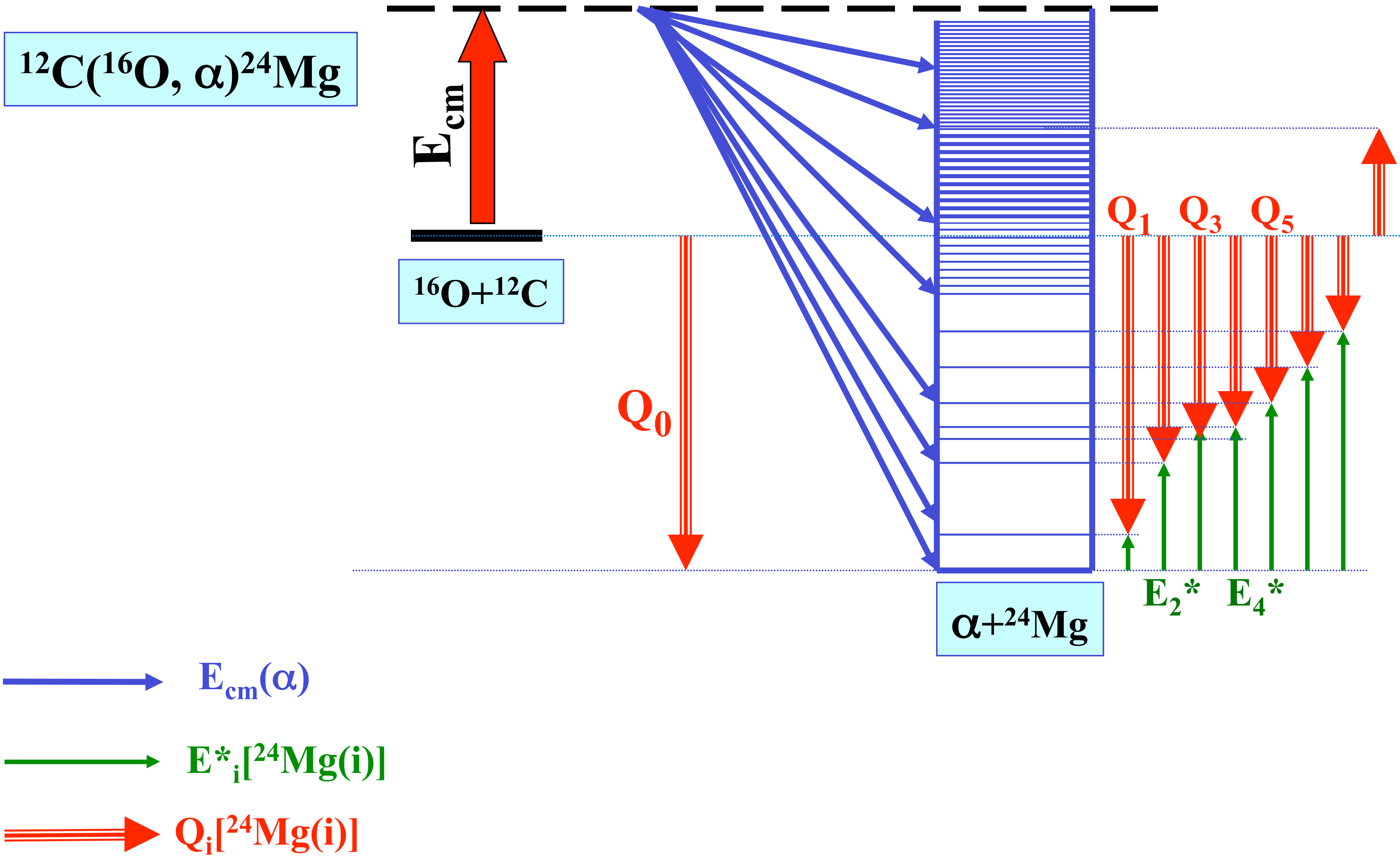


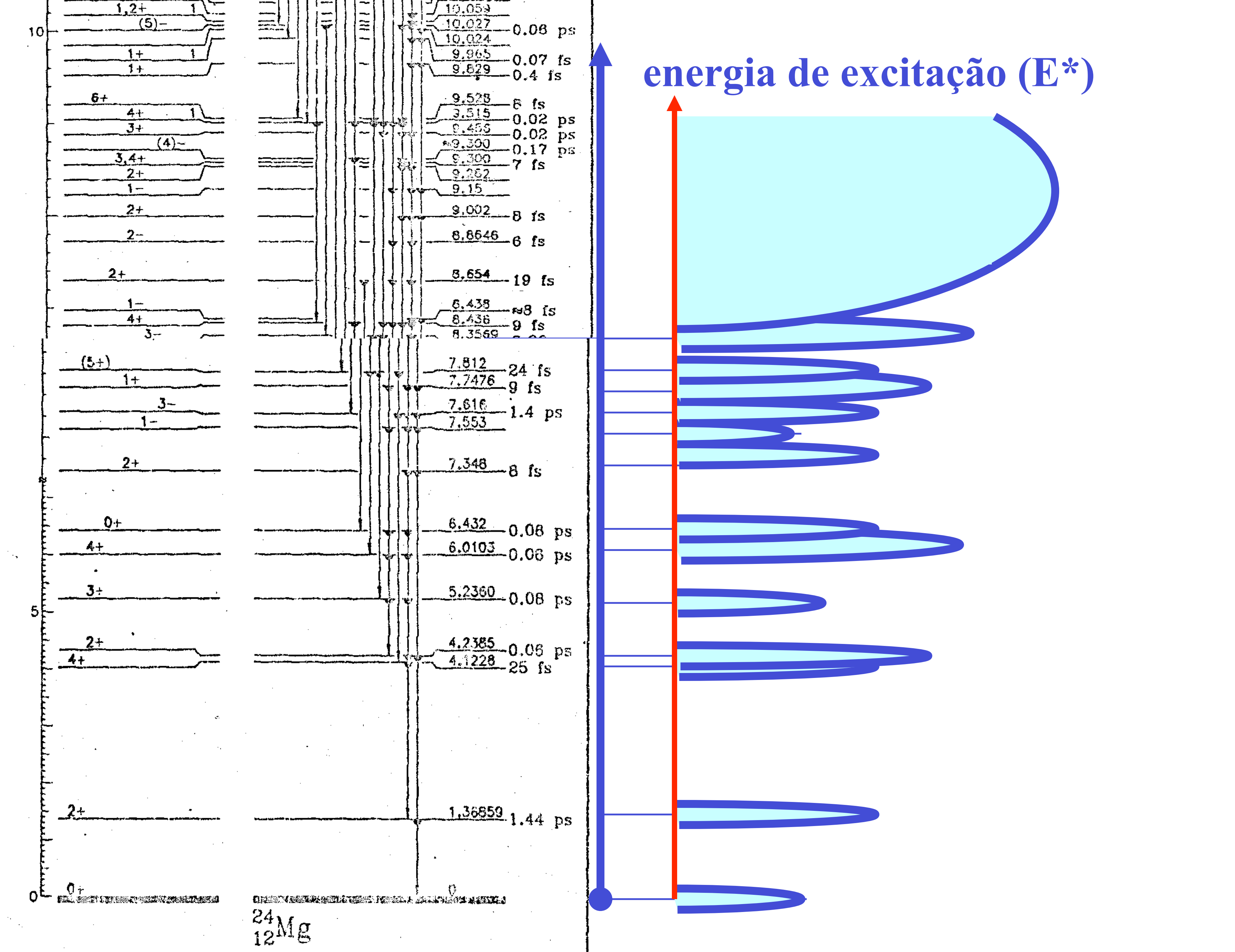
Processos estatísticos (lentos)



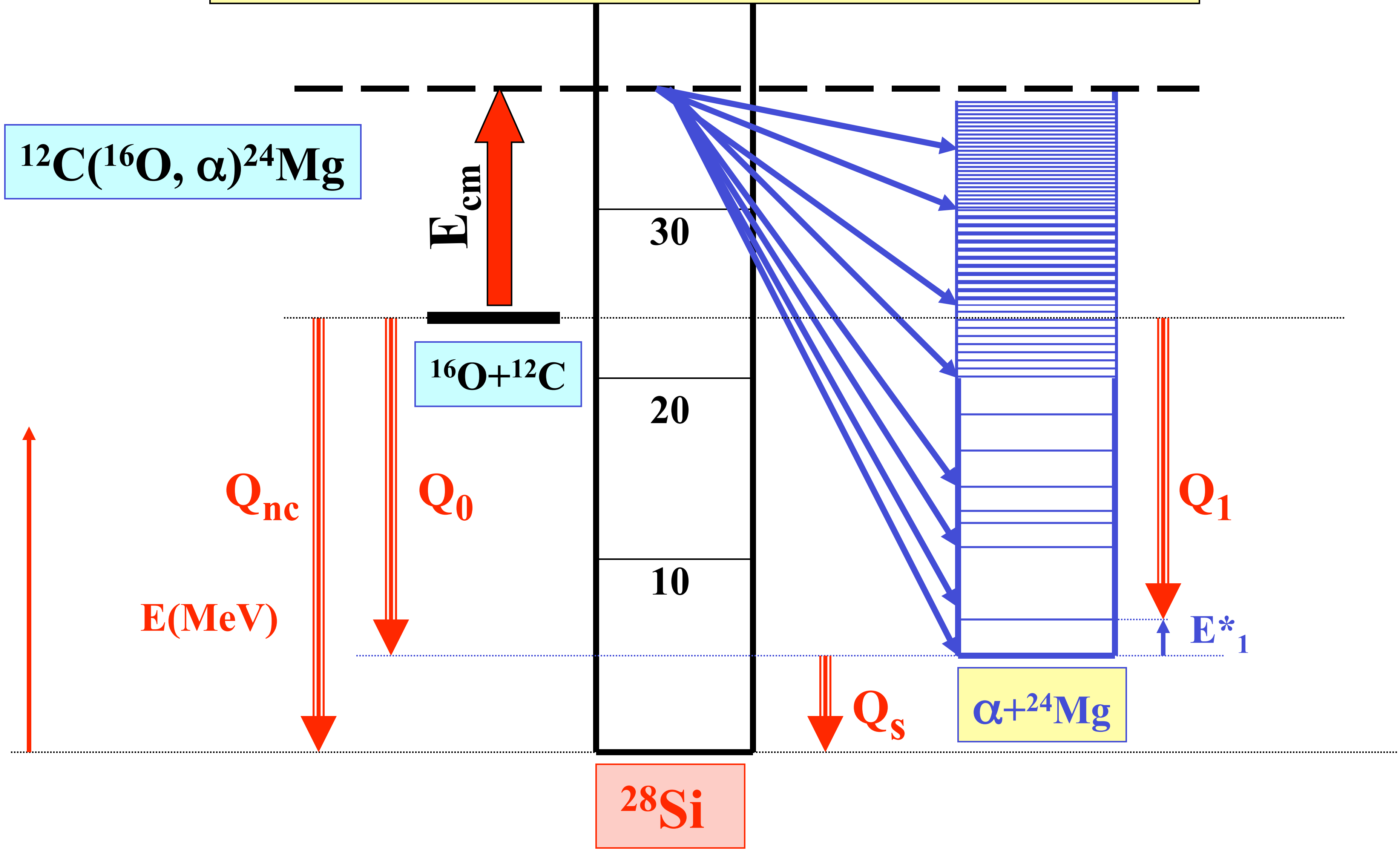
Processos estatísticos (lentos)

REAÇÃO DIRETA




energia de excitação (E*)
$^{24}_{12}Mg$
10
5
0
10.059
10.027 0.08 ps
10.024
9.965 0.07 fs
9.829 0.4 fs
9.528 8 fs
9.515 0.02 ps
9.458 0.02 ps
≈9.300 0.17 ps
9.300 7 fs
9.262
9.15
9.002 8 fs
8.8646 6 fs
8.654 19 fs
8.438 ≈8 fs
8.436 9 fs
8.3569
7.812 24 fs
7.7476 9 fs
7.616 1.4 ps
7.553
7.348 8 fs
6.432 0.08 ps
6.0103 0.08 ps
5.2360 0.08 ps
4.2385 0.06 ps
4.1228 25 fs
1.36859 1.44 ps
0

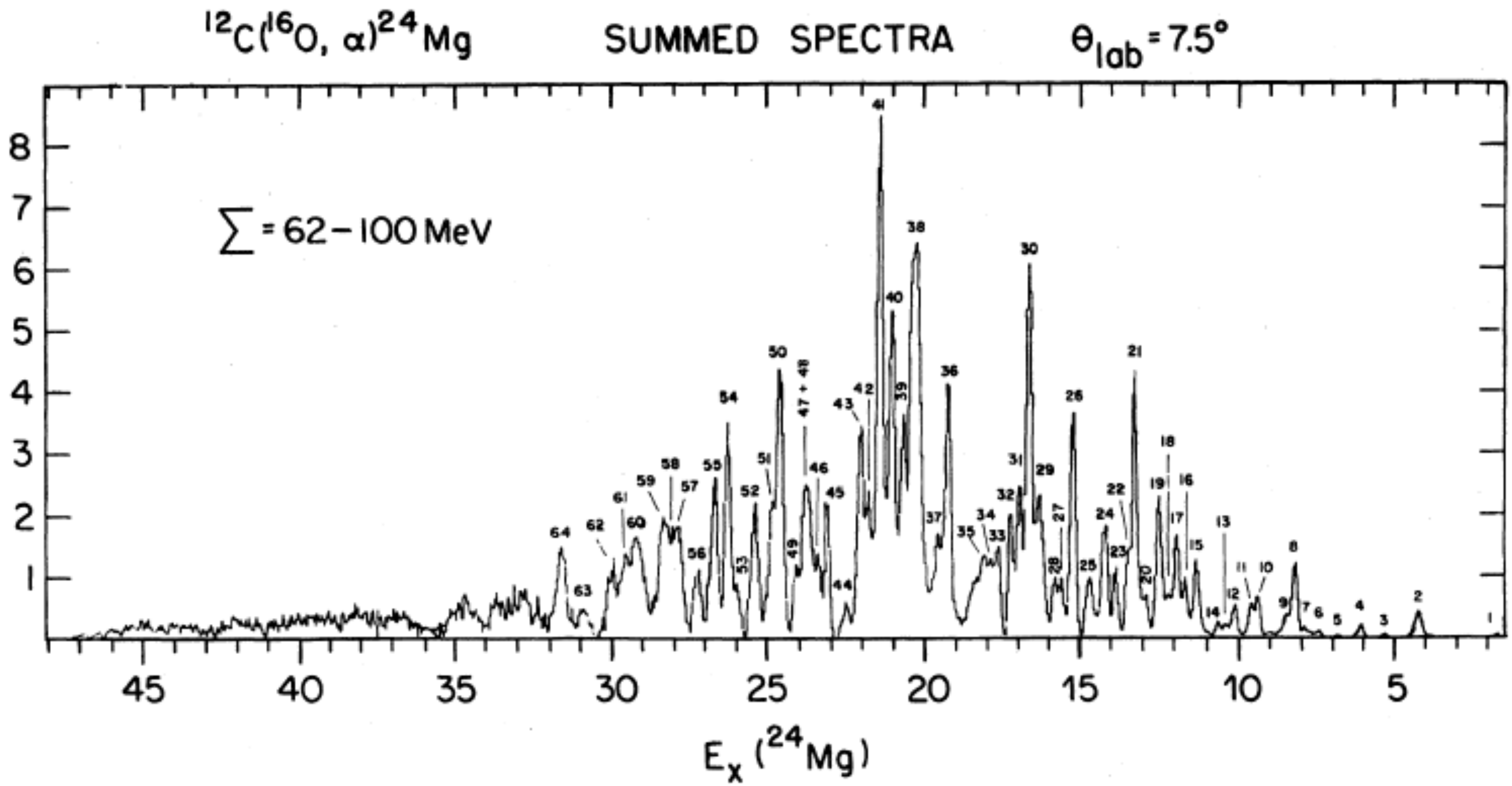
REAÇÃO VIA NÚCLEO COMPOSTO



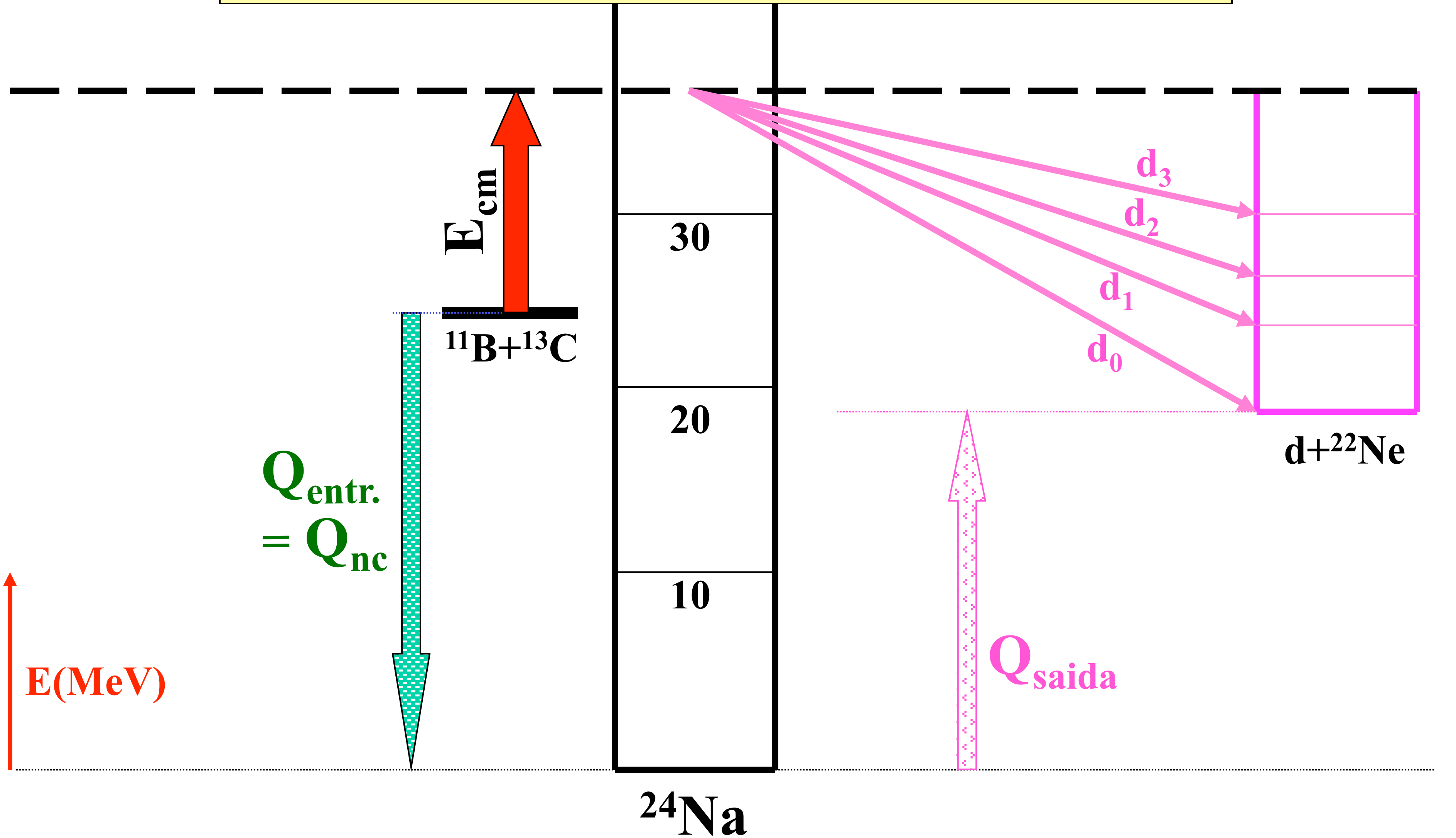
PHYSICAL REVIEW C VOLUME 27, NUMBER 4 APRIL 1983

$^{12}C(^{16}O,\alpha)^{24}Mg^*$ reaction in the energy region $E_{c.m.} = 26.6$ to 42.9 MeV

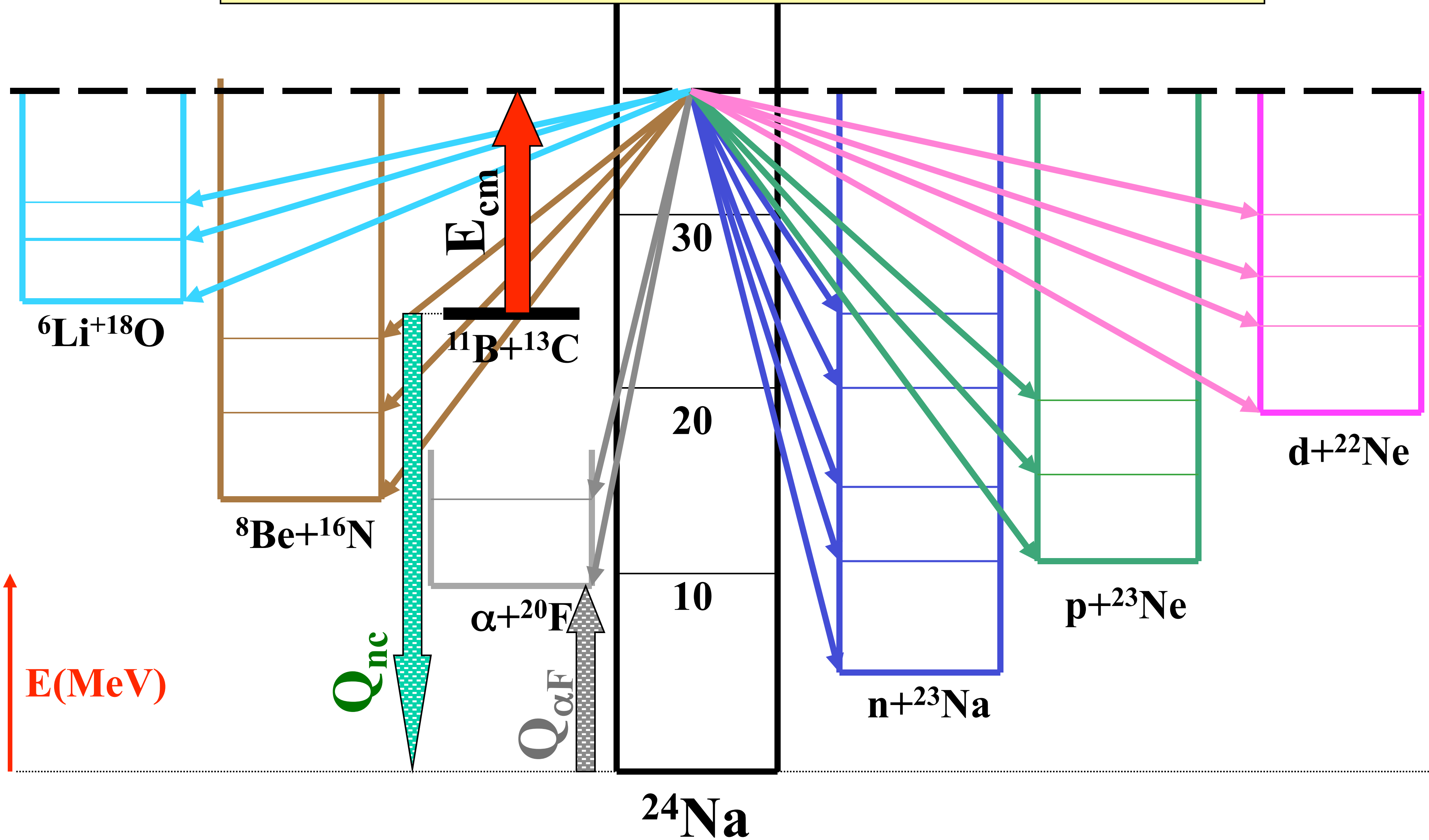
M. J. Bechara,* A. J. Lazzarini,† R. J. Ledoux, and E. R. Cosman
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology,
Cambridge, Massachusetts 02139
(Received 21 December 1981)

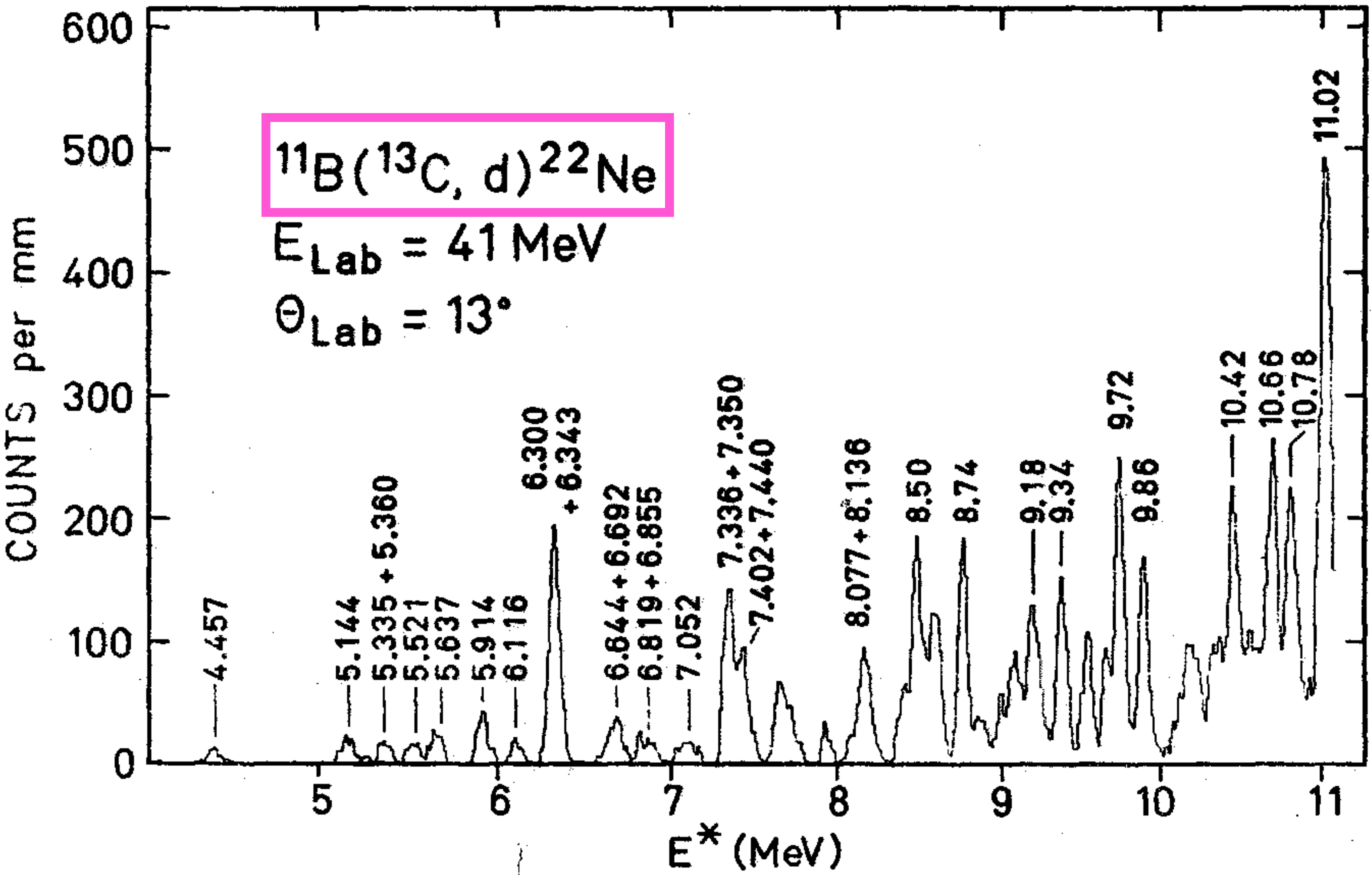


REAÇÃO VIA NÚCLEO COMPOSTO



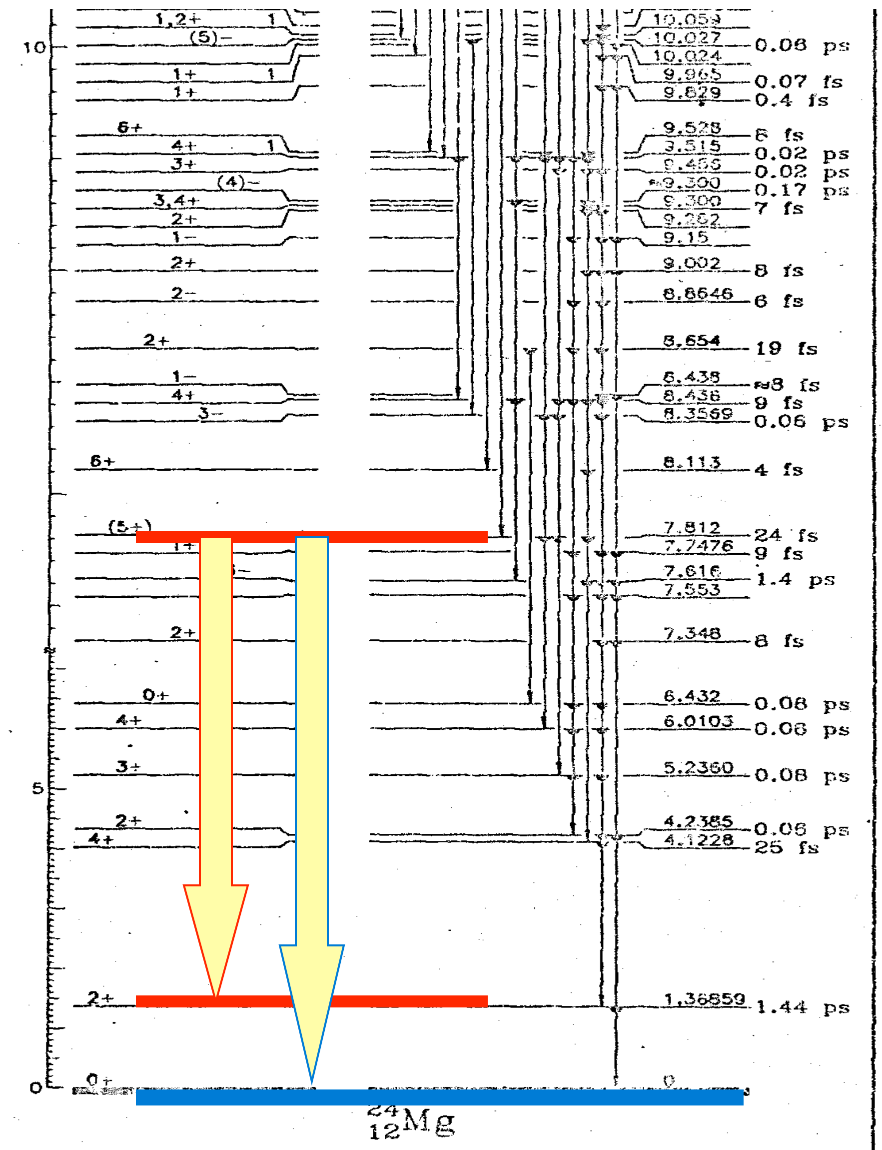
REAÇÕES VIA NÚCLEO COMPOSTO

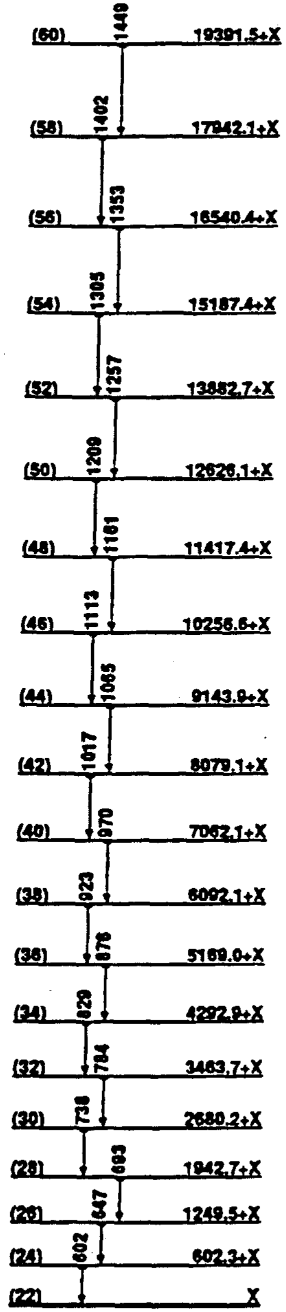




11B(13C, d)22Ne
ELab = 41 MeV
ΘLab = 13°
COUNTS per mm
E* (MeV)
0
100
200
300
400
500
600
5
6
7
8
9
10
11
4.457
5.144
5.335+5.360
5.521
5.637
5.914
6.116
6.300
+6.343
6.644+6.692
6.819+6.855
7.052
7.336+7.350
7.402+7.440
8.077+8.136
8.50
8.74
9.18
9.34
9.72
9.86
10.42
10.66
10.78
11.02

DECAIMENTO GAMMA (γ)





SD BAND [91BE12]

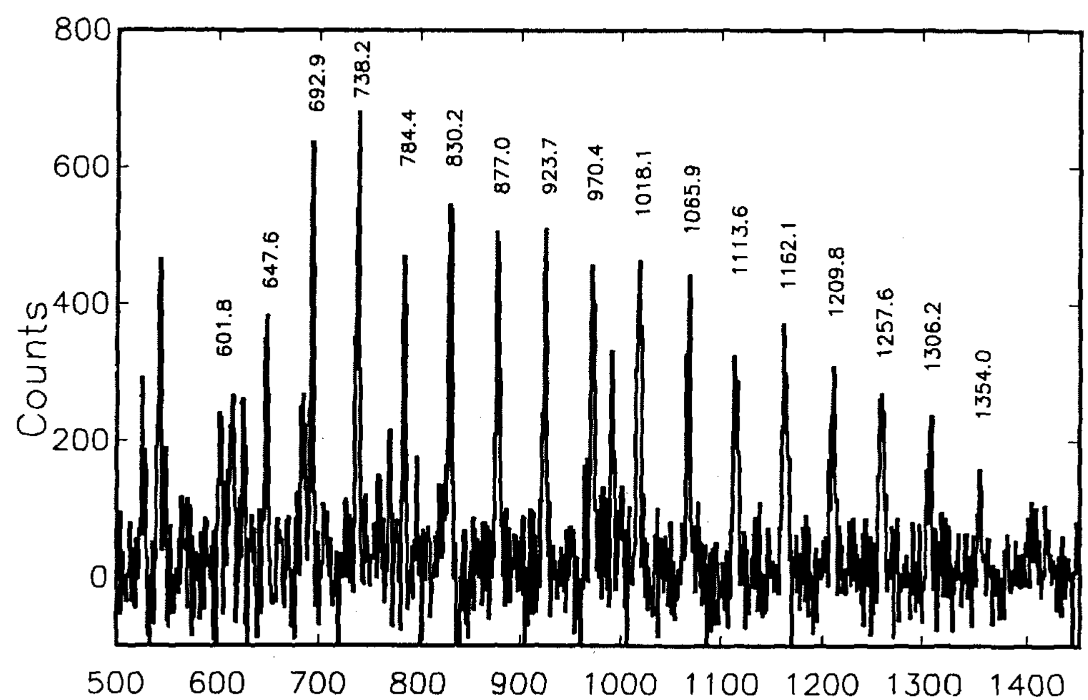
$^{152}_{66}$Dy Rotational Band

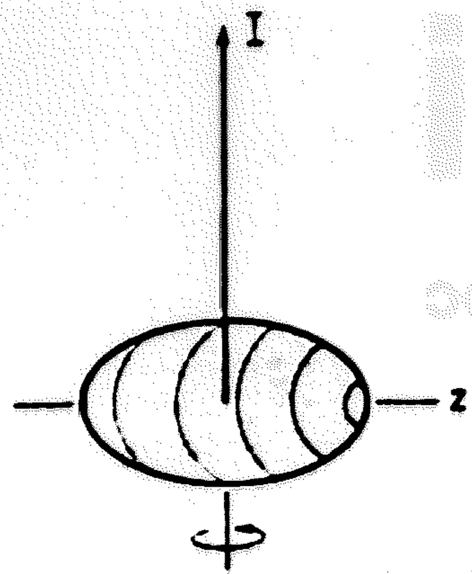
$$E^{\gamma}_{rot} = \frac{\hbar^2}{2\Im}\left[L(L+1)\right]$$

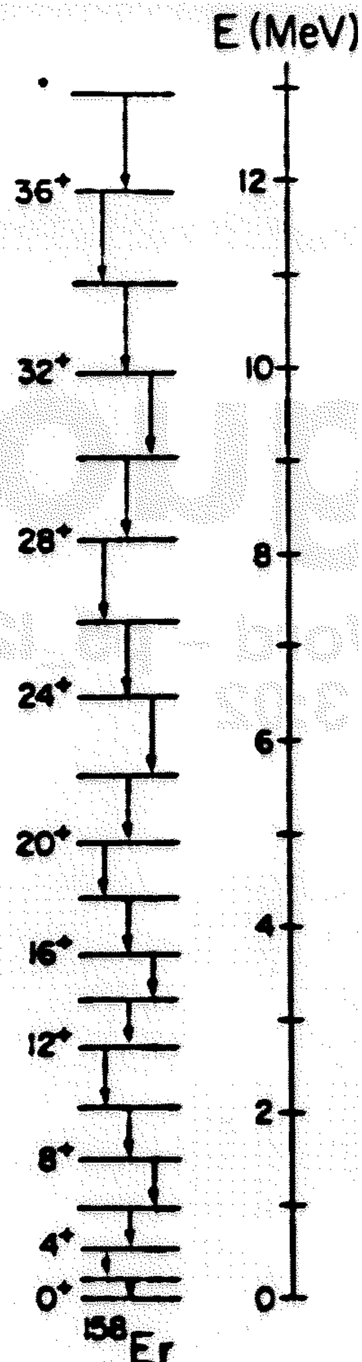
$$E^{\gamma}_{(L+2)\to L} = \frac{\hbar^2}{\Im}(2L+3) = E^{\gamma}_{(L+2)}$$

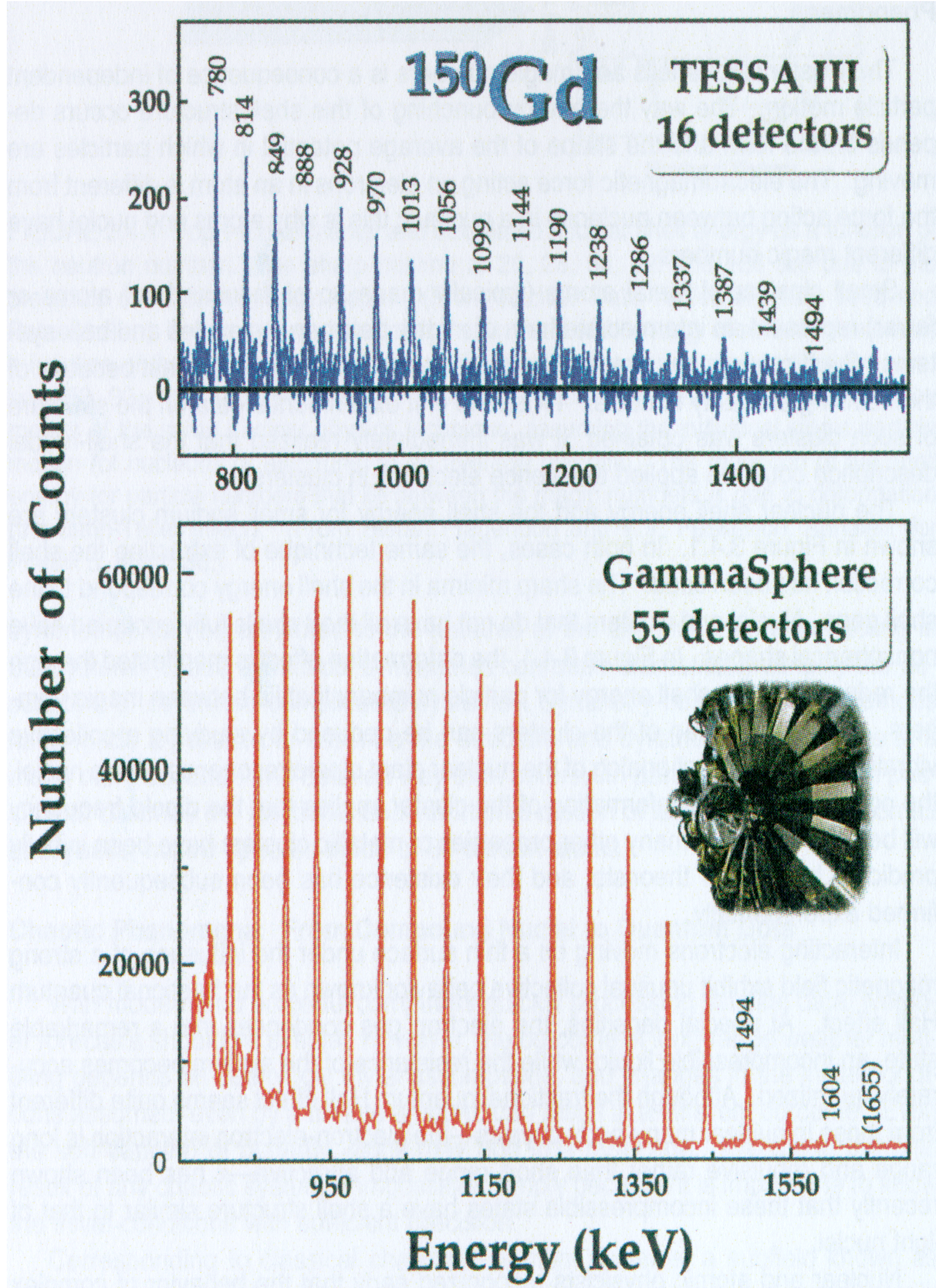
$$\Delta E^{\gamma} = E^{\gamma}_{(L+2)} - E^{\gamma}_{L} = 4\frac{\hbar^2}{\Im}$$

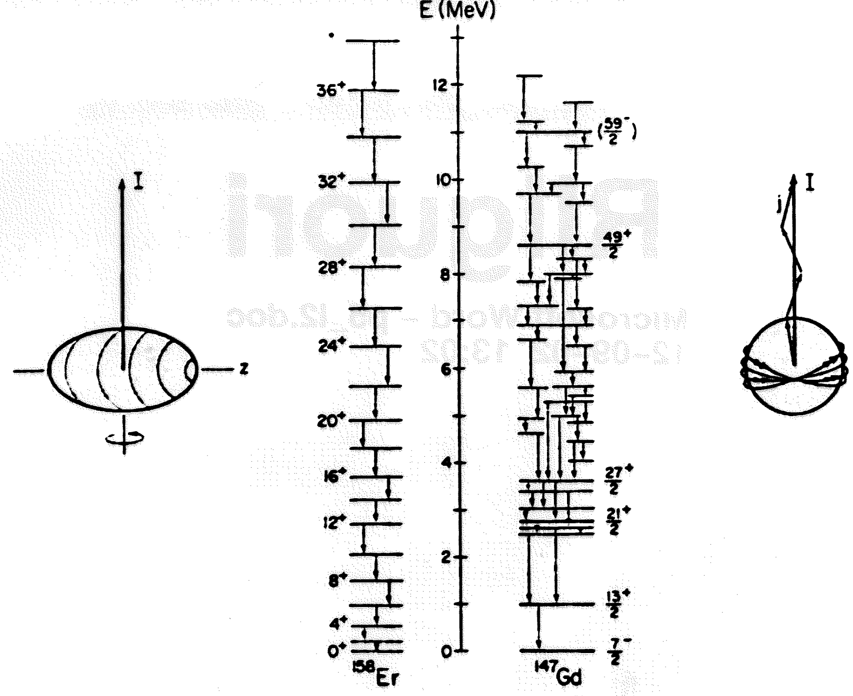
152Dy SD Band

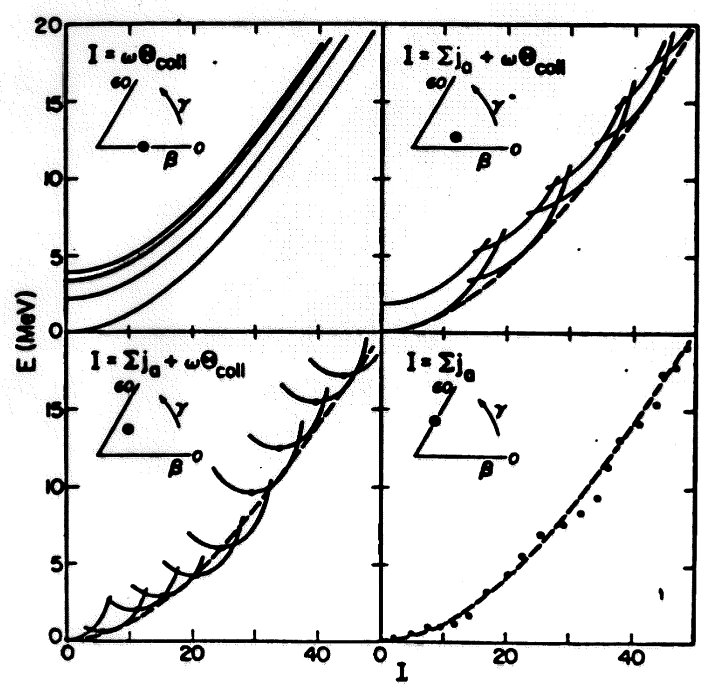


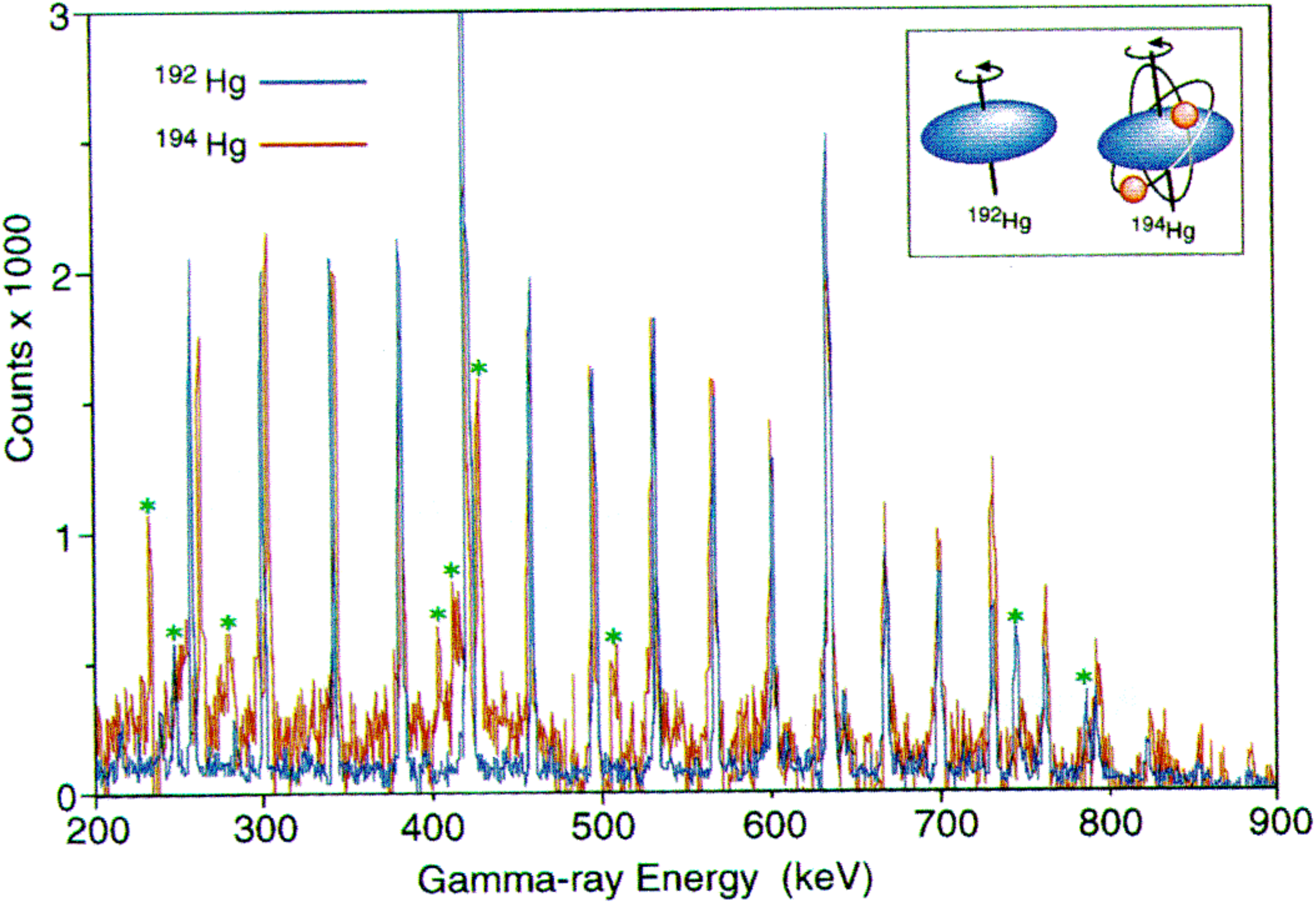

I
z


E (MeV)
36+
32+
28+
24+
20+
16+
12+
8+
4+
0+
158Er
12
10
8
6
4
2
0

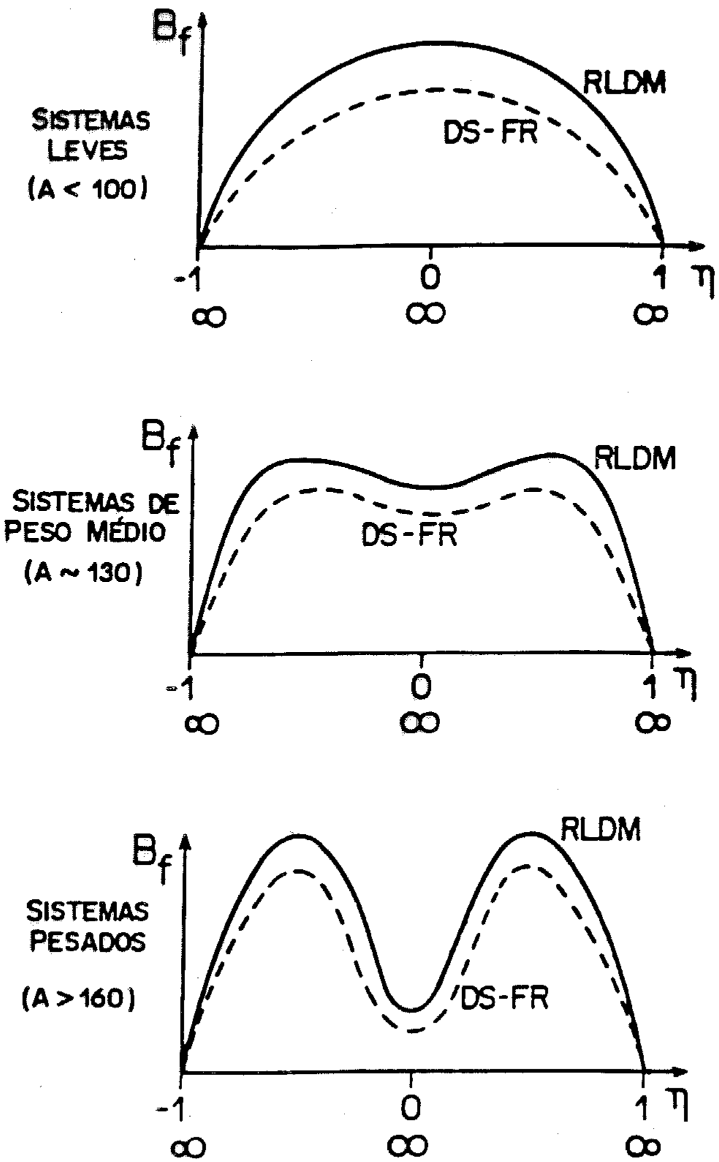

150Gd
TESSA III
16 detectors
780
814
849
888
928
970
1013
1056
1099
1144
1190
1238
1286
1337
1387
1439
1494
300
200
100
0
800
1000
1200
1400
GammaSphere
55 detectors
60000
40000
20000
0
1494
1604
(1655)
950
1150
1350
1550
Number of Counts
Energy (keV)


E (MeV)
I
z
j
36+
32+
28+
24+
20+
16+
12+
8+
4+
0+
158Er
147Gd
(59-/2)
49+/2
27+/2
21+/2
13+/2
7-/2
0
2
4
6
8
10
12

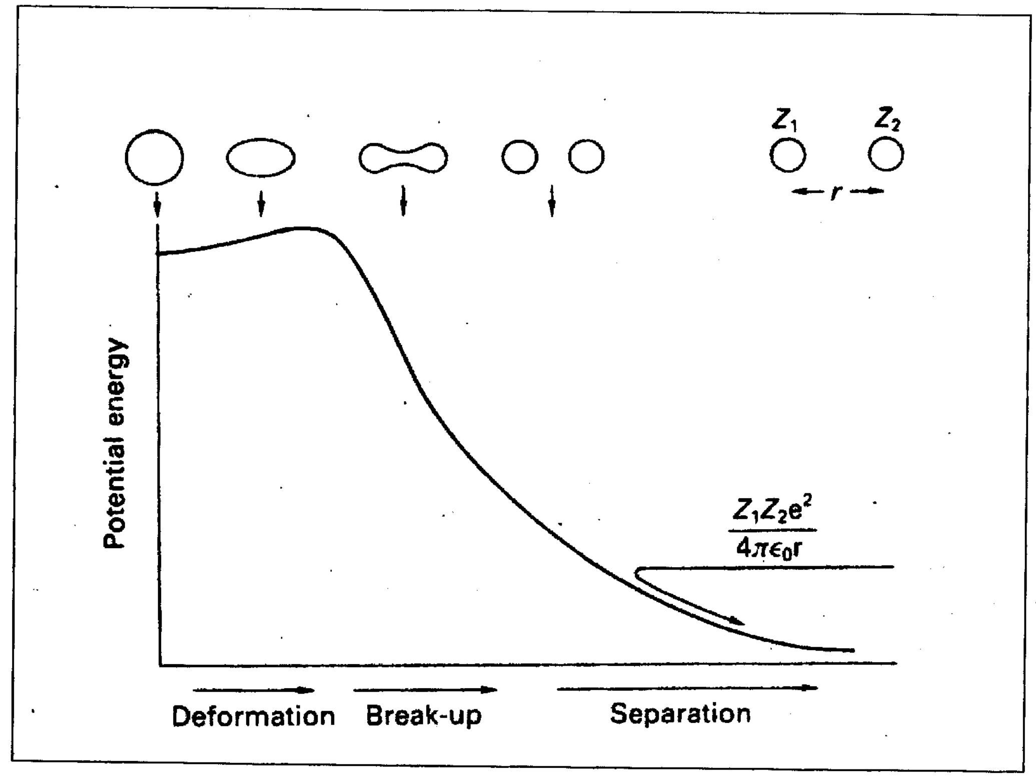

I = ωΘcoll
I = Σja + ωΘcoll
I = Σja + ωΘcoll
I = Σja
60
γ
β
0
E (MeV)
I
0
20
40
5
10
15
20

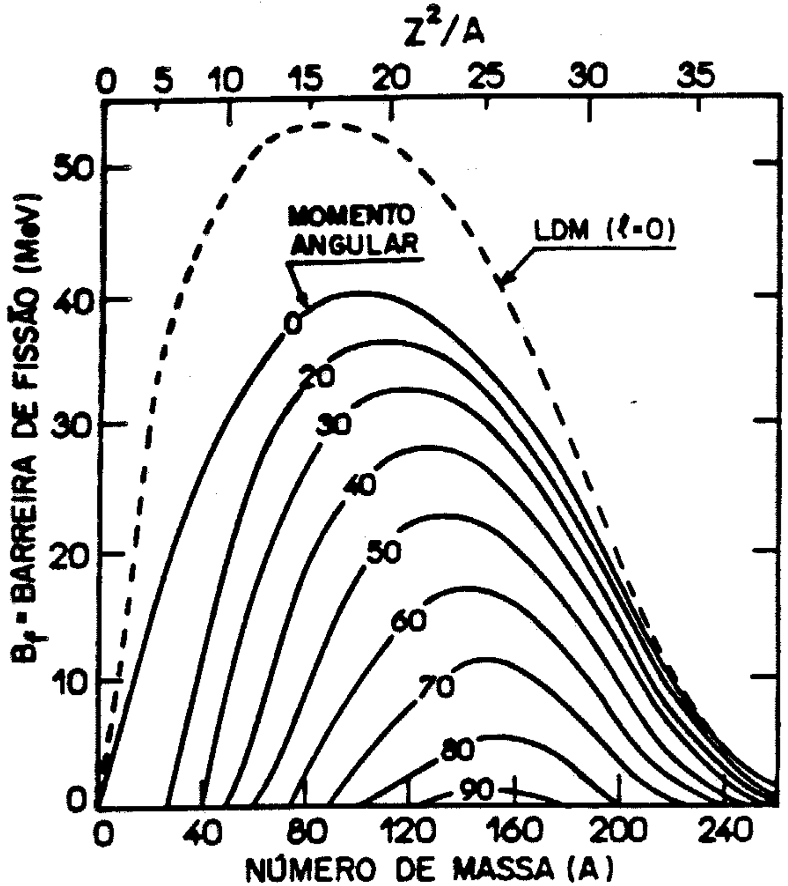


192 Hg
194 Hg
192Hg
194Hg
Counts x 1000
Gamma-ray Energy (keV)
0
1
2
3
200
300
400
500
600
700
800
900

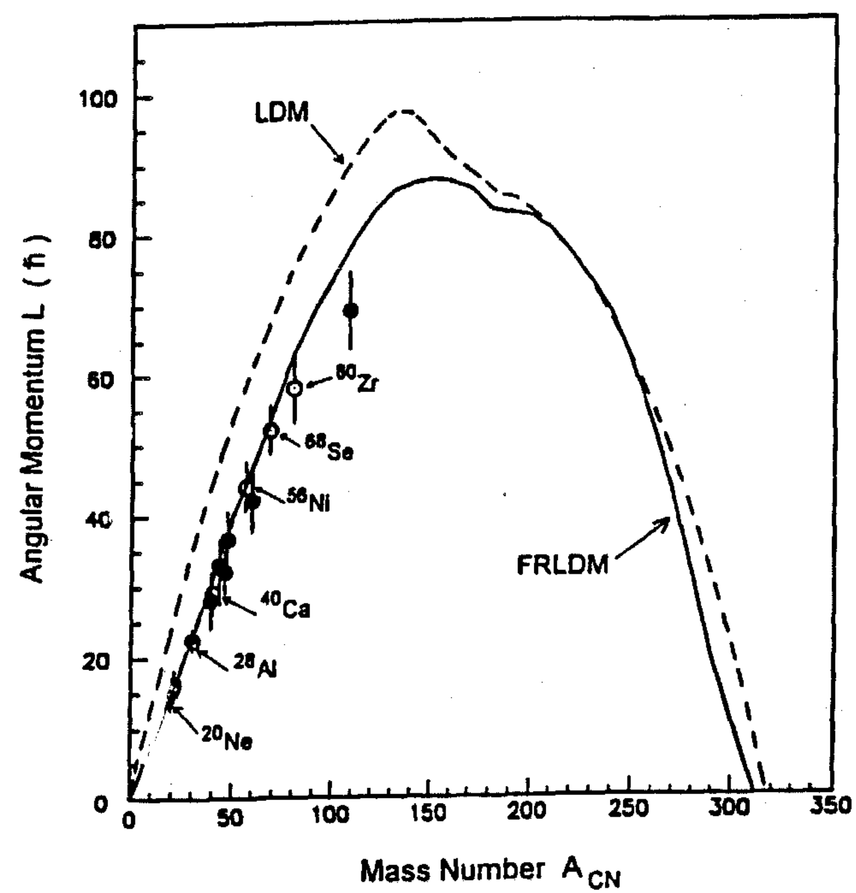


$$\eta = \frac{A_1 - A_2}{A_1 - A_2}$$





Z²/A
0 5 10 15 20 25 30 35
MOMENTO ANGULAR
LDM (ℓ=0)
0
20
30
40
50
60
70
80
90
B_f = BARREIRA DE FISSÃO (MeV)
50
40
30
20
10
0
0 40 80 120 160 200 240
NÚMERO DE MASSA (A)



LDM
FRLDM
80Zr
68Se
56Ni
40Ca
28Al
20Ne
Angular Momentum L (ħ)
100
80
60
40
20
0
0 50 100 150 200 250 300 350
Mass Number A CN

momentos eletromagneticos nucleares

Para um eletron orbitando em uma orbita de raio r e area A

$$|\mu| = iA$$

$$|\mu| = \frac{e}{(2\pi r)/v}\pi r^2 = \frac{evr}{2}$$

quanticamente

Como $|\ell| = r.p = r.m.v$
$r = |\ell|/mv$

$$|\mu| = \frac{e}{2m}|\ell|$$

ou

$$|\mu| = \frac{e\hbar}{2m}|\ell|$$

$$\frac{e\hbar}{2m} \equiv \textbf{magneton}$$

$\mu_B = 5.7884 \times 10^{-5}$ eV/T

$\mu_N = 3.1525 \times 10^{-8}$ eV/T

reescrevendo $\mu = g_\ell \, \ell \, \mu_N$

$g_\ell = 1$ *para protons*
$g_\ell = 0$ *para neutrons*

o calculo de $\mathbf{g}_{\boldsymbol{\ell}}$ *considera exclusivamente o momento angular orbital. No caso do momento angular intrínseco (spin) s*

$$\mu = g_s s\, \mu_N$$ *onde s= ½ para protons, neutrons e elétrons*

Previsões de Dirac para $g_s = 2$

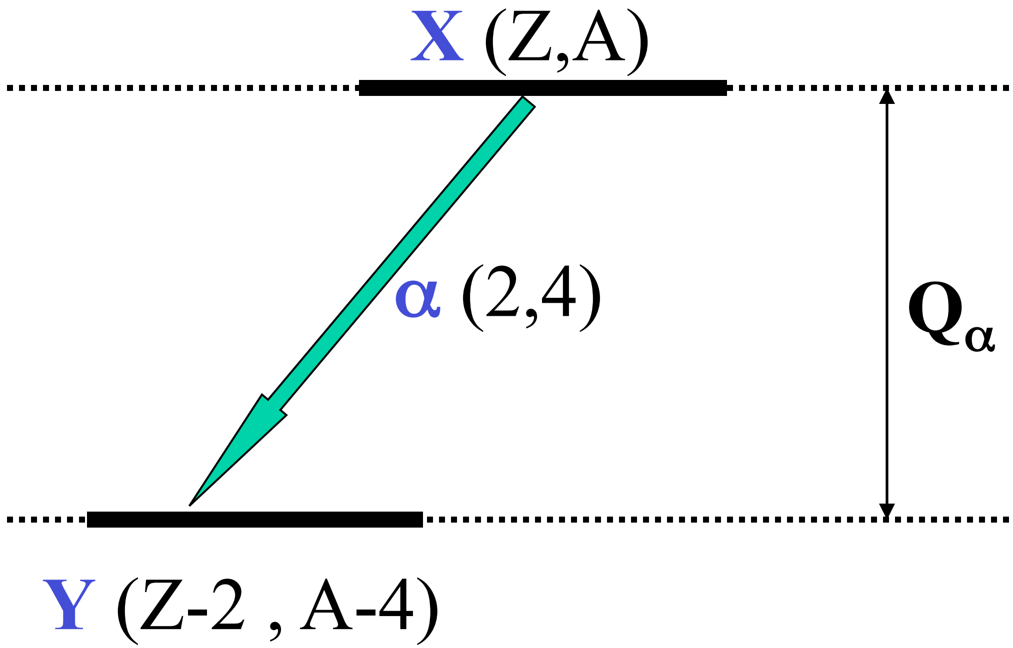
Valores experimentais: $\mathbf{g_s}$ **(elétron)** = 2.0023

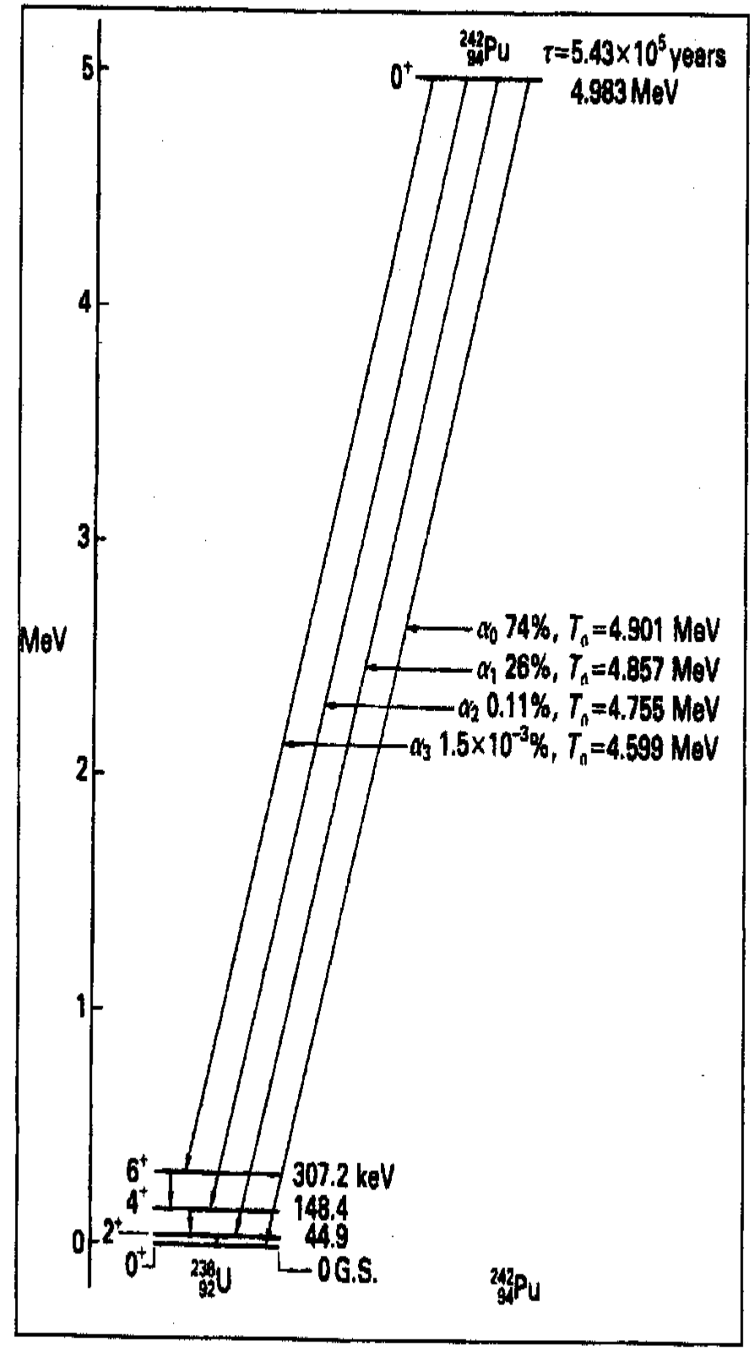
$\mathbf{g_s}$ **(proton)** = 5.5856912

$\mathbf{g_s}$ **(neutron)** = **-**3.8260837

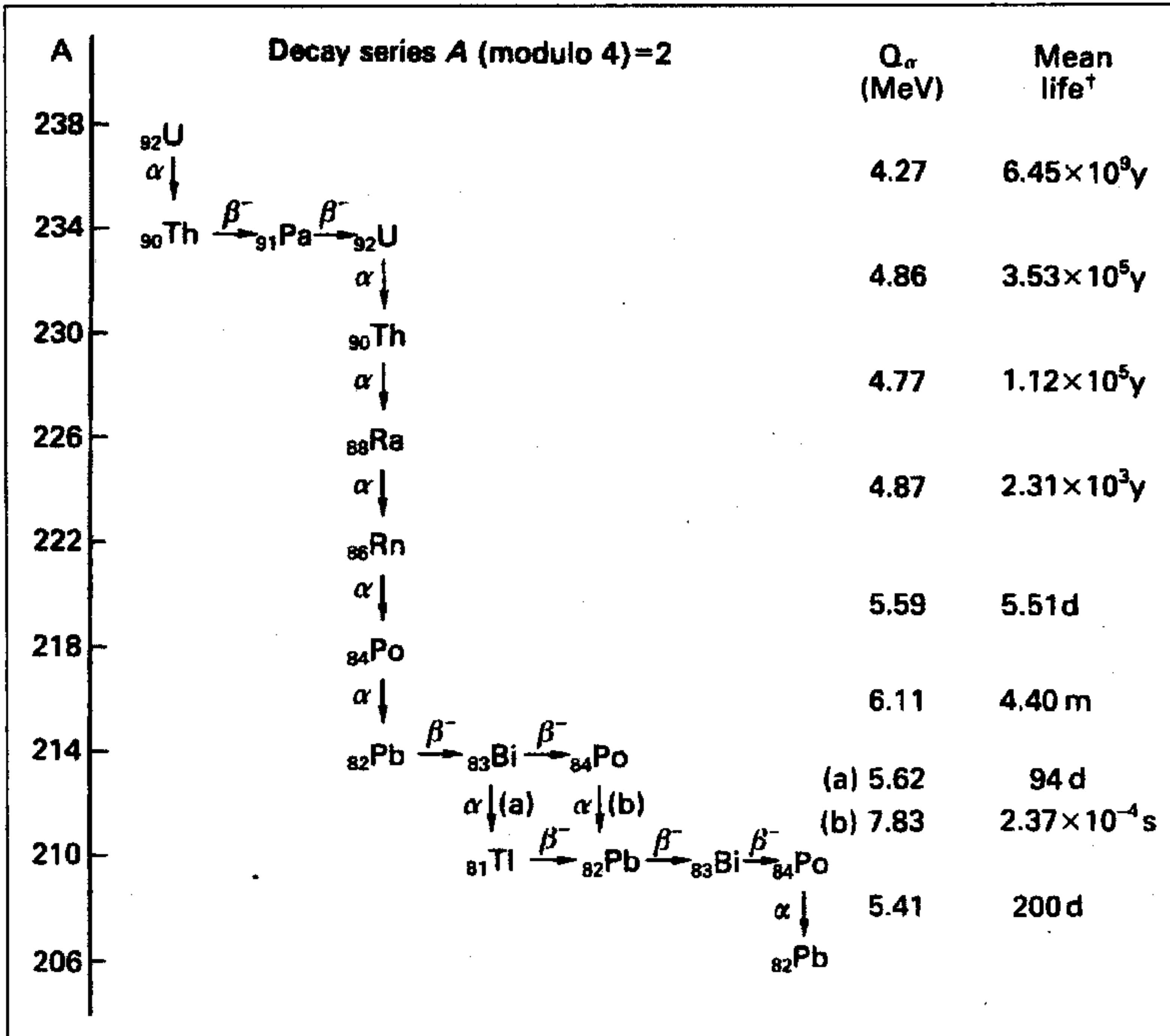
Note que $\mathbf{g_s}$ **(neutron) ≠ 0**

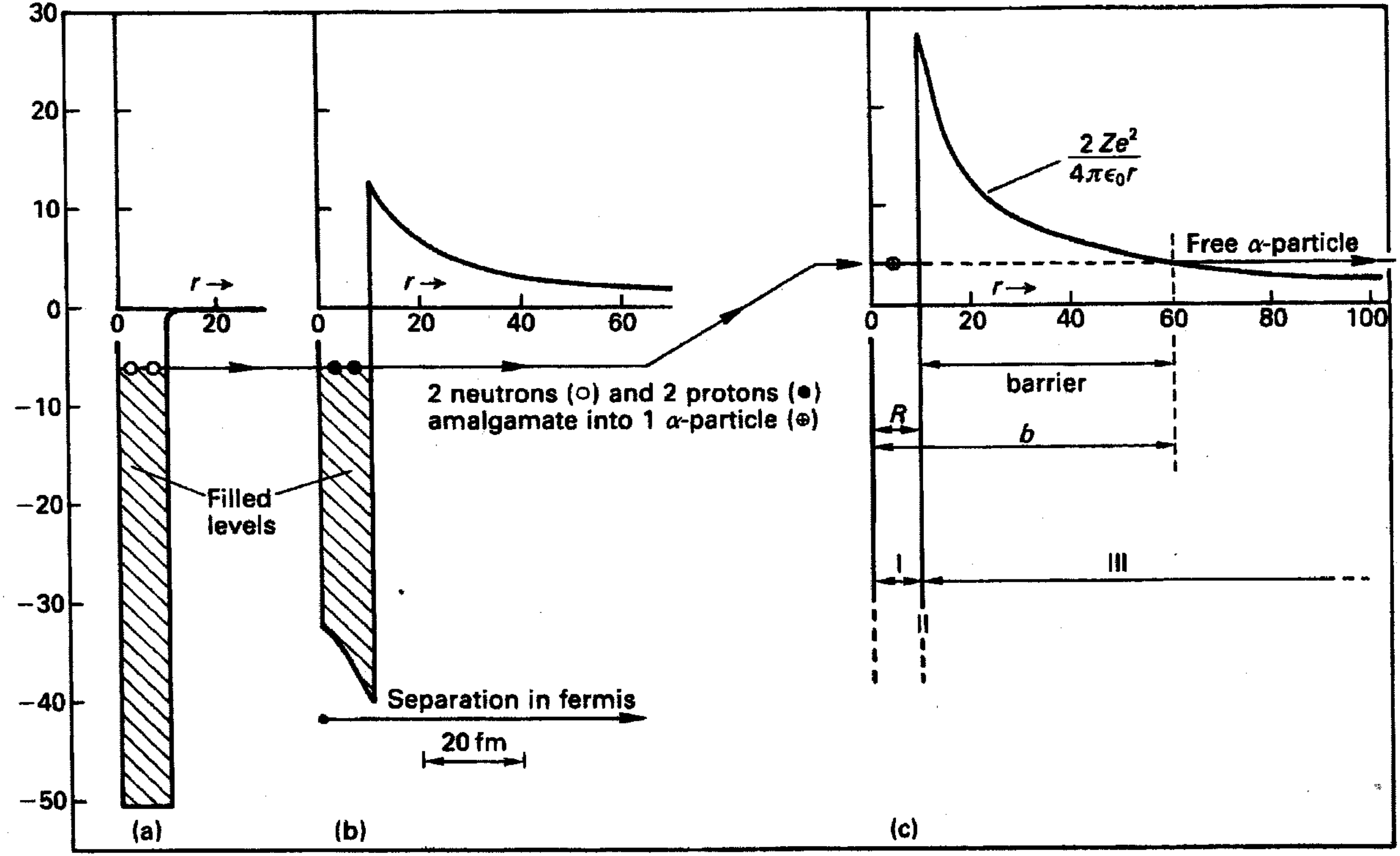
decaimento radiativo





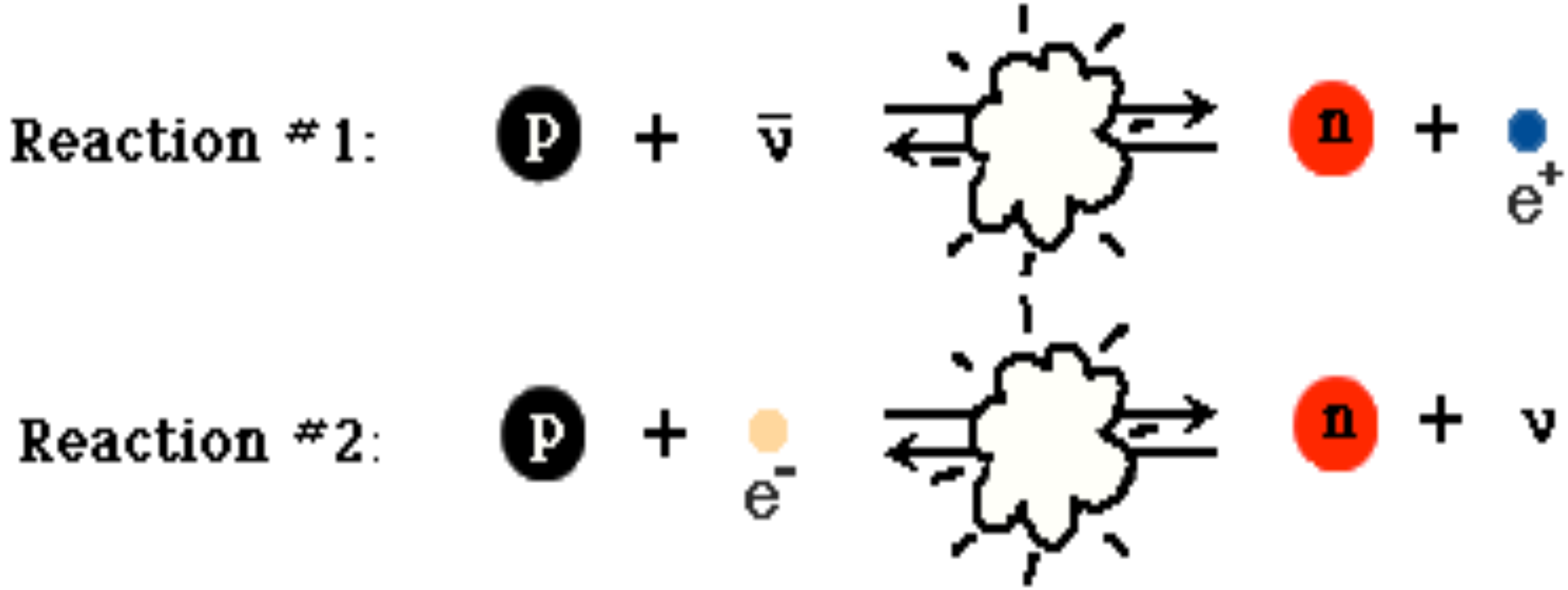
DECAIMENTO ALPHA (α)


A
Decay series A (modulo 4)=2
Q_α (MeV)
Mean life†
238
234
230
226
222
218
214
210
206
$_{92}U$
α
4.27
6.45×10^{9}y
$_{90}Th$ $\xrightarrow{\beta^-}$ $_{91}Pa$ $\xrightarrow{\beta^-}$ $_{92}U$
α
4.86
3.53×10^{5}y
$_{90}Th$
α
4.77
1.12×10^{5}y
$_{88}Ra$
α
4.87
2.31×10^{3}y
$_{86}Rn$
α
5.59
5.51 d
$_{84}Po$
α
6.11
4.40 m
$_{82}Pb$ $\xrightarrow{\beta^-}$ $_{83}Bi$ $\xrightarrow{\beta^-}$ $_{84}Po$
(a) 5.62
94 d
α (a)
α (b)
(b) 7.83
2.37×10^{-4}s
$_{81}Tl$ $\xrightarrow{\beta^-}$ $_{82}Pb$ $\xrightarrow{\beta^-}$ $_{83}Bi$ $\xrightarrow{\beta^-}$ $_{84}Po$
α
5.41
200 d
$_{82}Pb$



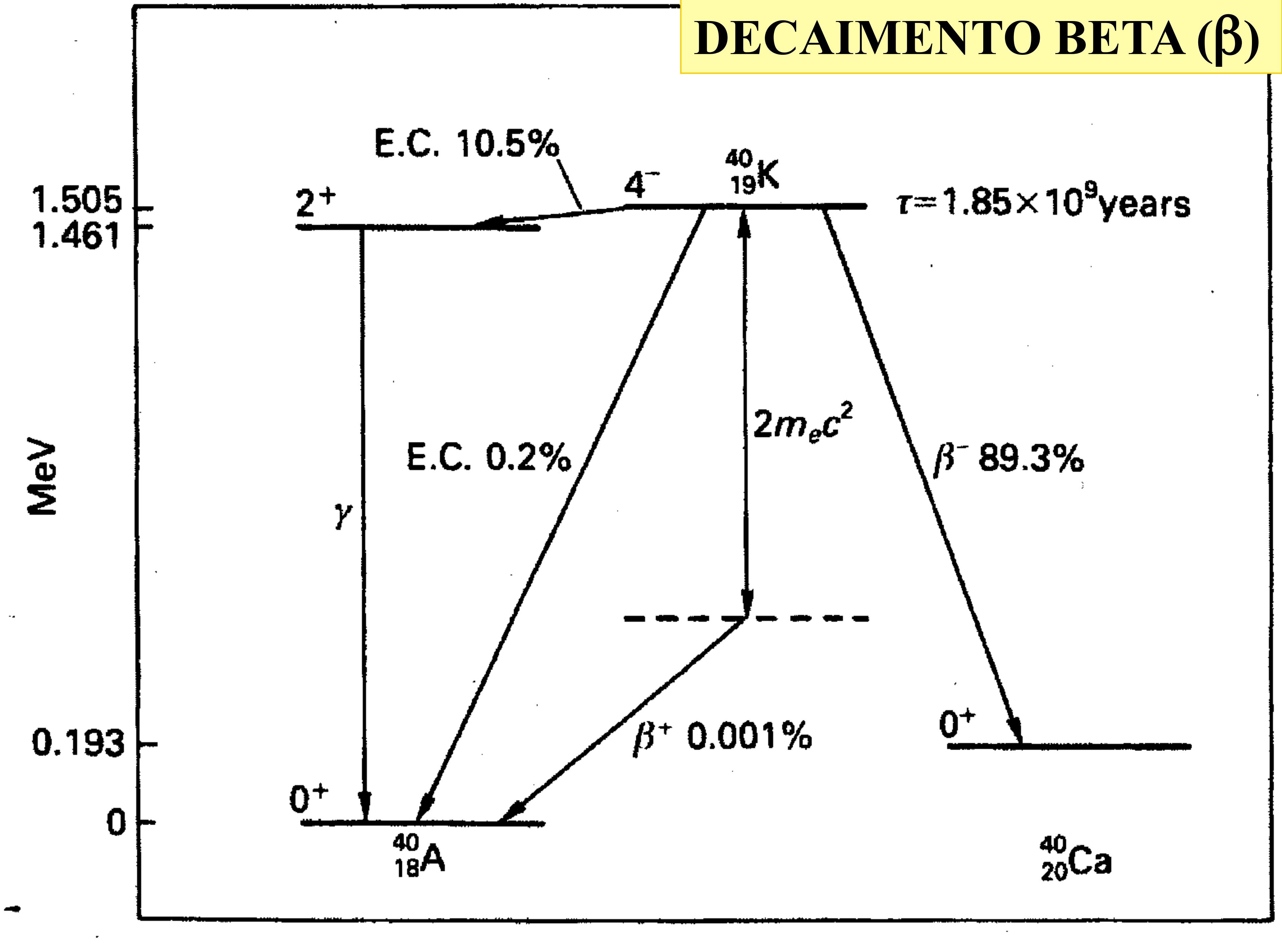
30
20
10
0
−10
−20
−30
−40
−50
r →
0
20
40
60
80
100
Filled levels
2 neutrons (○) and 2 protons (●)
amalgamate into 1 α-particle (⊕)
Separation in fermis
20 fm
$\frac{2Ze^2}{4\pi\epsilon_0 r}$
Free α-particle
barrier
R
b
I
II
III
(a)
(b)
(c)

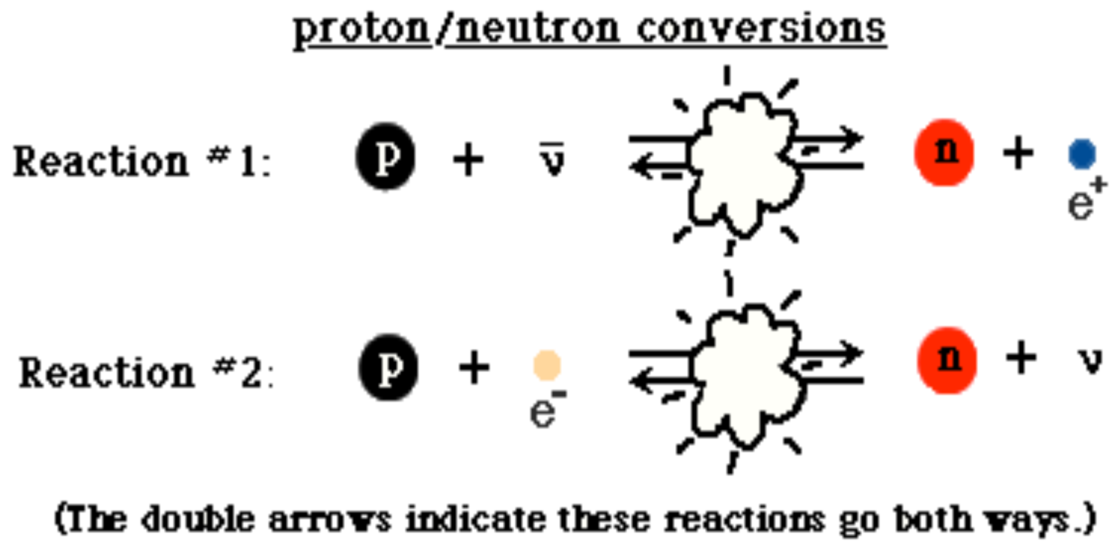
proton/neutron conversions

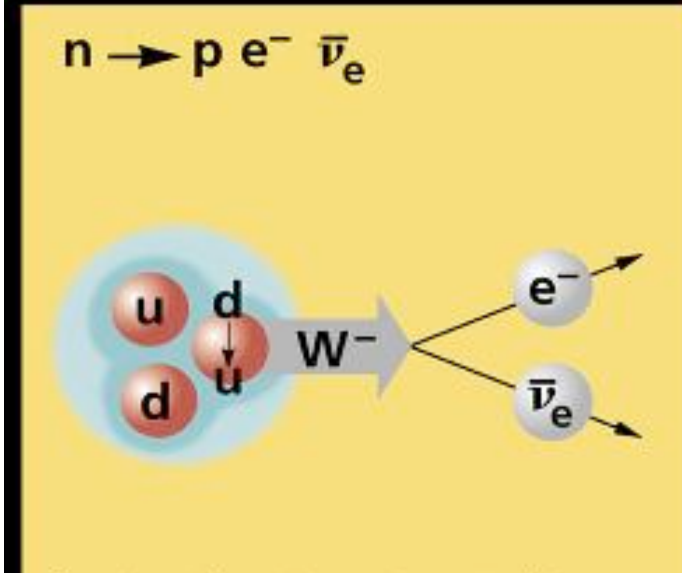


(The double arrows indicate these reactions go both ways.)

DECAIMENTO BETA (β)







A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

BOSONS

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c²	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c²	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	−1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	−1	1.672	3/2

BOSONS

force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c²	Electric charge
g gluon	0	0

Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c²	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	−1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

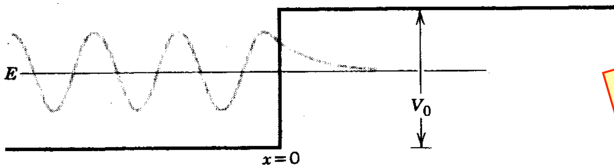


Figure 2.3 The wave function of a particle of energy E encountering a step of height V_0, for the case $E < V_0$. The wave function decreases exponentially in the classically forbidden region, where the classical kinetic energy would be negative. At $x = 0$, ψ and $d\psi/dx$ are continuous.

the classically forbidden region. All (classical) *particles* are reflected at the boundary; the quantum mechanical *wave packet*, on the other hand, can penetrate a short distance into the forbidden region. The (classical) particle is never directly *observed* in that region; since $E < V_0$, the kinetic energy would be negative in region 2. The solution is illustrated in Figure 2.3

Barrier Potential, $E > V_0$

The potential is

$$\begin{aligned} V(x) &= 0 & x &< 0 \\ &= V_0 & 0 &\le x \le a \\ &= 0 & x &> a \end{aligned} \tag{2.35}$$

In the three regions 1, 2, and 3, the solutions are

$$\begin{aligned} \psi_1 &= A e^{ik_1 x} + B e^{-ik_1 x} \\ \psi_2 &= C e^{ik_2 x} + D e^{-ik_2 x} \\ \psi_3 &= F e^{ik_3 x} + G e^{-ik_3 x} \end{aligned} \tag{2.36}$$

where $k_1 = k_3 = \sqrt{2mE/\hbar^2}$ and $k_2 = \sqrt{2m(E - V_0)/\hbar^2}$.

Using the continuity conditions at $x = 0$ and at $x = a$, and assuming again that particles are incident from $x = -\infty$ (so that G can be set to zero), after considerable algebraic manipulation we can find the transmission coefficient $T = |F|^2/|A|^2$:

$$T = \frac{1}{1 + \dfrac{1}{4}\dfrac{V_0^2}{E(E - V_0)}\sin^2 k_2 a} \tag{2.37}$$

The solution is illustrated in Figure 2.4.

Barrier Potential, $E < V_0$

For this case, the ψ_1 and ψ_3 solutions are as above, but ψ_2 becomes

$$\psi_2 = C e^{k_2 x} + D e^{-k_2 x} \tag{2.38}$$

where now $k_2 = \sqrt{2m(V_0 - E)/\hbar^2}$. Because region 2 extends only from $x = 0$ to $x = a$, the question of an exponential solution going to infinity does not arise, so we cannot set C or D to zero.

Again, applying the boundary conditions at $x = 0$ and $x = a$ permits the solution for the transmission coefficient:

$$T = \frac{1}{1 + \dfrac{1}{4}\dfrac{V_0^2}{E(V_0 - E)}\sinh^2 k_2 a} \tag{2.39}$$

Classically, we would expect $T = 0$—the particle is not permitted to enter the forbidden region where it would have negative kinetic energy. The quantum wave can penetrate the barrier and give a nonzero probability to find the particle beyond the barrier. The solution is illustrated in Figure 2.5.

This phenomenon of *barrier penetration* or quantum mechanical *tunneling* has important applications in nuclear physics, especially in the theory of α decay, which we discuss in Chapter 8.

ref. Krane

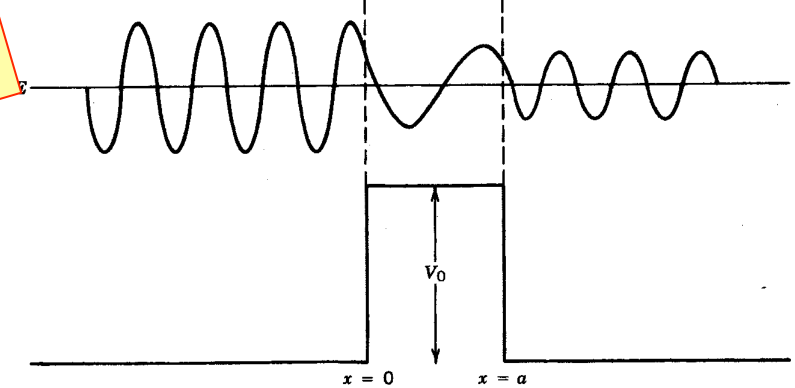


Figure 2.4 The wave function of a particle of energy $E > V_0$ encountering a barrier potential. The particle is incident from the left. The wave undergoes reflections at both boundaries, and the transmitted wave emerges with smaller amplitude.

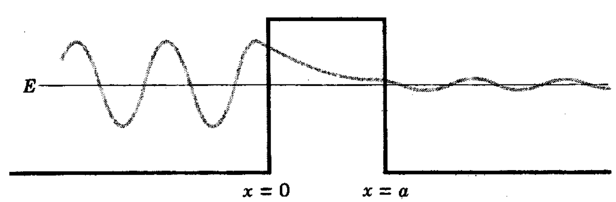
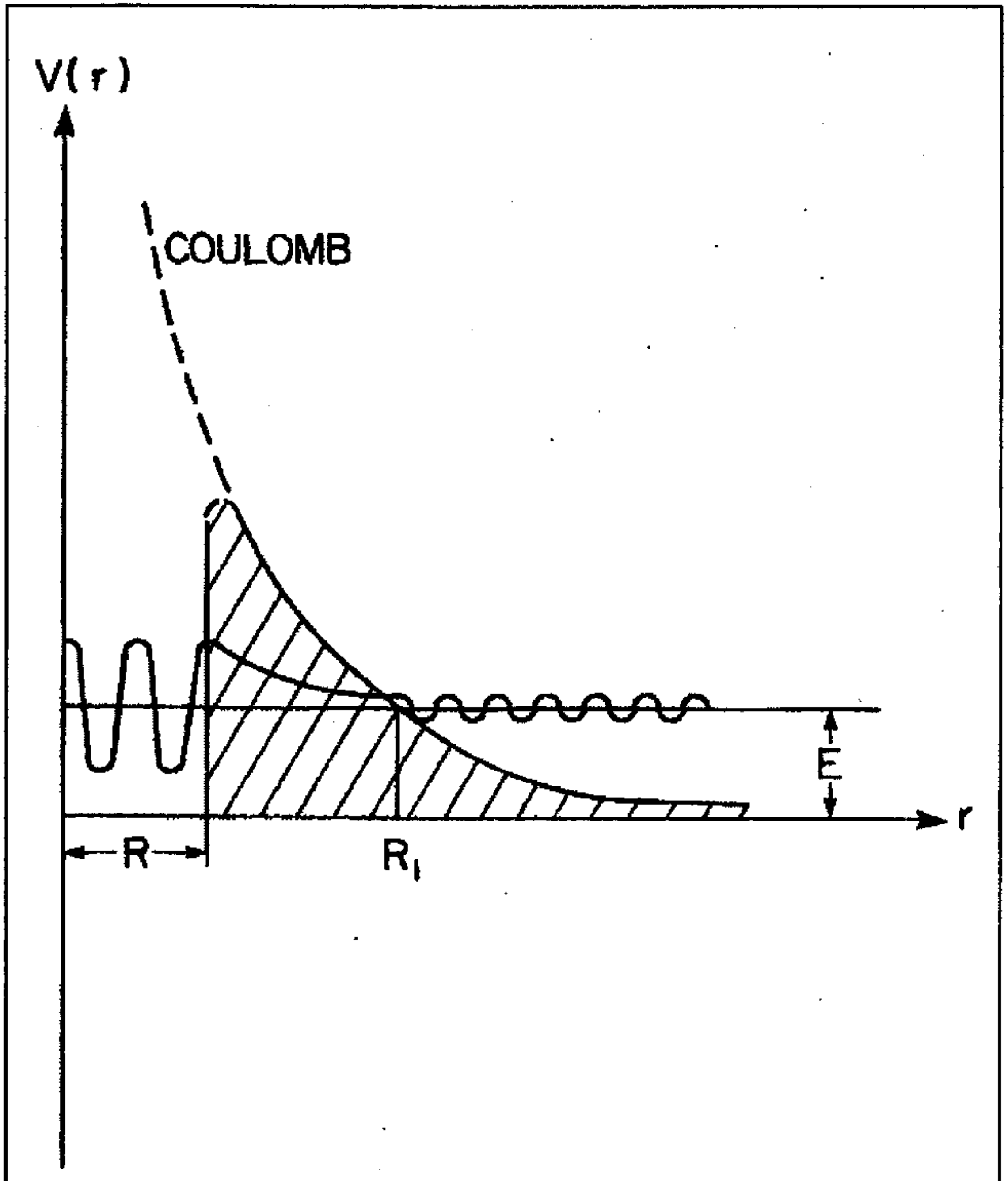
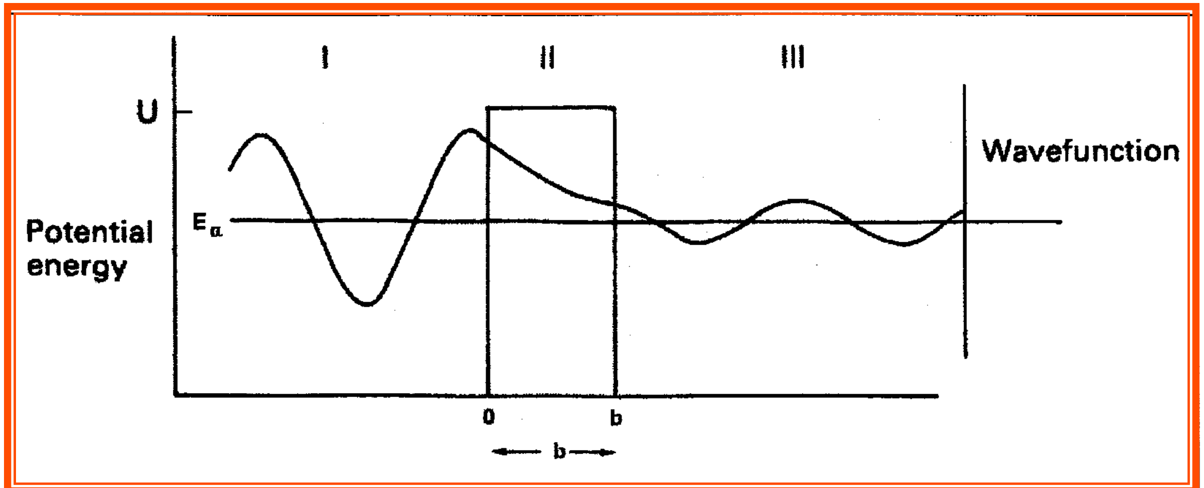


Figure 2.5 The wave function of a particle of energy $E < V_0$ encountering a barrier potential (the particle would be incident from the left in the figure). The wavelength is the same on both sides of the barrier, but the amplitude beyond the barrier is much less than the original amplitude. The particle can never be observed, inside the barrier (where it would have negative kinetic energy) but it can be observed *beyond* the barrier.


V(r)
COULOMB
E
r
R
R_1

Barrier Potential, $E > V_0$



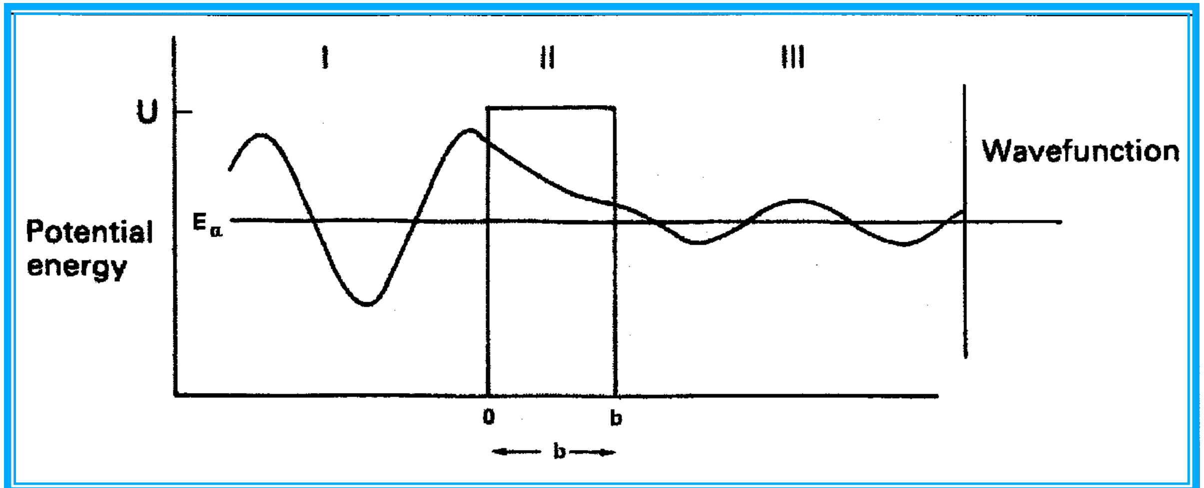
$$\psi_1 = A e^{ik_1 x} + B e^{-ik_1 x}$$

$$\psi_2 = C e^{ik_2 x} + D e^{-ik_2 x}$$

$$\psi_3 = F e^{ik_3 x} + \cancel{G} e^{-ik_3 x}$$

$$k_1 = k_3 = \sqrt{2mE/\hbar^2} \quad \text{and} \quad k_2 = \sqrt{2m(E - V_0)/\hbar^2}$$

Barrier Potential, $E < V_0$



$$T = |F|^2/|A|^2$$

$$T = \frac{1}{1 + \frac{1}{4}\frac{V_0^2}{E(E - V_0)}\sin^2 k_2 a}$$

$$\psi_1 = A e^{ik_1 x} + B e^{-ik_1 x}$$

$$\psi_2 = C e^{k_2 x} + D e^{-k_2 x}$$

$$\psi_3 = F e^{ik_3 x} + G e^{-ik_3 x}$$

$k_1 = k_3 = \sqrt{2mE/\hbar^2}$ and $k_2 = \sqrt{2m(V_0 - E)/\hbar^2}$

BLOCO 2

GRANDEZAS MACROSCÓPICAS DO NUCLEO

ESPALHAMENTO RUTHERFORD

- PARÂMETRO DE IMPACTO
- ÂNGULO DE DEFLEXÃO
- BARREIRA COULOMBIANA

RAIO NUCLEAR

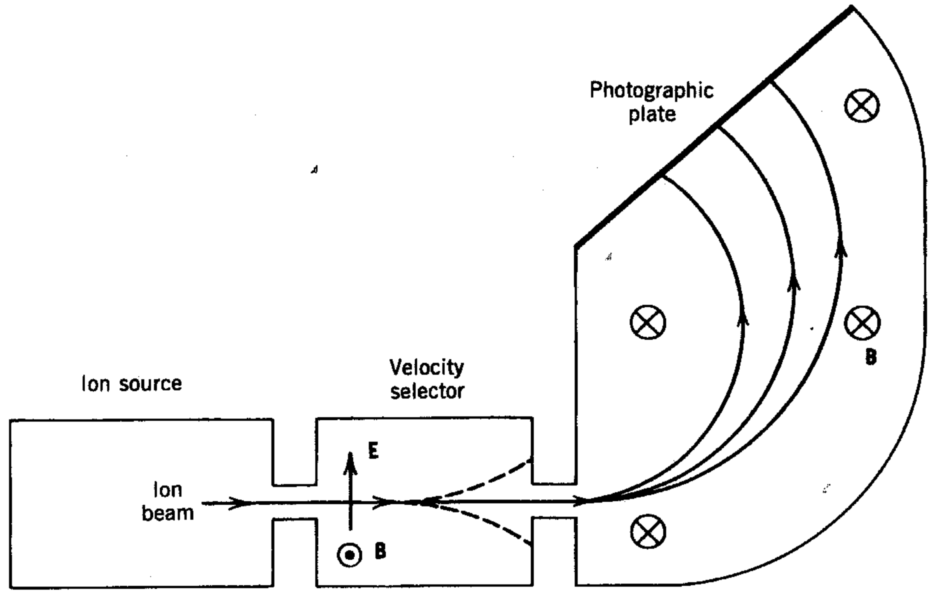
- ESPALHAMENTO DE ELETRONS
- DIFUSIVIDADE

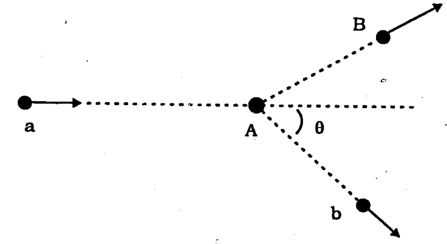
MASSA NUCLAR:

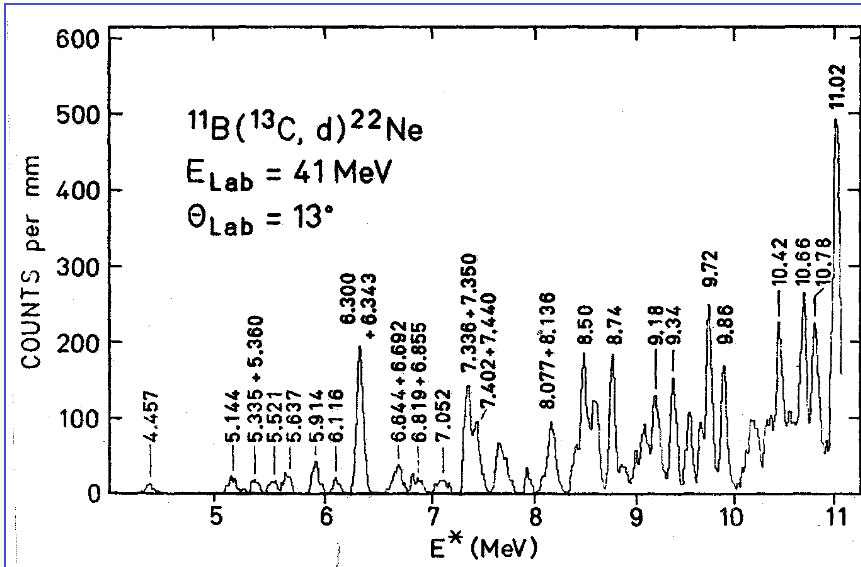
- ENERGIA DE LIGAÇÃO
- EXCESSO DE MASSA
- ENERGIA DE SEPARAÇÃO
- VALOR "Q" de REAÇÃO

CINEMÁTICA DE REAÇÃO - CINEMÁTICA INVERSA

massa dos nucleos







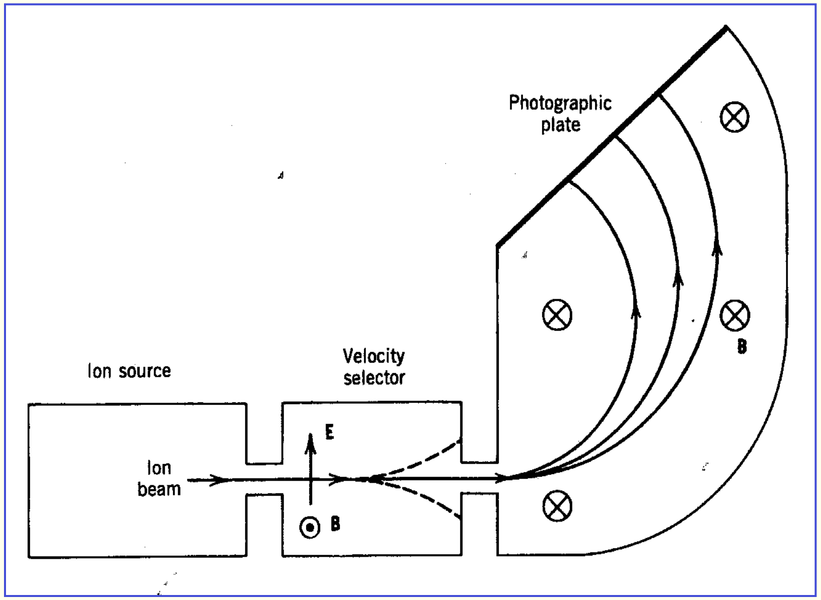
$$\frac{mv^2}{r} = qvB \quad \Rightarrow \quad m = \frac{qrB^2}{E}$$

m_e	0,511 MeV
m_n	939,566 MeV
m_p	938,272 MeV
m_d	1875,613 MeV
$m\ (^3He)$	2808,350 MeV
m_α	3727,323 MeV
u	931,494 MeV

$$(m_{12C}) = 12.0000\ u$$

$$u = m_u = (m_{12C})/12$$

$$1u = 1.66056 \times 10^{-24}\ g$$
$$\Rightarrow m_u c^2 = 931.50\ MeV/c^2$$



$$Z + N = A$$

$$m(Z,N) = Zm_H + Nm_n - B(Z,N)/c^2$$

$$B(Z,N) = [\, Zm_H + Nm_n - m(Z,N)]\, c^2$$
$$B(Z,N) = [\, \Delta m\,]\, c^2$$

m_e	0,511 MeV
m_n	939,566 MeV
m_p	938,272 MeV
m_d	1875,613 MeV
$m\ (^3He)$	2808,350 MeV
m_α	3727,323 MeV
u	931,494 MeV

Excesso de massa (mass excess)
Δ (MeV)

$$\Delta_A \text{ (MeV)} = (m_A - A)u \cdot c^2$$

$$\Delta(^{12}C) = 0$$

dada a reação: A(B,C)D

$$A\ (Z_A,N_A) + B\ (Z_B,N_B) \rightarrow C\ (Z_C,N_C) + D\ (Z_D,N_D)$$

Definimos o valor de "Q" de uma reação

$$(M_A + M_B) = (M_C + M_D) + Q(A+B \rightarrow C+D)$$

$$Q_{(A+B \rightarrow C+D)} = (B_C + B_D) - (B_A + B_B)$$

$$Q_{(A+B \rightarrow C+D)} = (\Delta_C + \Delta_D) - (\Delta_A + \Delta_B)$$

N	Z	A	EL	O	MASS EXCESS (KEV)		B/A	BINDING ENERGY (KEV)		ATOMIC MASS (U)	
1	0	1	N		8071.69	0.10		0.0	0.0	1.00866522	0.00000006
0	1		H		7289.22	0.09		0.0	0.0	1.00782522	0.00000004
1	1	2	H		13136.27	0.16	1.11	2224.64	0.04	2.01410222	0.00000007
2	1	3	H		14950.38	0.22	2.42	8482.22	0.15	3.01604972	0.00000016
1	2		HE		14931.73	0.22	2.57	7718.40	0.14	3.01602970	0.00000016
3	1	4	H	-N	25920	500		5580	500	4.02783	0.00054
2	2		HE		2424.94	0.25	7.07	28296.9	0.4	4.00260326	0.00000027
1	3		LI	+NN	25130	300		4810	300	4.02697	0.00032
4	1	5	H	+	33790	800		5790	800	5.03627	0.00086
3	2		HE	-N	11390	50		27410	50	5.01222	0.00005
2	3		LI	-P	11680	50		26330	50	5.01254	0.00005
4	2	6	HE		17597.3	3.6		29267.9	3.6	6.0188913	0.0000039
3	3		LI		14087.5	0.7	5.33	31995.2	0.8	6.0151234	0.0000008
2	4		BE	-	18375	5		26926	5	6.019726	0.000006
5	2	7	HE	+	26111	30		28826	30	7.028031	0.000032
4	3		LI		14908.6	0.8	5.60	39245.9	0.9	7.0160048	0.0000008
3	4		BE		15770.3	0.8		37601.6	0.9	7.0169299	0.0000008
2	5		B	-	27940	100		24650	100	7.02999	0.00011
6	2	8	HE	+	31650	120		31360	120	8.03397	0.00013
5	3		LI	-N	20947.5	1.0		41278.6	1.2	8.0224879	0.0000011
4	4		BE		4941.8	0.5	7.06	56501.9	0.8	8.0053052	0.0000005
3	5		B	-PP	22922.3	1.2		37738.8	1.3	8.0246079	0.0000013
6	3	9	LI	+	24966	5		45331	5	9.026802	0.000005
5	4		BE		11348.4	0.6	6.46	58167.0	0.9	9.0121828	0.0000006
4	5		B	-	12415.7	0.9		56317.1	1.1	9.0133287	0.0000010
3	6		C		28912	5		39038	5	9.031038	0.000006
7	3	10	LI	-N	35340	SYST		43030	SYST	10.03794	SYST
6	4		BE		12608.1	0.7		64978.9	1.0	10.0135352	0.0000008
5	5		B		12052.3	0.4	6.47	64752.3	0.9	10.0129385	0.0000004
4	6		C		15702.7	1.8		60319.4	2.0	10.0168573	0.0000020
8	3	11	LI	-N	43310	SYST		43130	SYST	11.04649	SYST
7	4		BE		20177	6		65482	6	11.021660	0.000007
6	5		B		8667.95	0.2	6.93	76208.3	1.0	11.00930533	0.00000030
5	6		C	-	10650.2	1.1		73443.6	1.4	11.0114333	0.0000011
4	7		N	-	25450	SYST		57860	SYST	11.02732	SYST
8	4	12	BE	-N	24950	SYST		68780	SYST	12.02678	SYST
7	5		B		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]
6	6		C		0.0	0.0	7.68	92165.5	1.1	12.00000000	0.0
5	7		N		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]

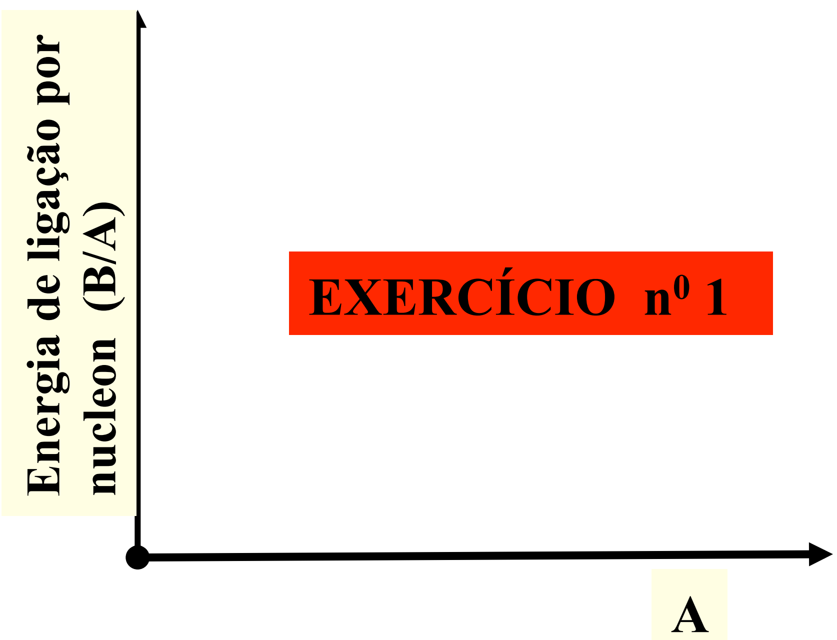
Nuclear Wallet Cards

Isotope Z	El	A	Jπ	Δ (MeV)	T½, Γ, or Abundance	Decay Mode
0	n	1	1/2+	8.071	10.24 m *2*	β−
1	H	1	1/2+	7.289	99.985% *1*	
		2	1+	13.136	0.015% *1*	
		3	1/2+	14.950	12.33 y *6*	β−
		4	2−	25.9	4.6 MeV *9*	n
		5		36.8	5.7 MeV *21*	n
		6		41.9	1.4 MeV *3*	3n ?
2	He	3	1/2+	14.931	0.000137% *3*	
		4	0+	2.425	99.999863% *3*	
		5	3/2−	11.39	0.60 MeV *2*	α, n
		6	0+	17.594	806.7 ms *15*	β−
		7	(3/2)−	26.11	160 keV *30*	n
		8	0+	31.598	119.0 ms *15*	β−, β−n 16%
		9	(1/2−)	40.82	65 keV *37*	n
		10	0+	48.81	0.17 MeV *11*	2n ?
3	Li	4	2−	25.3	6.03 MeV	p
		5	3/2−	11.68	~1.5 MeV	α, p
		6	1+	14.086	7.59% *4*	
		7	3/2−	14.908	92.41% *4*	
		8	2+	20.946	838 ms *6*	β−, β−α
		9	3/2−	24.954	178.3 ms *4*	β−, β−n 50.8%
		10	(1−,2−)	33.05	1.2 MeV *3*	n
		11	3/2−	40.80	8.5 ms *2*	β−, β−nα 0.027%, β−n
		12		50.1s	<10 ns	n ?
4	Be	5	(1/2+)	38.s		p
		6	0+	18.375	92 keV *6*	α, 2p
		7	3/2−	15.769	53.29 d *7*	ε
		8	0+	4.942	6.8 eV *17*	α
		9	3/2−	11.348	100.%	
		10	0+	12.607	1.51×10^{6} y *6*	β−

http://ie.lbl.gov/toimass.html

http://ie.lbl.gov/mass/2003AWMass_3.pdf

http://www.phy.ornl.gov/hribf/calculators/mass-diff.shtml



EXERCÍCIO nº 1

ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT

Nuclear Physics A 729 (2003) 337–676

NUCLEAR PHYSICS A

www.elsevier.com/locate/npe

The AME2003 atomic mass evaluation *

(II). Tables, graphs and references

G. Audi[a,§], A.H. Wapstra[b] and C. Thibault[a]

[a] *Centre de Spectrometrie Nucleaire et de Spectrometrie de Masse, CSNSM, IN2P3-CNRS&UPS, Batiment 108, F-91405 Orsay Campus, France*

[b] *National Institute of Nuclear Physics and High-Energy Physics, NIKHEF, PO Box 41882, 1009DB Amsterdam, The Netherlands*

Abstract

This paper is the second part of the new evaluation of atomic masses AME2003. From the results of a least-squares calculation described in Part I for all accepted experimental data, we derive here tables and graphs to replace those of 1993. The first table lists atomic masses. It is followed by a table of the influences of data on primary nuclides, a table of separation energies and reaction energies, and finally, a series of graphs of separation and decay energies. The last section in this paper lists all references to the input data used in Part I of this AME2003 and also to the data entering the NUBASE2003 evaluation (first paper in this volume).

AMDC: http://csnwww.in2p3.fr/AMDC/

1. Introduction

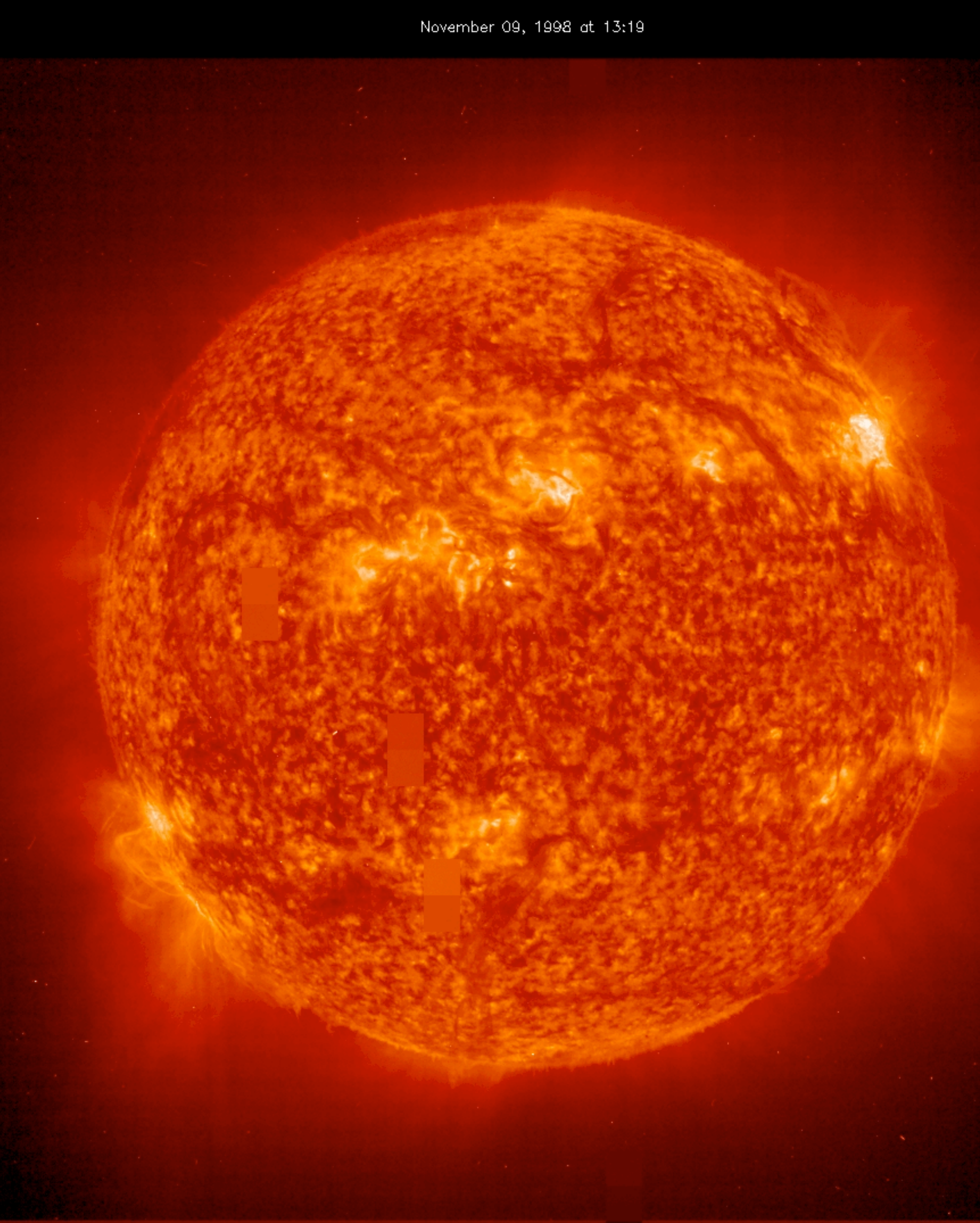
The description of the general procedures and policies are given in Part I of this series of two papers, where the input data used in the evaluation are presented. In this paper we give tables and graphs derived from the evaluation of the input data in Part I.

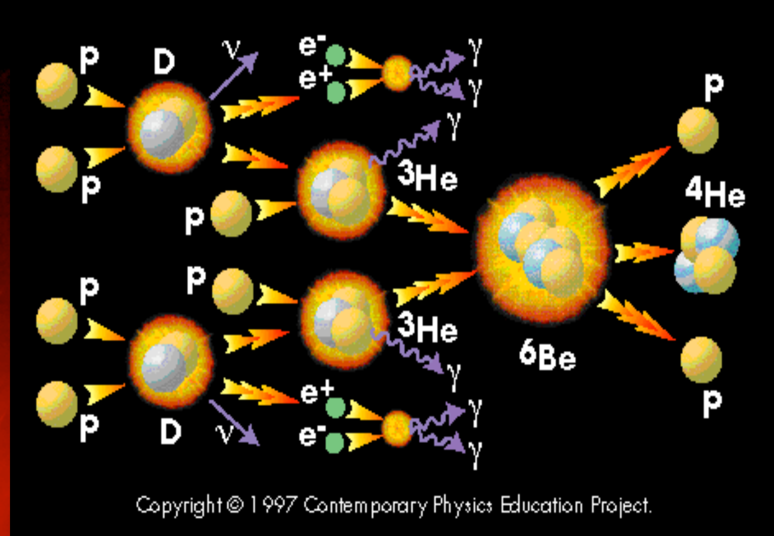
Firstly, we present the table of atomic masses (Table I) expressed as mass excesses in energy units, together with the binding energy per nucleon, the beta-decay energy and the full atomic mass in mass units.

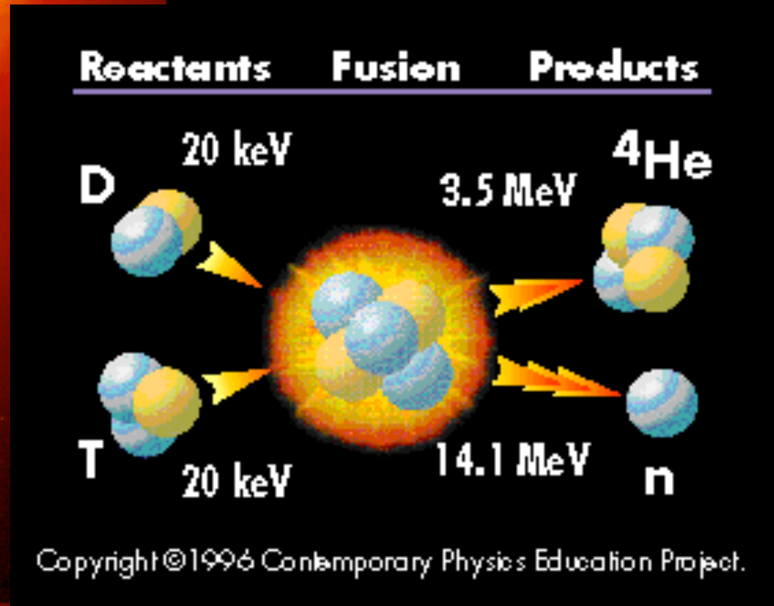
* This work has been undertaken with the encouragement of the IUPAP Commission on Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants (SUN-AMCO).

§ Corresponding author. *E-mail address:* audi@csnsm.in2p3.fr (G. Audi).

doi:10.1016/j.nuclphysa.2003.11.003

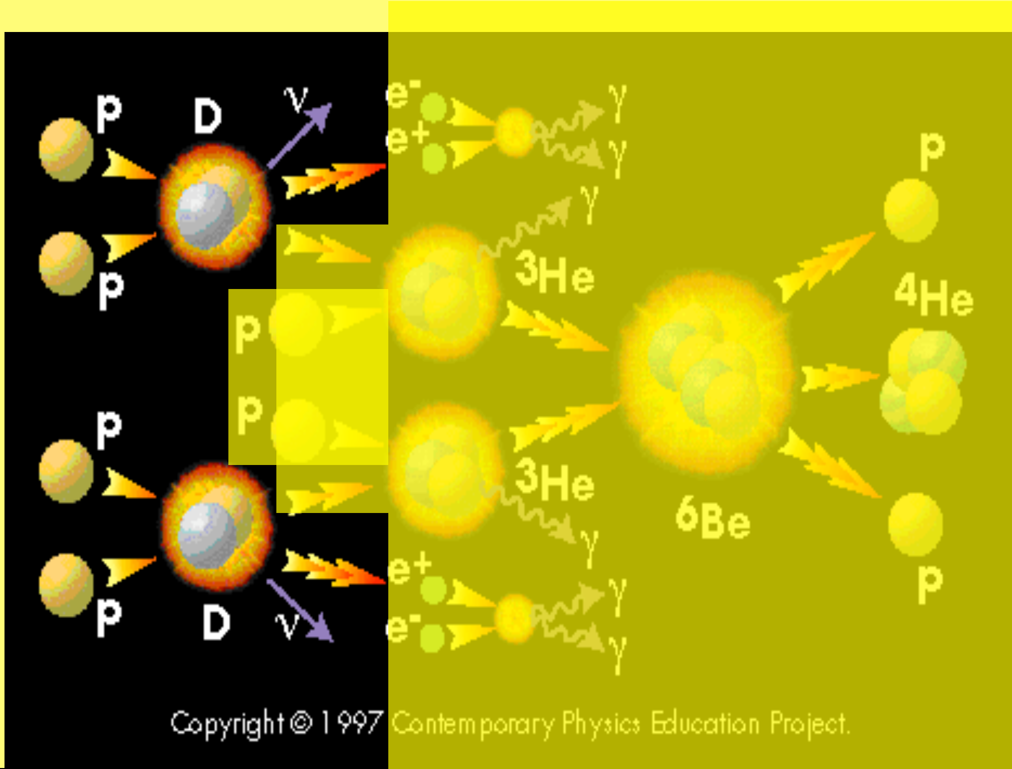

November 09, 1998 at 13:19


p
p
D
ν
e-
e+
γ
γ
γ
3He
p
p
p
p
3He
γ
6Be
4He
p
p
D
ν
e+
e-
γ
γ
Copyright © 1997 Contemporary Physics Education Project.


Reactants
Fusion
Products
D
20 keV
3.5 MeV
4He
T
20 keV
14.1 MeV
n
Copyright ©1996 Contemporary Physics Education Project.

$p + p \rightarrow d + e^+ + \nu$

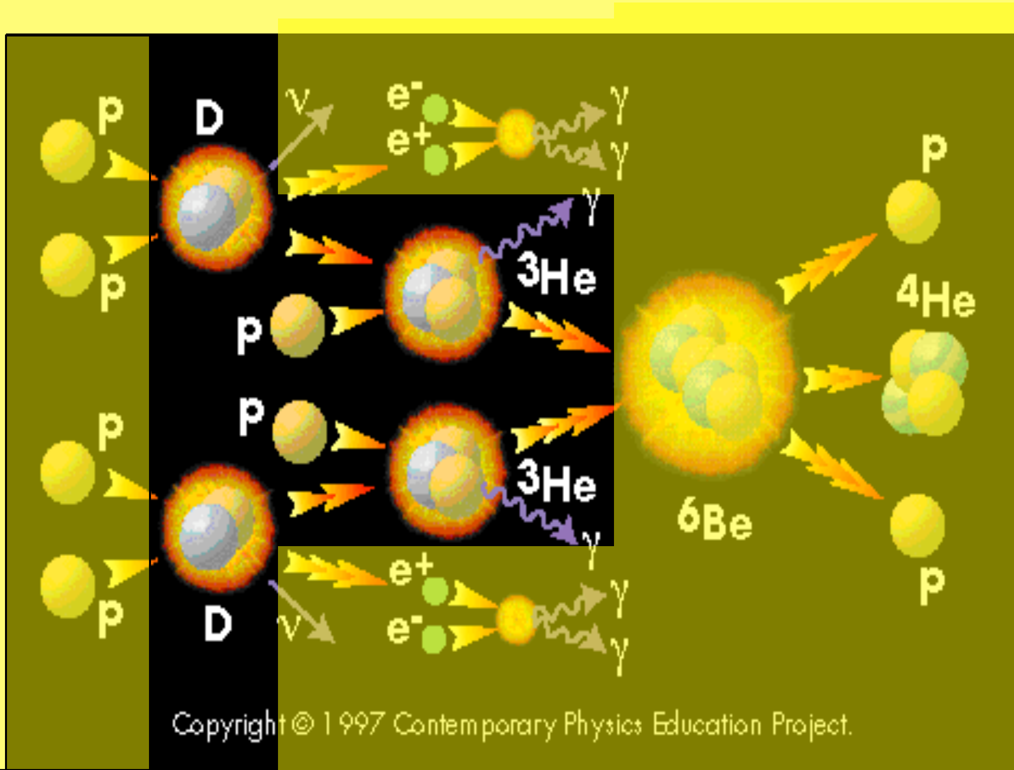
n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He=	14.931
^{4}He=	2.425
^{6}Be=	18.375



$$p + p \rightarrow d + e^+ + \nu$$

$$p + d \rightarrow {}^3He + \gamma$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He = 14.931
^{4}He = 2.425
^{6}Be = 18.375

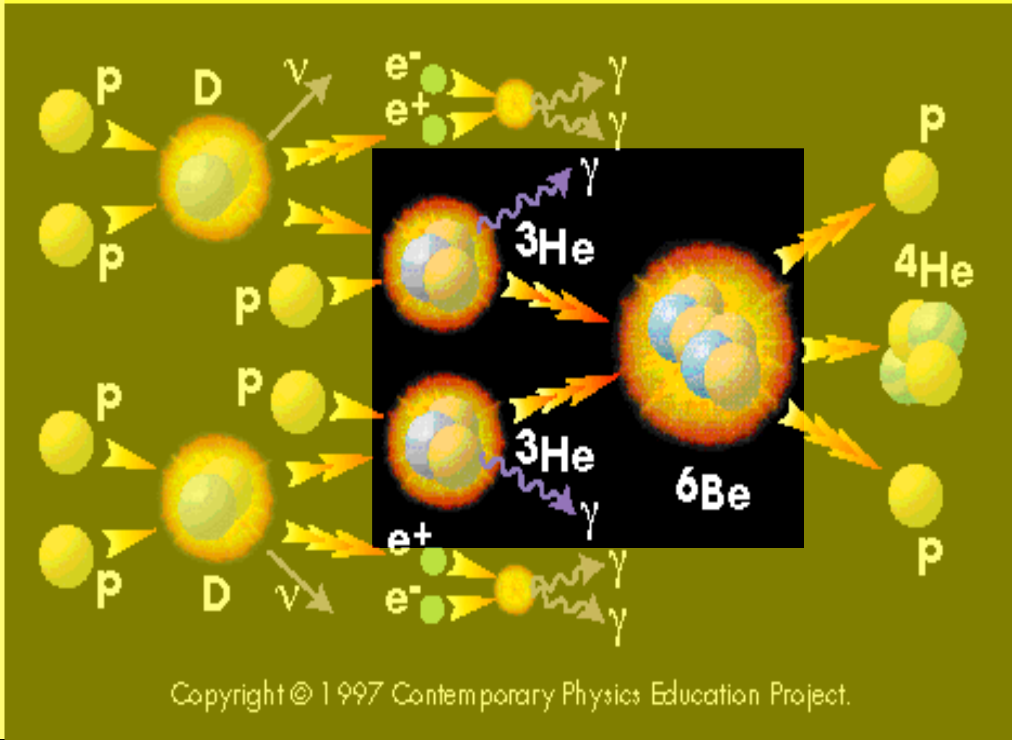


$$p + p \rightarrow d + e^+ + \nu$$

$$p + d \rightarrow {}^3He + \gamma$$

$${}^3He + {}^3He \rightarrow {}^6Be + \gamma$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He= 14.931
^{4}He= 2.425
^{6}Be= 18.375



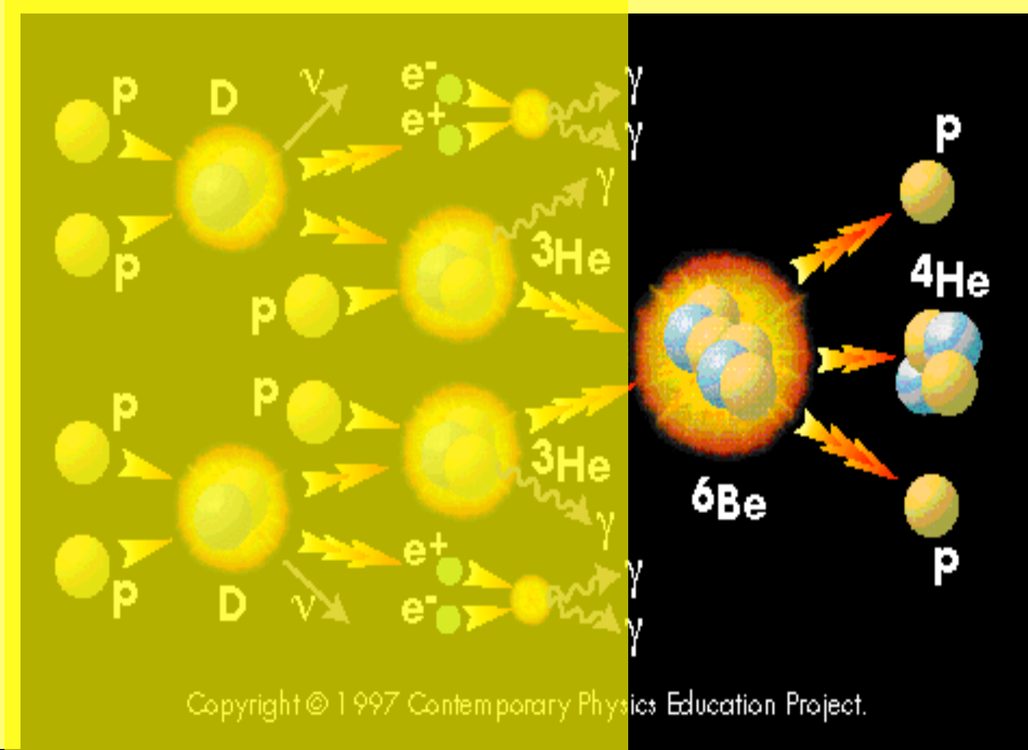
$p + p \rightarrow d + e^{+} + \nu$

$p + d \rightarrow {}^{3}He + \gamma$

${}^{3}He + {}^{3}He \rightarrow {}^{6}Be + \gamma$

${}^{6}Be \rightarrow {}^{4}He + p + p$

n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He=	14.931
^{4}He=	2.425
^{6}Be=	18.375



$p + p \rightarrow d + e^{+} + \nu$

$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931$ MeV

$p + d \rightarrow {}^{3}He + \gamma$

$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494$ MeV

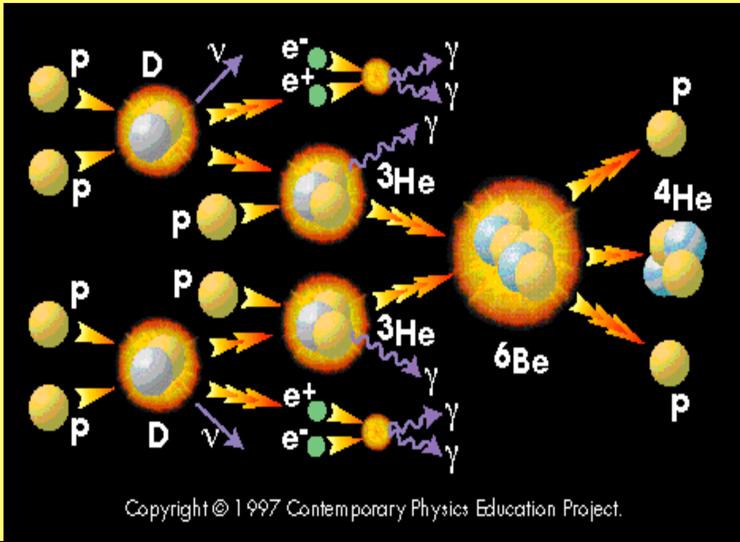
${}^{3}He + {}^{3}He \rightarrow {}^{6}Be + \gamma$

$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487$ MeV

${}^{6}Be \rightarrow {}^{4}He + p + p$

$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372$ MeV

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He = 14.931
^{4}He = 2.425
^{6}Be = 18.375



$p + p \rightarrow d + e^+ + \nu$

$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931$ MeV

$p + d \rightarrow {}^3He + \gamma$

$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494$ MeV

${}^3He + {}^3He \rightarrow {}^6Be + \gamma$

$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487$ MeV

${}^6Be \rightarrow {}^4He + p + p$

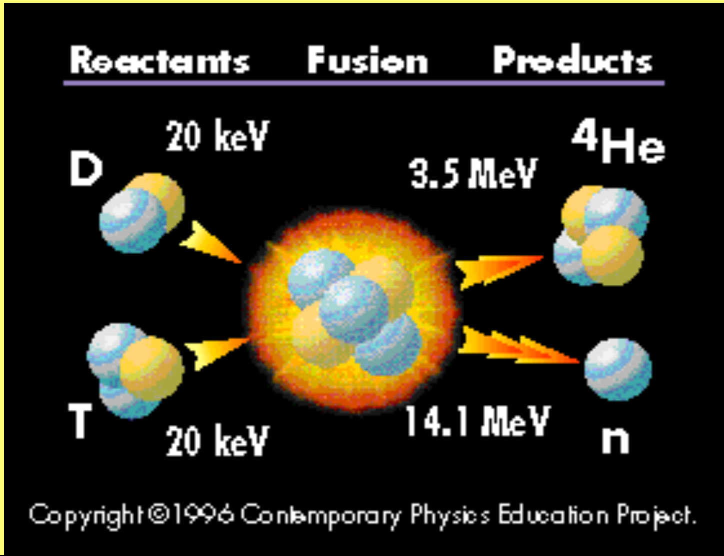
$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372$ MeV

n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He=	14.931
^{4}He=	2.425
^{6}Be=	18.375

$d + t \rightarrow {}^4He + n$

$13.136 + 14.950 \rightarrow 2.425 + 8.071 + Q$

Q = 17.59 MeV



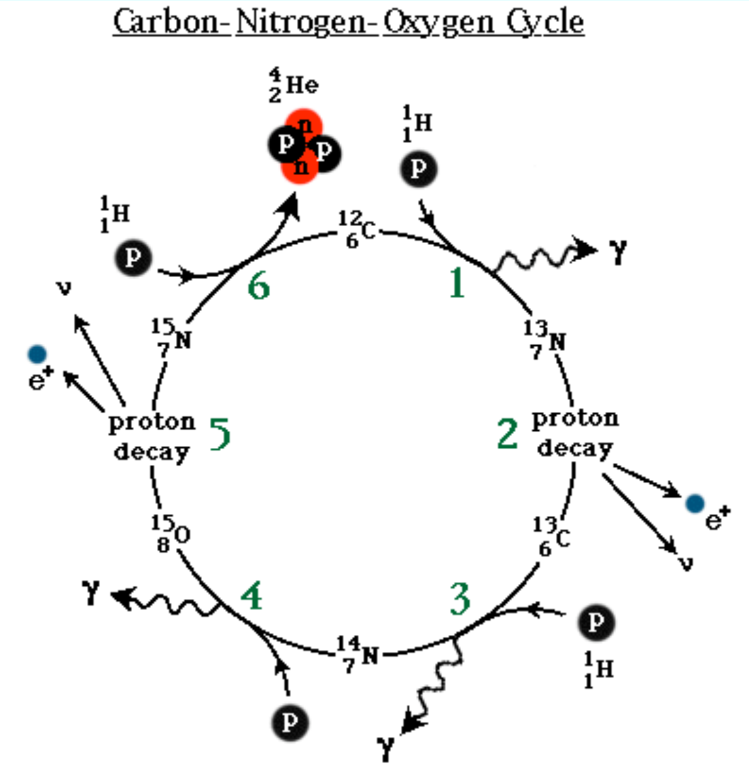
EXERCÍCIOS

CALCULAR O BALANÇO ENERGÉTICO NAS SEGUINTES REAÇÕES

$\Delta = (M-A)c^2$ (MeV)

Exemplo== $^{12}C + p \rightarrow {}^{13}N + \gamma$

n = 8.071	^{12}C = 0.00
p = 7.289	^{13}C = 3.125
d = 13.136	^{13}N = 5.345
t = 14.950	^{14}N = 2.863
^{3}He = 14.931	^{15}N = 0.011
^{4}He = 2.425	^{15}O = 2.855
^{6}Li = 14.086	^{16}O = -4.737
^{7}Li = 14.908	^{17}O = -0.809
^{6}Be = 18.375	^{18}O = -0.782



$^{12}C + p \rightarrow {}^{13}N + \gamma$

$0 + 7.289 \rightarrow 5.345 + Q \Rightarrow Q = 1.944$ MeV

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$

$5.345 \rightarrow 3.125 + 0.511 + Q \Rightarrow Q = 1.709$ MeV

$^{13}C + p \rightarrow {}^{14}N + \gamma$

$3.125 + 7.289 \rightarrow 2.863 + Q \Rightarrow Q = 7.551$ MeV

$^{14}N + p \rightarrow {}^{15}O + \gamma$

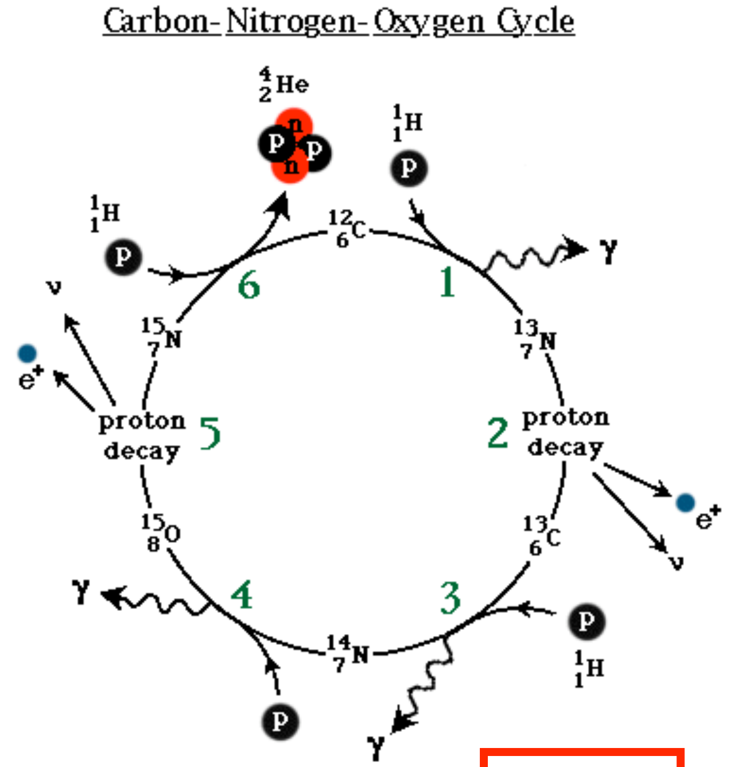
$2.863 + 7.289 \rightarrow 2.855 + Q \Rightarrow Q = 7.297$ MeV

$^{15}O \rightarrow {}^{15}N + e^+ + \nu$

$2.855 \rightarrow 0.101 + 0.511 + Q \Rightarrow Q = 2.243$ MeV

$^{15}N + p \rightarrow {}^{12}C + \alpha + \gamma$

$0.101 + 7.289 \rightarrow 0 + 2.425 + Q \Rightarrow Q = 4.965$ MeV

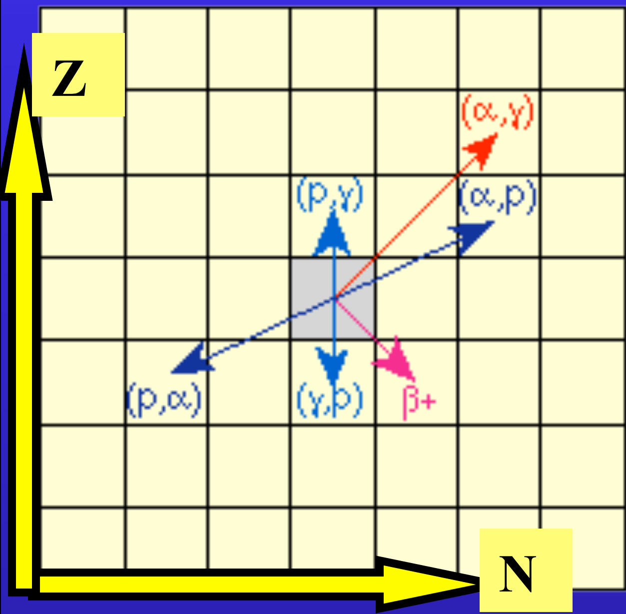


$$\begin{array}{r} 1.944 \\ +1.709 \\ +7.551 \\ +7.297 \\ +2.243 \\ +4.965 \\ \hline 25.709 \end{array}$$

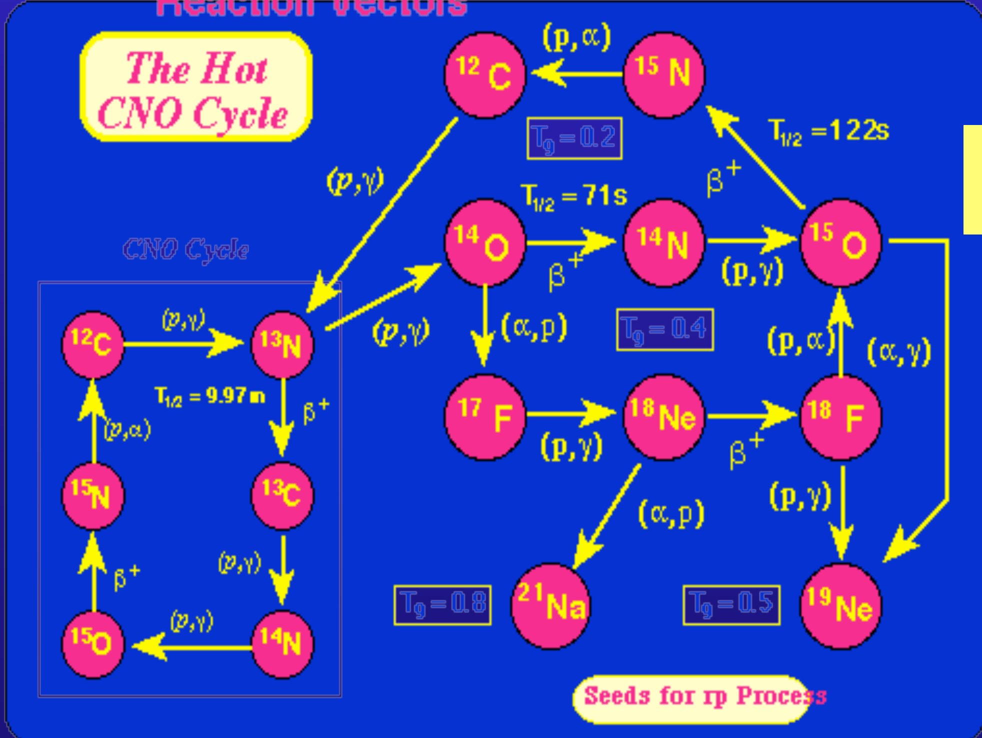
$^{12}C + p + p + p + p \rightarrow {}^{12}C + {}^{4}He + 2e^+ + 2\nu$

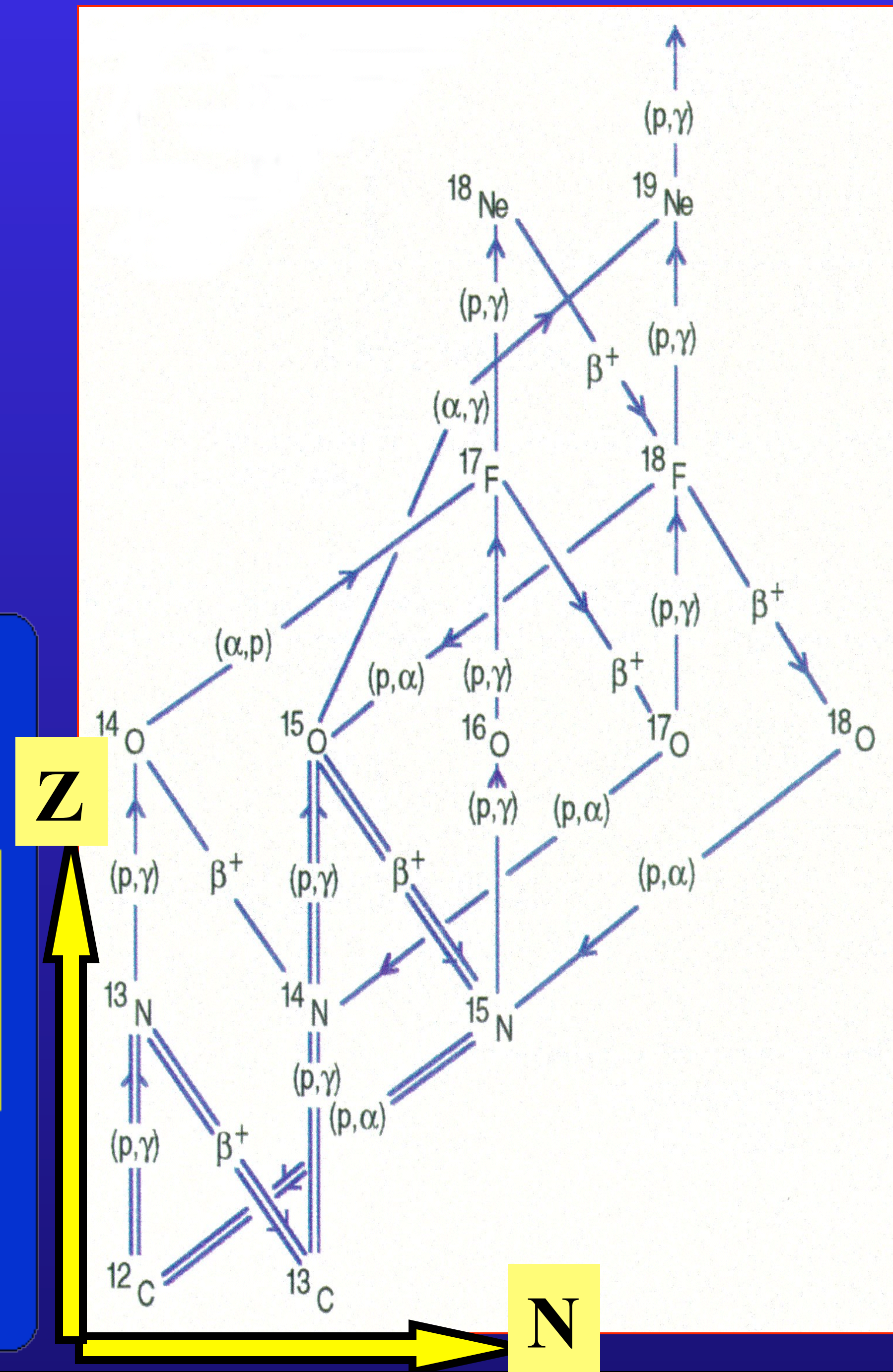
$0 + 4x\,(7.289) \rightarrow 0 + 2.425 + 1.022 + Q$

Q = 25.709 MeV



Reaction Vectors





BLOCO 2

GRANDEZAS MACROSCÓPICAS DO NUCLEO

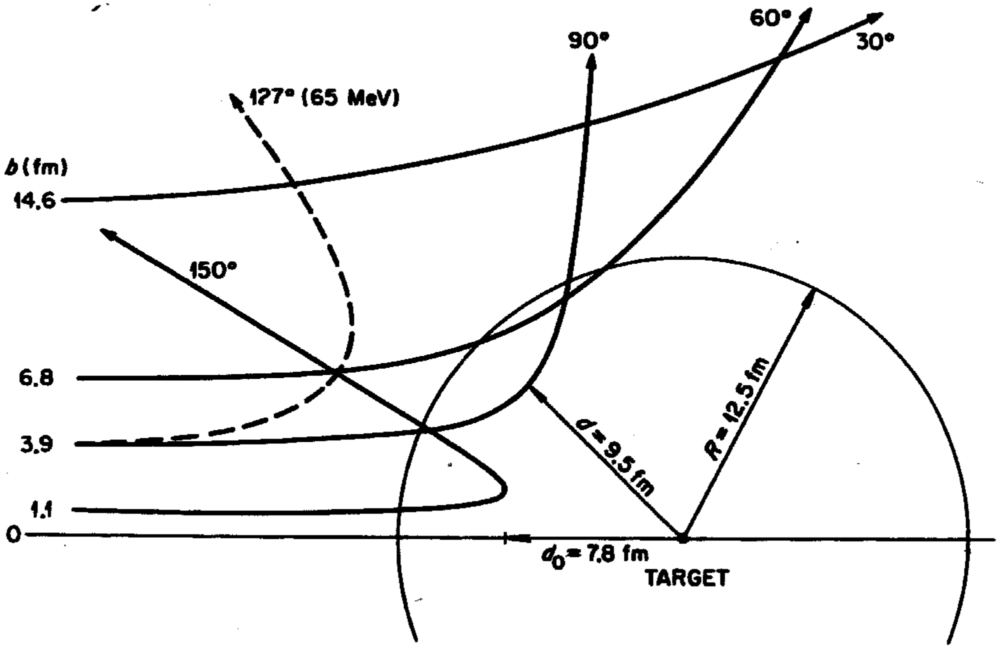
ESPALHAMENTO RUTHERFORD
- PARÂMETRO DE IMPACTO
- ÂNGULO DE DEFLEXÃO
- BARREIRA COULOMBIANA

RAIO NUCLEAR
- ESPALHAMENTO DE ELETRONS
- DIFUSIVIDADE

MASSA NUCLAR:
- ENERGIA DE LIGAÇÃO
- EXCESSO DE MASSA
- ENERGIA DE SEPARAÇÃO
- VALOR "Q" de REAÇÃO

CINEMÁTICA DE REAÇÃO - CINEMÁTICA INVERSA

espalhamento Rutherford



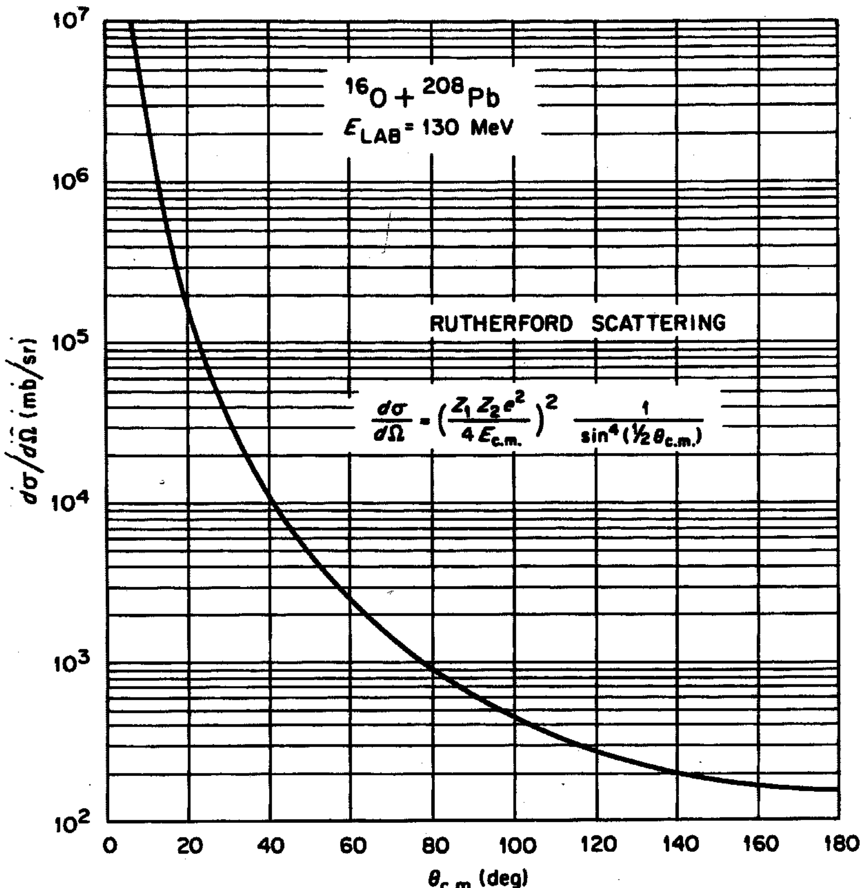
$$\frac{d\sigma}{d\Omega} = \left[\frac{Z_A Z_a e^2}{2E}\right]^2 \frac{1}{\mathrm{sen}^4 \frac{\theta}{2}}$$

espalhamento:

^{208}Pb(^{16}O,^{16}O)^{208}Pb

na energia **E** $_{16O}$= 130MeV

a curva pontilhada corresponde a $E_{(16O)} = 65MeV$



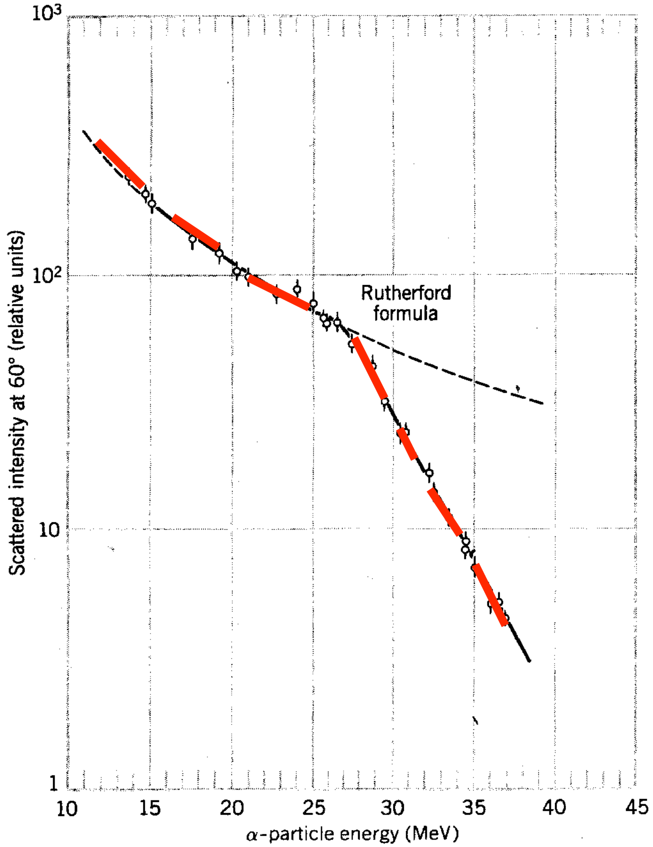


Figure 3.11 The breakdown of the Rutherford scattering formula. When the incident α particle gets close enough to the target Pb nucleus so that they can interact through the nuclear force (in addition to the Coulomb force that acts when they are far apart) the Rutherford formula no longer holds. The point at which this breakdown occurs gives a measure of the size of the nucleus. Adapted from a review of α particle scattering by R. M. Eisberg and C. E. Porter, *Rev. Mod. Phys.* **33**, 190 (1961).

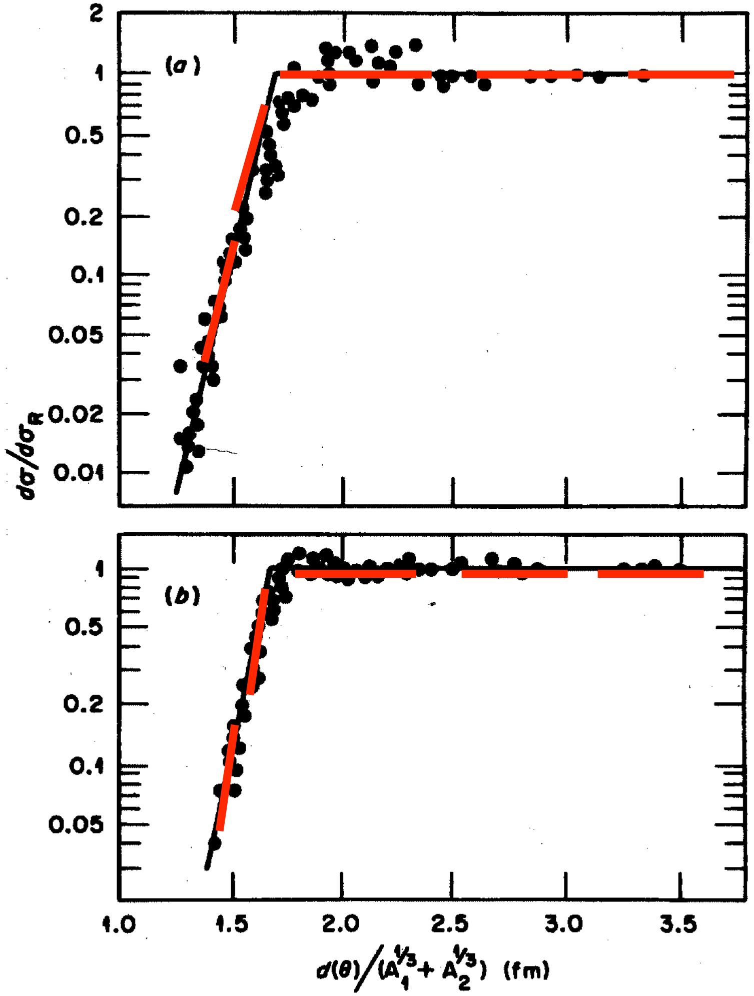


Figure 2.12 The differential cross-sections for O and C ions, in ratio to the Rutherford ones, scattering from various targets plotted against the distance of closest approach *d*, instead of the scattering angle θ, by using equation 2.20. The distance *d* has been divided by $(A_1^{1/3}+A_2^{1/3})$, where A_i is the mass number of nucleus i. The measured cross-sections then fall on a universal curve, showing that nuclear radii are approximately proportional to $A^{1/3}$. (a) $^{16}O + {}^{40,48}Ca$ at 49 MeV, $^{16}O + {}^{40,48}Ca$, ^{50}Ti, ^{52}Cr, ^{54}Fe, ^{62}Ni at 60 MeV and $^{18}O + {}^{60}Ni$ at 60 MeV; (b) $^{12}C + {}^{96}Zr$ at 38 MeV, $^{16}O + {}^{96}Zr$ at 47, 49 MeV, $^{16}O + {}^{88}Sr$, ^{93}Zr at 60 MeV and $^{18}O + {}^{90}Zr$ at 60, 66 MeV. (After Christensen *et al.*, 1973)

BARREIRA COULOMBIANA

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R}$$

Para a colisão razante:

$$R = R_A + R_a$$

$$V_{Coul} = \frac{Z_A Z_a e^2}{R}$$

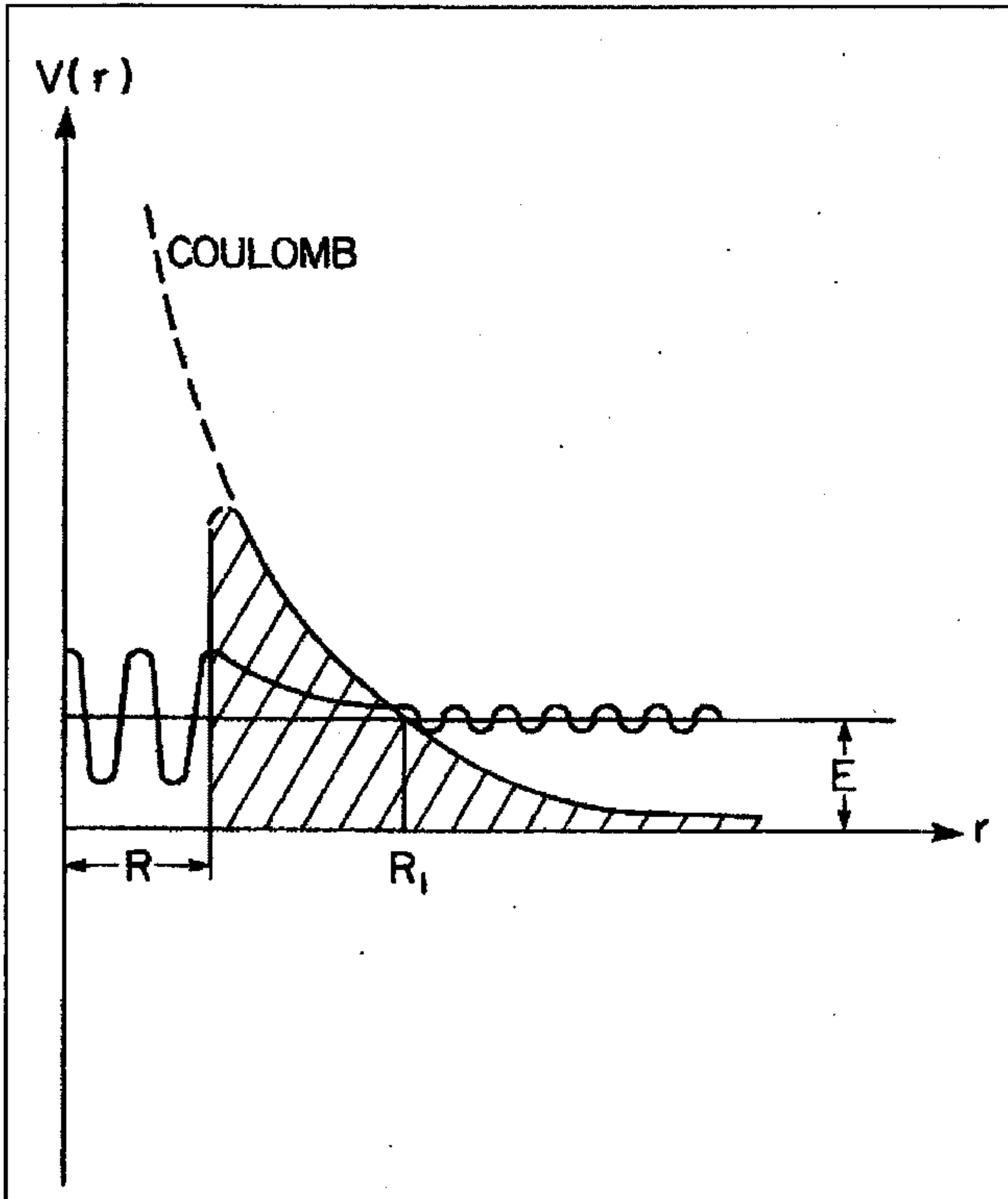
$E_{cm} > V_{coul} \implies T_l(E_{cm}) = 1$

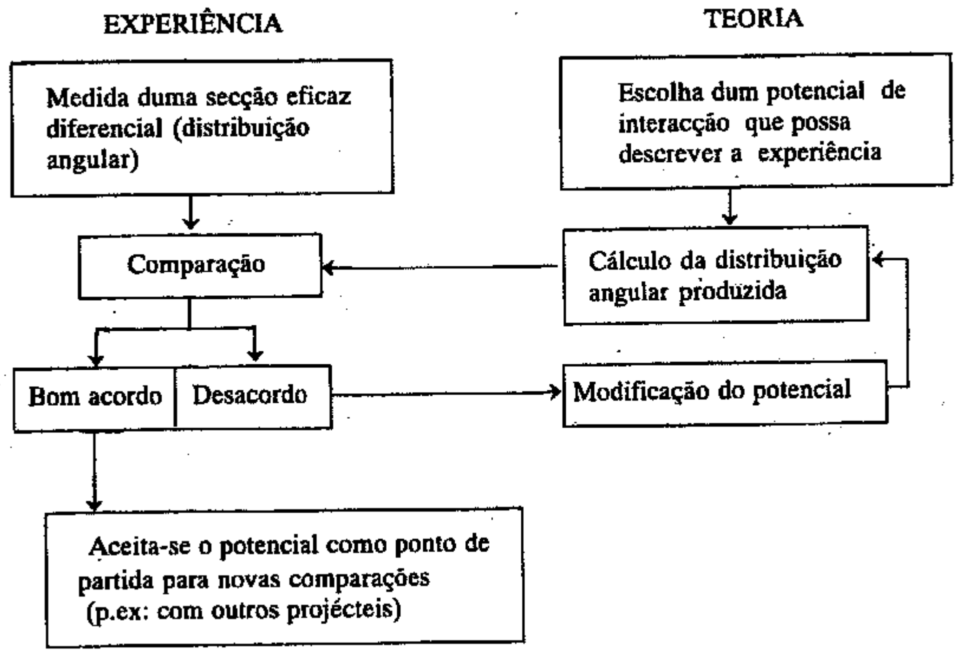
$E_{cm} < V_{coul} \implies T_l(E_{cm}) = 0$

$e^2 = 1.44$ MeV.fm

$r_0 \sim 1.25$ fm

V_{coul} (MeV)


V(r)
COULOMB
E
r
R
R_1



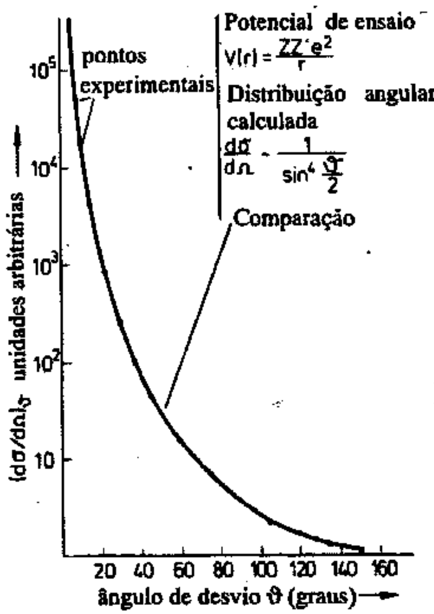


Fig.5 a) Método usado para estudar um potencial dispersor a partir duma distribuição angular.
b) Exemplo: Resultado de experiências de Rutherford realizadas com partículas α de 5,5 MeV sobre um alvo de ouro (Geiger e Marsden, 1913)

BLOCO 2

GRANDEZAS MACROSCÓPICAS DO NUCLEO

ESPALHAMENTO RUTHERFORD
- PARÂMETRO DE IMPACTO
- ÂNGULO DE DEFLEXÃO
- BARREIRA COULOMBIANA

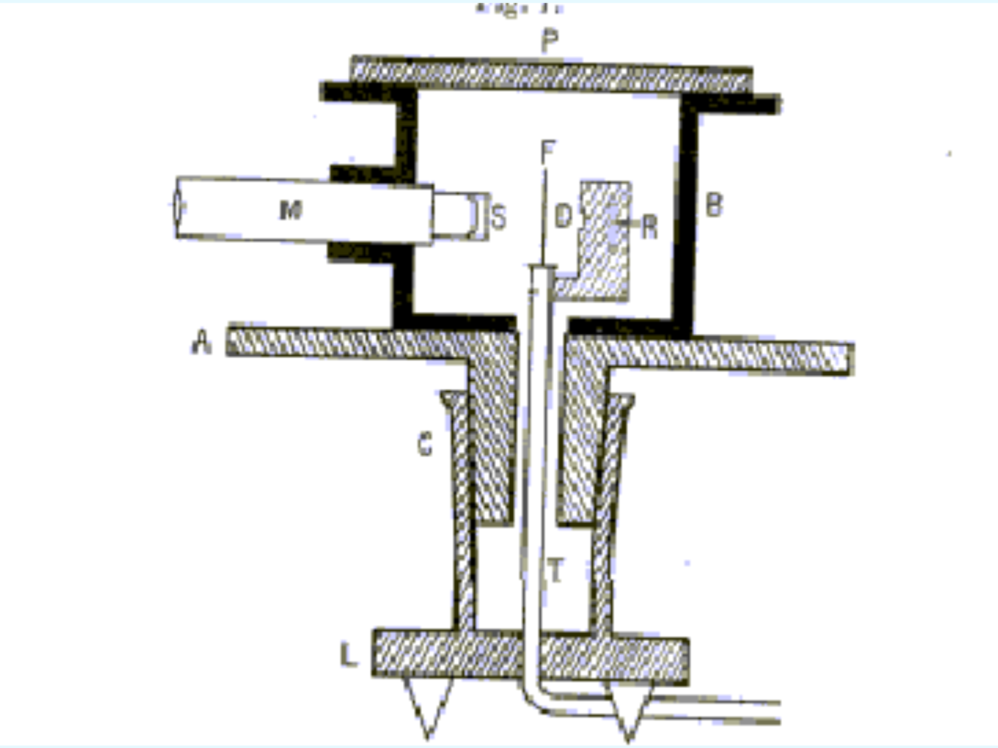
RAIO NUCLEAR
- ESPALHAMENTO DE ELETRONS
- DIFUSIVIDADE

MASSA NUCLAR:
- ENERGIA DE LIGAÇÃO
- EXCESSO DE MASSA
- ENERGIA DE SEPARAÇÃO
- VALOR "Q" de REAÇÃO

CINEMÁTICA DE REAÇÃO - CINEMÁTICA INVERSA

espalhamento Rutherford

NUCLEO DE RUTHERFORD → *ANIMAÇÃO*



http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/rutherford/rutherford.html

http://www.nat.vu.nl/~pwgroen/projects/sdm/applets.htm

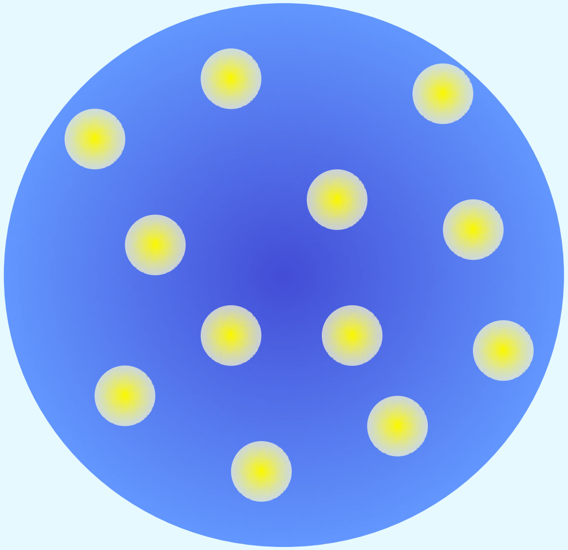
http://www.nat.vu.nl/~pwgroen/sdm/hyper/anim/anim_DI.html

http://physics.uwstout.edu/physapplets/

http://micro.magnet.fsu.edu/electromag/java/rutherford/

NUCLEO DE THOMSON

ANIMAÇÃO



^{24}Mg

Disproof of the Pudding

The back scattered alpha-particles proved fatal to the plum pudding model. A central assumption of that model was that both the positive charge and the mass of the atom were more or less uniformly distributed over its size, approximately 10-10 meters across or a little more. It is not difficult to calculate the magnitude of electric field from this charge distribution. (Recall that this is the field that must scatter the alphas, the electrons are so light they will jump out of the way with negligible impact on an alpha.)

To be specific, let us consider the gold atom, since the foil used by Rutherford was of gold, beaten into leaf about 400 atoms thick. The gold atom has a positive charge of 79e (balanced of course by that of the 79 electrons in its normal state). Neglecting these electrons -- assume them scattered away -- the maximum electric force the alpha will encounter is that at the surface of the sphere of positive charge,

$$E.2e = \frac{1}{4\pi\varepsilon_0}.\frac{79e.2e}{r_0^2} = 9.10^9.\frac{158.(1.6.10^{-19})^2}{10^{-20}} = 3.64.10^{-6} \text{ newtons}$$

If the alpha particle initially has momentum *p*, for small deflections the angle of deflection (in radians) is given by (*delta_p*)/*p*, where *delta_p* is the sideways momentum resulting from the electrically repulsive force of the positive sphere of charge. Assuming the atomic sphere itself moves negligibly -- it is much heavier than the alpha, so this is reasonable -- the trajectory of the alpha in the inverse square electric field can be found by standard methods. It is the same mathematical problem as finding the elliptic orbits of planets around the sun. Replacing inverse square attraction with inverse square repulsion changes the orbit from an ellipse (or a hyperbola branch swinging around the sun for a comet) to a hyperbola branch lying on one side of the center of repulsion.

Note that since the alpha particle has mass 6.7x10-27 kg, from $F = ma$, the electric force at the atomic surface above will give it a sideways acceleration of 5.4x1020 meters per sec per sec (compare g = 10!). But the force doesn't have long to act - the alpha is moving at 1.6x107 meters per second. So the time available for the force to act is the time interval a particle needs to cross an atom if the particle gets from New York to Australia in one second.

The time $t0 = 2r0/v$ = 2x10-10/1.6x107 = 1.25x10-17 seconds.

Thus the magnitude of the total sideways velocity picked up is the sideways acceleration multiplied by the time,

1.25x10-17x5.4x1020 = 6750 meters per second.

This is a few ten-thousandths of the alpha's forward speed, so there is only a very tiny deflection. Even if the alpha hit 400 atoms in succession and they all deflected it the same way, an astronomically improbable event, the deflection would only be of order a
In fact, one can get a clear idea of how much deflection comes about without going into the details of the trajectory. Outside the atom, the repulsive electrical force falls away as the inverse square. Inside the atom, the force drops to zero at the center, just as the gravitational force is zero at the center of the earth. The force is maximum right at the surface. Therefore, a good idea of the sideways deflection is given by assuming the alpha experiences that maximal force for a time interval equal to the time it takes the alpha to cross the atom -- say, a distance $2r0$.
degree. Therefore, the observed deflection through *ninety* degrees and more was completely inexplicable using Thomson's pudding model!

ref: williams

1.2 The Rutherford scattering formula

Although Rutherford derived this formula with α-particles as the incident, to-be-scattered particle, we shall be slightly more general and assume that incident particle carries positive charge ze, where e is the magnitude of the electronic charge. Firstly the model:

1. The atom contains a nucleus with positive charge Ze and almost the entire mass of the atom.
2. The electrically neutral atom contains Z electrons moving around the nucleus.

It is easy to show that the electrons cannot cause a single scattering with significant deflection of α-particles of the kinetic energy that Rutherford considered, so we shall now neglect them. Our other assumptions are:

3. That the target nucleus is very much more massive than the incident particle and therefore does not recoil significantly in the collision.
4. That classical mechanics can be used to describe the collision. (And, of course, we thereby include the conservation of momentum, angular momentum and energy.)
5. That the target nucleus and the incident particle have point-like charge distributions so that the Coulomb potential $V(r) = Zze^2/4\pi\varepsilon_0 r$ acts between them, where r is the distance separating their centres. We will be treating the orbit of the incident particle classically and will work out the case of like charges (as is the case in α-particle-nucleus scattering) and therefore of a repulsive force.
6. That there is no other force acting other than that due to the Coulomb potential.
7. That there is no excitation of incident or target particle: each remains unchanged. This is elastic scattering.

The symbols we shall use are defined in Table 1.1. Figure 1.5 shows an orbit. The incident particle, if undeflected, would pass the centre (at O) of the target nucleus at a distance b, the **impact parameter**. In fact, the orbit is hyperbolic and at D the incident particle is at its distance of closest approach, d. The orbit is

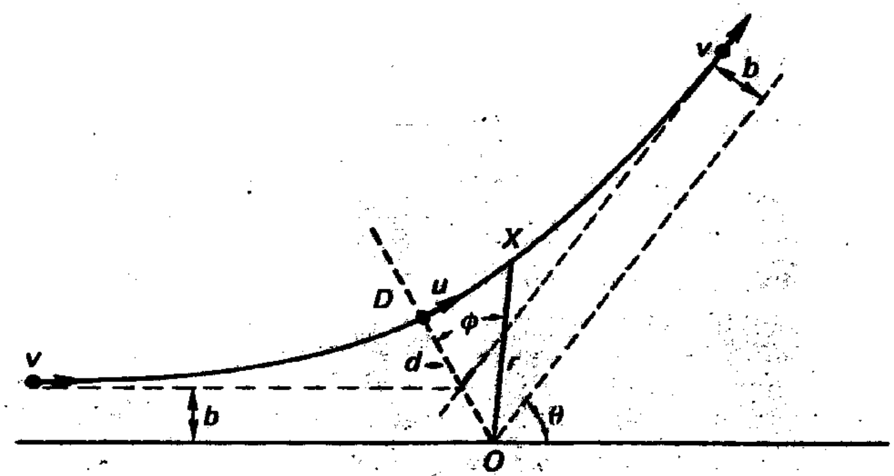


Fig. 1.5 The classical orbit of the incident particle in Rutherford scattering for non-zero impact parameter b.

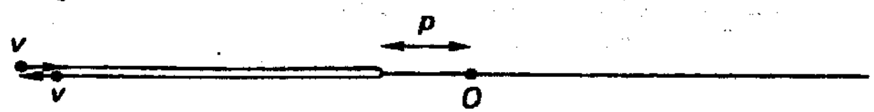


Fig. 1.6 The classical orbit in Rutherford scattering for zero impact parameter. Conservation of energy requires that the incident particle's distance of closest approach p, is given by

$$p = Zze^2/4\pi\varepsilon_0 T.$$

clearly symmetric about the line OD. If b was zero the incident particles would approach to a distance p (see Fig. 1.6). At this point the incident kinetic energy is transformed into mechanical potential energy in the Coulomb field, therefore:

$$\tfrac{1}{2}mv^2 p = Zze^2/4\pi\varepsilon_0. \tag{1.1}$$

Step 1 To find the connection between b and θ.

We use the conservation of angular momentum about O to connect the incident velocity to the component of the velocity transverse to OX at X:

$$mvb = mr^2\frac{d\varphi}{dt}, \tag{1.2}$$

hence

$$\frac{dt}{r^2} = \frac{d\varphi}{vb}. \tag{1.3}$$

Consider now the component of the linear momentum in the direction OD. This changes from $-mv\sin(\theta/2)$ to $+mv\sin(\theta/2)$. At X the rate of change of this momentum is the component of the Coulomb repulsion in the direction OD. Hence

$$2mv\sin\frac{\theta}{2} = \int_{-\infty}^{+\infty} (Zze^2/4\pi\varepsilon_0 r^2)\cos\varphi\, dt.$$

We use equation (1.3) to change the variable of integration from time, t, to φ, obtaining

$$2mv\sin\frac{\theta}{2} = \frac{Zze^2}{4\pi\varepsilon_0 vb}\int_{\varphi=-(\pi-\theta)/2}^{\varphi=(\pi-\theta)/2} \cos\varphi\, d\varphi = \frac{p}{2}\frac{mv}{b}\Big[\sin\varphi\Big]_{-(\pi-\theta)/2}^{(\pi-\theta)/2}$$

which gives

$$\tan\frac{\theta}{2} = \frac{p}{2b}. \tag{1.4}$$

This is the relation required from Step 1.

Step 2 To derive a first cross-section.

The relation (1.4) tells us that as b decreases θ increases. Therefore to suffer an angle of scatter greater than Θ the impact parameter b must be less than $(p/2)\cot(\Theta/2)$. That means the incident particle must strike a disc of this radius centred at O and perpendicular to v. The area, σ, presented by the nucleus for scattering through an angle greater than Θ is the area of this disc. That is

$$\sigma(\theta>\Theta)=\frac{\pi p^2}{4}\cot^2\frac{\Theta}{2}, \tag{1.5}$$

or in its full glory:

$$\sigma(\theta>\Theta)=\frac{\pi}{4}\left(\frac{Zze^2}{4\pi\varepsilon_0 T}\right)^2\cot^2\frac{\Theta}{2}.$$

The area σ is called a cross-section: if the reader is concerned about the meaning and use of this term we suggest reading Section 2.10, where a fuller description of the concept is given, before proceeding.

Step 3 To obtain the angular differential cross-section.

What we want is $d\sigma/d\Omega$, which is the cross-section per unit solid angle located at an angle θ. The element of solid angle $d\Omega$ between θ and $\theta+d\theta$ is given by

$$d\Omega=2\pi\sin\theta\, d\theta.$$

Therefore

$$\frac{d\sigma}{d\Omega}=\frac{1}{2\pi\sin\theta}\frac{d\sigma}{d\theta}.$$

The $d\sigma/d\theta$ we need is $(d/d\Theta)\sigma(\theta>\Theta)$ from Equation (1.5) and hence we obtain

$$\frac{d\sigma}{d\Omega}=\left(\frac{Zze^2}{16\pi\varepsilon_0 T}\right)^2\operatorname{cosec}^4\frac{\theta}{2}. \tag{1.6}$$

Dado um processo: $A + a \rightarrow b + B$

$$A\,(a,b)B$$

Definimos:

$$\sigma = \frac{\text{Número de partículas b emitidas}}{(\text{n}^o\ \text{particulas a/unidade de área})(\text{n}^o\ \text{de partículas A no alvo que iterceptam o alvo})}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_\theta = \frac{\text{n}^o\ \text{partículas no ângulo } \theta \text{ (no ângulo sólido } d\Omega)}{(\text{n}^o\ \text{partículas a/unidade de área})(\text{n}^o\ \text{partículas A no alvo})}$$

$\sigma \rightarrow$ 1 barn = $10^{-28} m^2 = 100\ fm^2$

1 fm = 10^{-16}m

$$dS = 2\pi R\ \text{sen}\theta\ .\ R d\theta\ =\ 2\pi R^2\ \text{sen}\theta\ d\theta$$

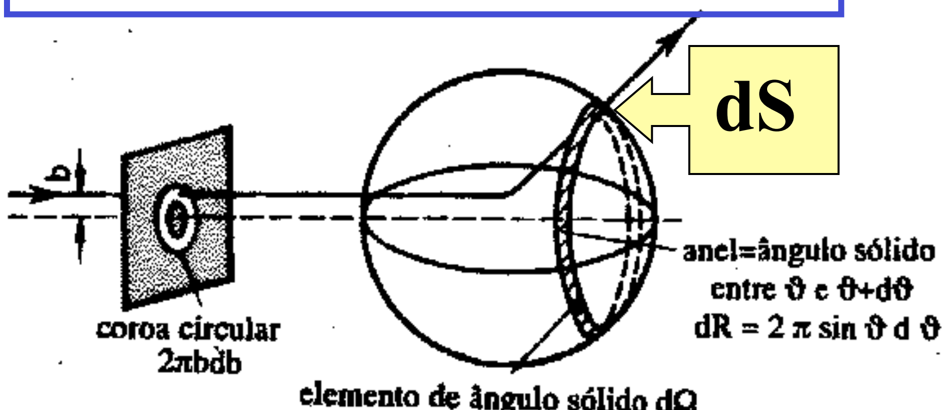


Fig.2 Relações geométricas na dispersão.

$$\Delta\Omega = \frac{\Delta S}{R^2} = 2\pi\,\text{sen}\,\theta\, d\theta \Rightarrow d\Omega$$

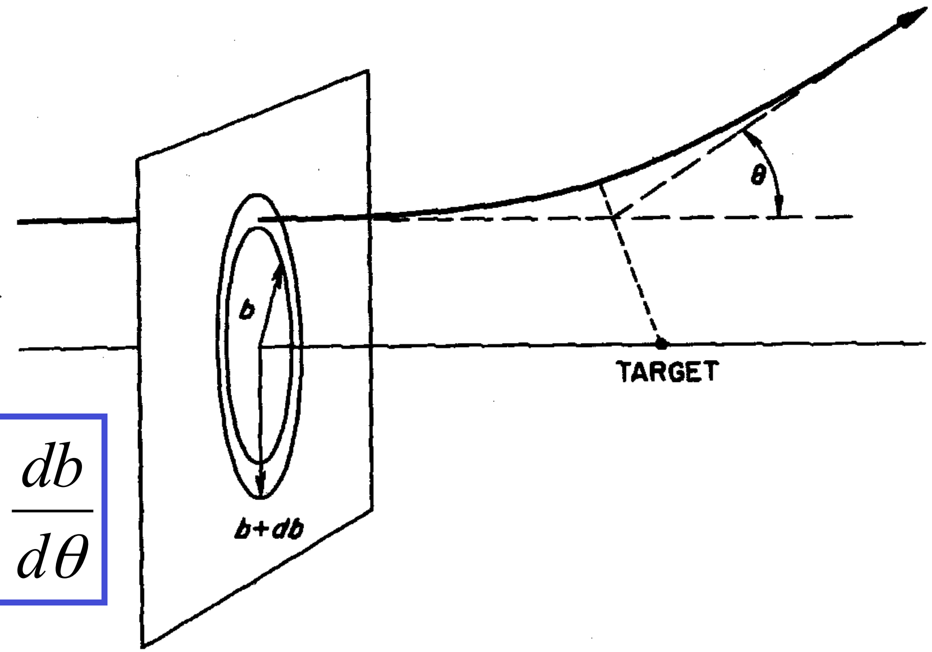
$$\sigma_{Total} = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega \qquad \frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\theta}\left(\frac{d\theta}{d\Omega}\right)$$

$$\frac{d\theta}{d\Omega} = \frac{1}{2\pi\,\text{sen}\,\theta} \Rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{2\pi\,\text{sen}\,\theta}\left(\frac{d\sigma}{d\theta}\right)$$

$$\frac{d\sigma}{d\theta} = \frac{d\left(\pi b^2\right)}{d\theta} = 2\pi b \frac{db}{d\theta}$$

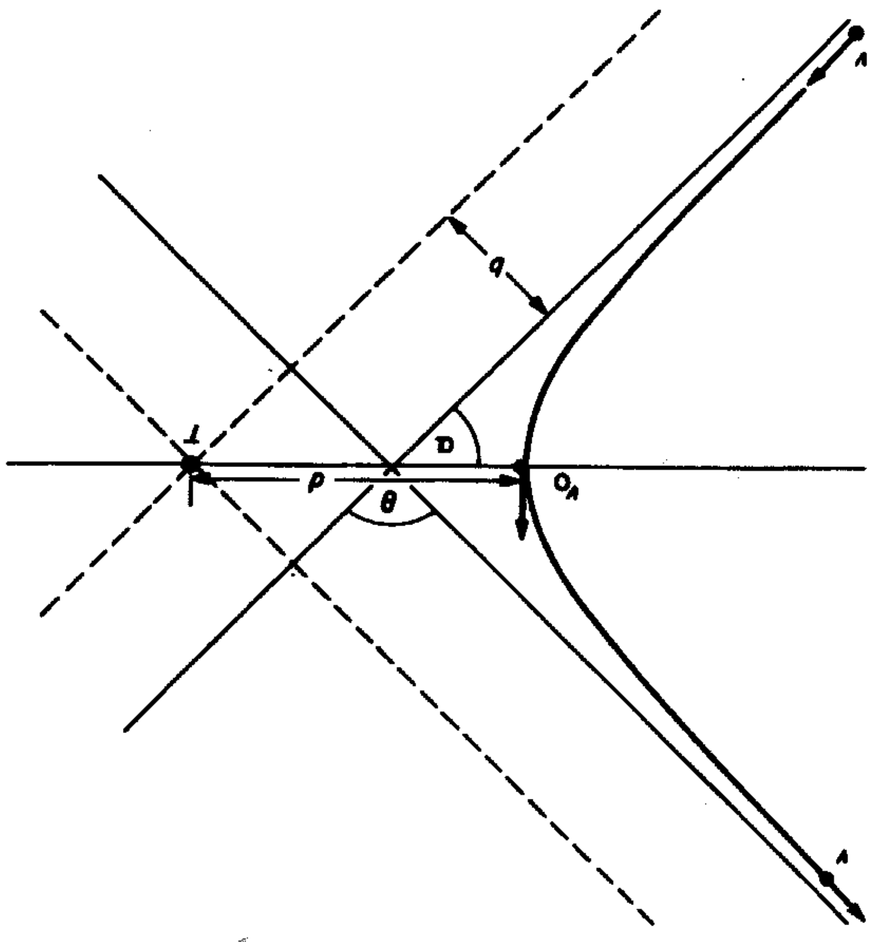
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\theta}\left(\frac{d\theta}{d\Omega}\right)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{2\pi \operatorname{sen}\theta}\right)\left(2\pi b \frac{db}{d\theta}\right) = \frac{b}{\operatorname{sen}\theta}\frac{db}{d\theta}$$



Para um potencial central

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R} \qquad M_A >> m_a$$



$$r = \frac{l/mC}{1 - E\cos\theta} \text{ onde } l = bp = (2mE)^{\frac{1}{2}}$$

$$E^2 = 1 + \frac{2El^2}{mc^2} = 1 + \frac{4E^2b^2}{c^2}$$

$$b = \frac{1}{4\pi E_0}\frac{Z_A Z_a}{2E}\cot^2\frac{\theta}{2}$$

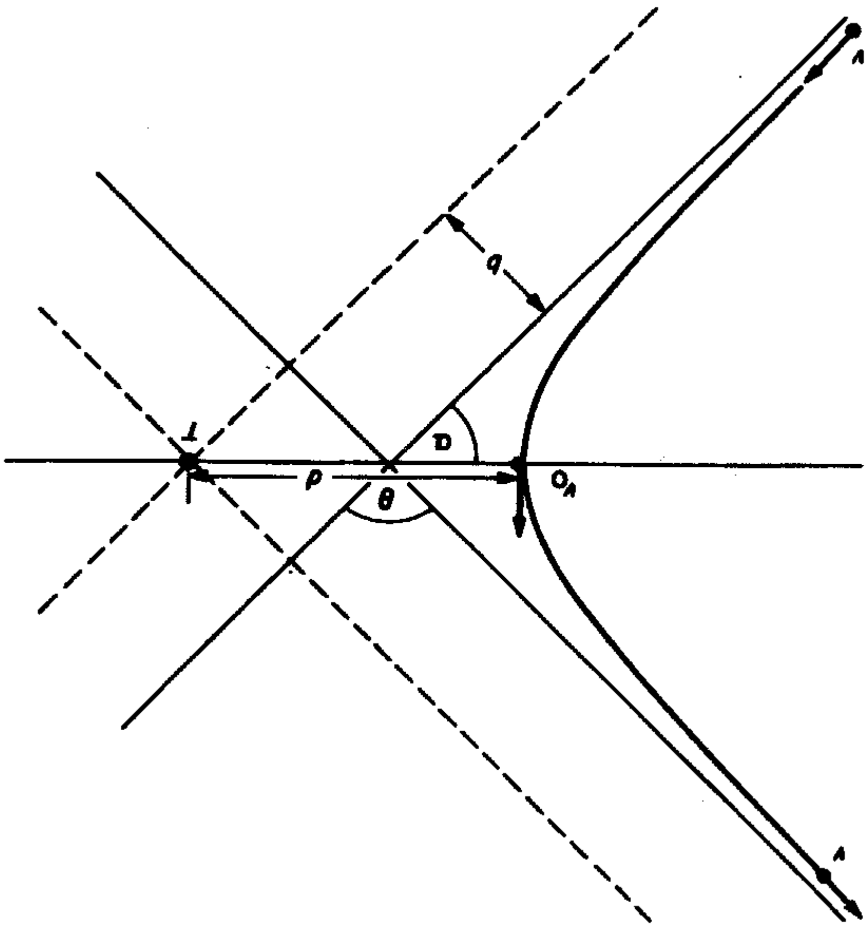
$$\Downarrow$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\operatorname{sen}\theta}\frac{db}{d\theta} = \left(\frac{Z_A Z_a e^2}{2E}\right)^2 \frac{1}{\operatorname{sen}^4\frac{\theta}{2}}$$

Para um potencial central

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R}$$

$M_A >> m_a$



(Distancia de máxima aproximação d)

$$\frac{1}{2} m v^2 = \frac{1}{2} m v_0^2 + \frac{Z_A Z_a e^2}{d}$$

Para uma colisão frontal (b=0) d = d_0

$$\left(\frac{v_0}{v}\right)^2 = 1 - \frac{d_0}{d} \text{ onde } d_0 = 2\frac{Z_A Z_a e^2}{mv^2} = \frac{Z_A Z_a e^2}{E}$$

onde

$$e^2 = 1{,}44\,MeVfm \Rightarrow d_0 = DMA \text{ para } b = 0$$

$mvb = mv_0 d$

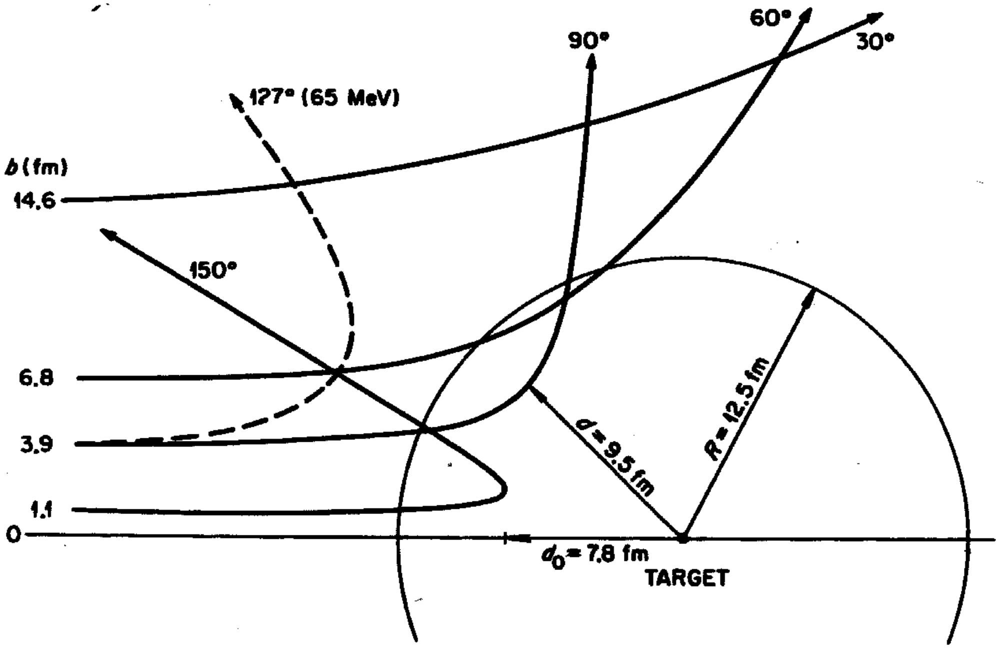
$b^2 = d(d-d_0)$

Propriedade de hiperbola: $d = b \cot \alpha/2$

$$\text{tg}\alpha = \frac{2b}{d_0} \qquad \theta = \pi - 2d \Rightarrow \cot g \frac{\theta}{2} = \frac{2b}{d_0}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{I_0}\frac{dI}{d\Omega} = \left(\frac{d_0}{4}\right)^2 \frac{1}{\text{sen}^4 \frac{\theta}{2}} = \left[\frac{Z_A Z_a e^2}{2E}\right]^2 \frac{1}{\text{sen}^4 \frac{\theta}{2}}$$

$$dI = I_0 2\pi b \, db \rightarrow dI = \frac{1}{4}\pi I_0 d_0^2 \frac{\cos\frac{\theta}{2}}{\sin^3\frac{\theta}{2}} d\theta$$

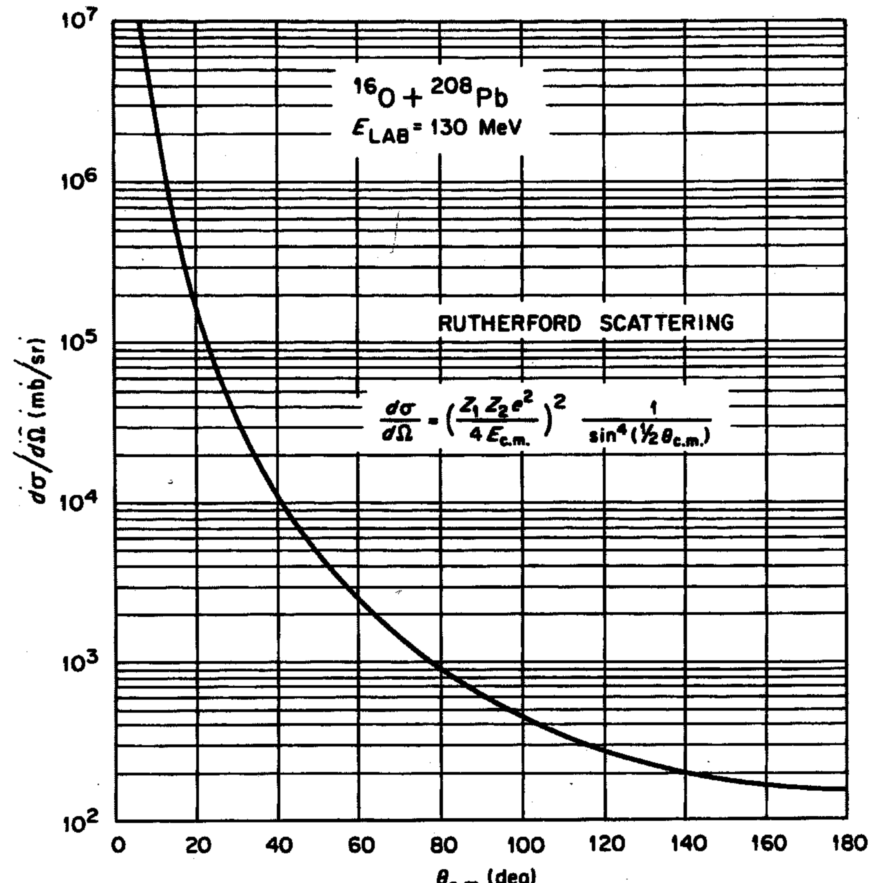


$$\frac{d\sigma}{d\Omega} = \left[\frac{Z_A Z_a e^2}{2E}\right]^2 \frac{1}{\text{sen}^4 \frac{\theta}{2}}$$

espalhamento:
^{208}Pb(^{16}O,^{16}O)^{208}Pb

na energia **E** $_{16O}$= 130MeV

a curva pontilhada corresponde a $E_{(16O)}$ = 65MeV



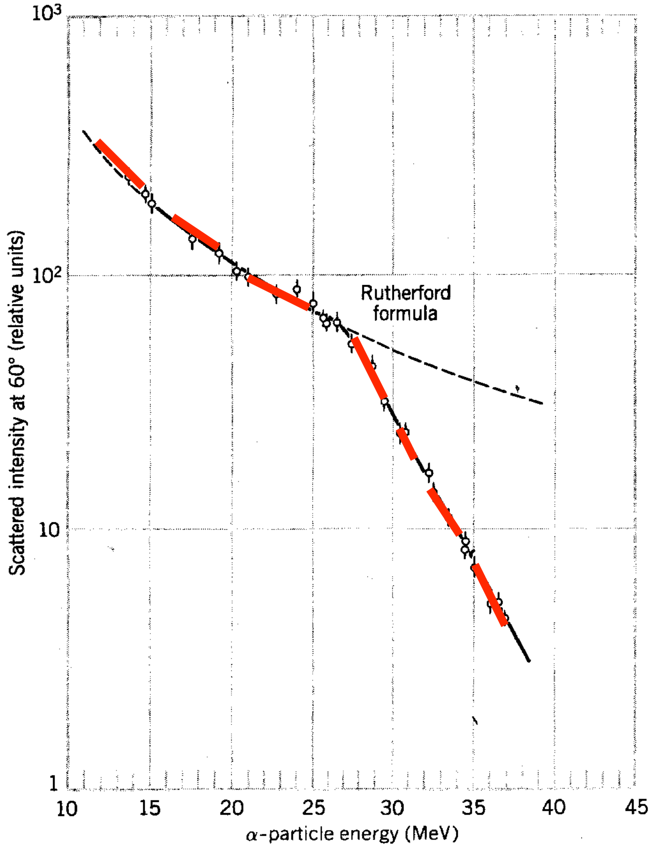


Figure 3.11 The breakdown of the Rutherford scattering formula. When the incident α particle gets close enough to the target Pb nucleus so that they can interact through the nuclear force (in addition to the Coulomb force that acts when they are far apart) the Rutherford formula no longer holds. The point at which this breakdown occurs gives a measure of the size of the nucleus. Adapted from a review of α particle scattering by R. M. Eisberg and C. E. Porter, *Rev. Mod. Phys.* **33**, 190 (1961).

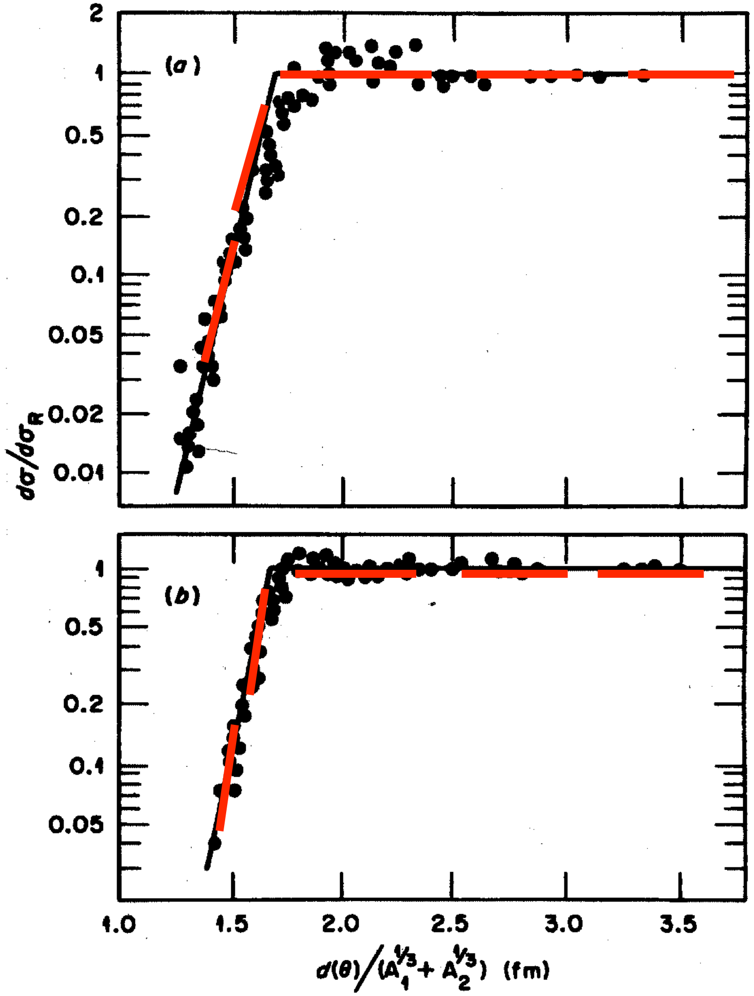


Figure 2.12 The differential cross-sections for O and C ions, in ratio to the Rutherford ones, scattering from various targets plotted against the distance of closest approach d, instead of the scattering angle θ, by using equation 2.20. The distance d has been divided by $(A_1^{1/3}+A_2^{1/3})$, where A_i is the mass number of nucleus i. The measured cross-sections then fall on a universal curve, showing that nuclear radii are approximately proportional to $A^{1/3}$. (a) $^{16}O + ^{40,48}Ca$ at 49 MeV, $^{16}O + ^{40,48}Ca$, ^{50}Ti, ^{52}Cr, ^{54}Fe, ^{62}Ni at 60 MeV and $^{18}O + ^{60}Ni$ at 60 MeV; (b) $^{12}C + ^{96}Zr$ at 38 MeV, $^{16}O + ^{96}Zr$ at 47, 49 MeV, $^{16}O + ^{88}Sr$, ^{93}Zr at 60 MeV and $^{18}O + ^{90}Zr$ at 60, 66 MeV. (After Christensen *et al.*, 1973)

BARREIRA COULOMBIANA

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R}$$

Para a colisão razante:

$$R_c = r_0 A_A^{1/3} + r_0 A_a^{1/3} = r_0 (A_A^{1/3} + A_a^{1/3})$$

$$V_{Coul} = \frac{Z_A Z_a e^2}{r_0 (A_A^{1/3} + A_a^{1/3})}$$

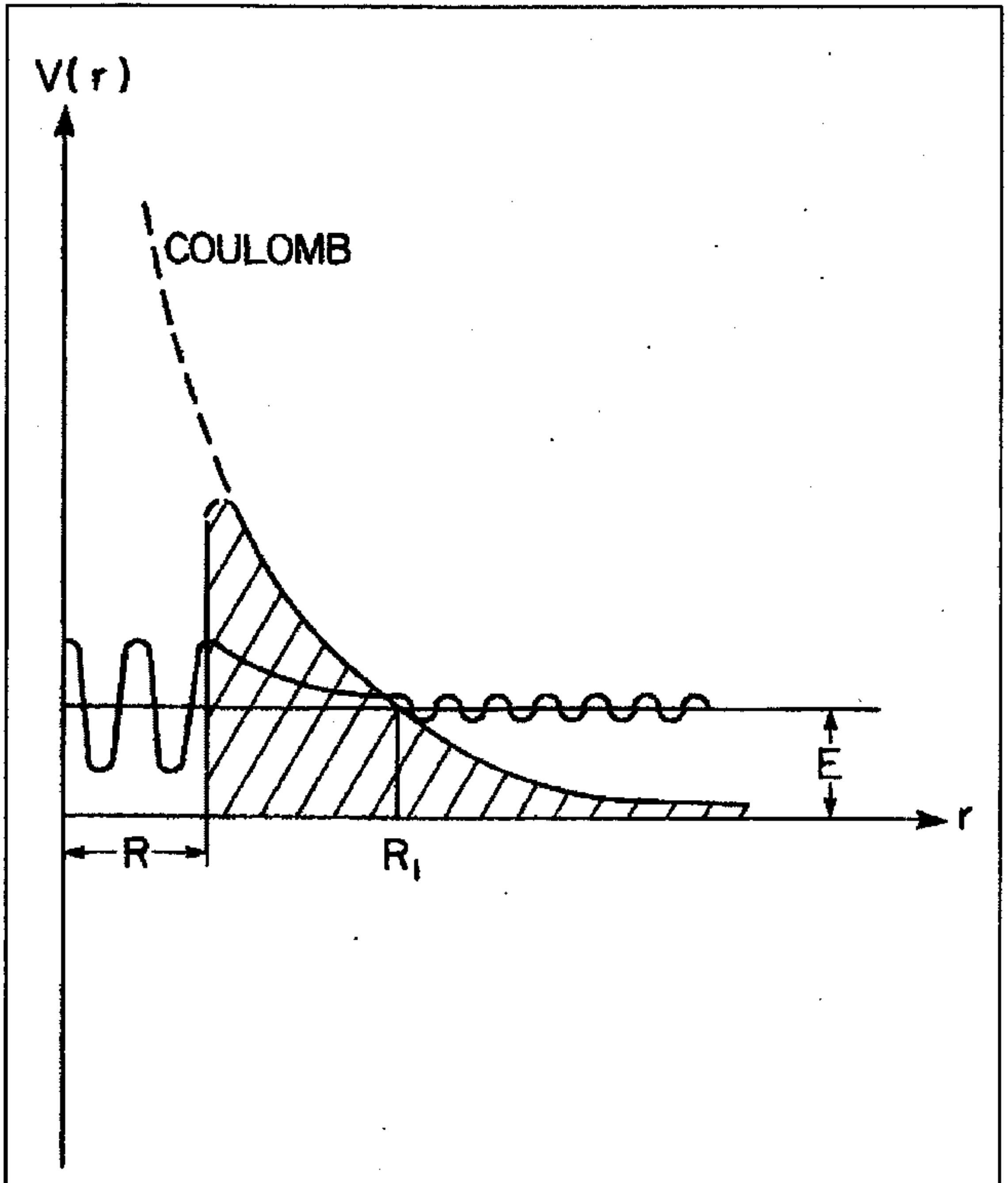
$E_{cm} > V_{coul}$ ==> $T_l(E_{cm}) = 1$

$E_{cm} < V_{coul}$ ==> $T_l(E_{cm}) = 0$

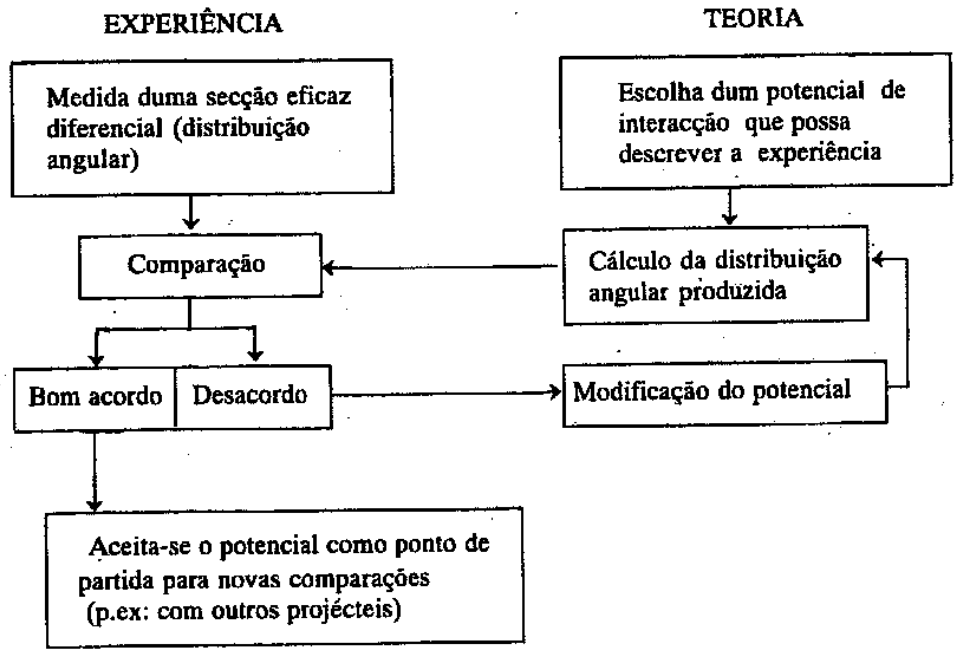
$e^2 = 1.44$ MeV.fm

$r_0 \sim 1.25$ fm

V_{coul} (MeV)



V(r)
COULOMB
E
r
R
R_1



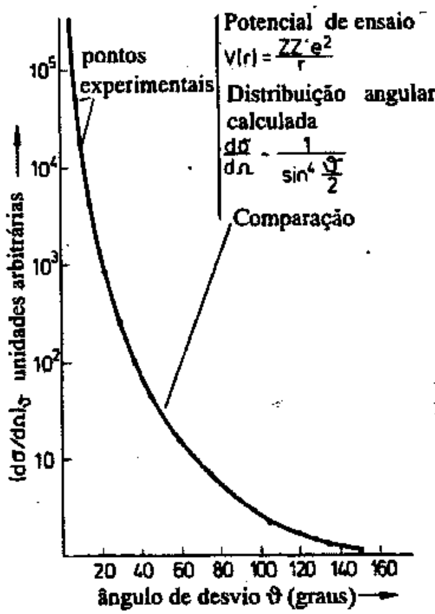
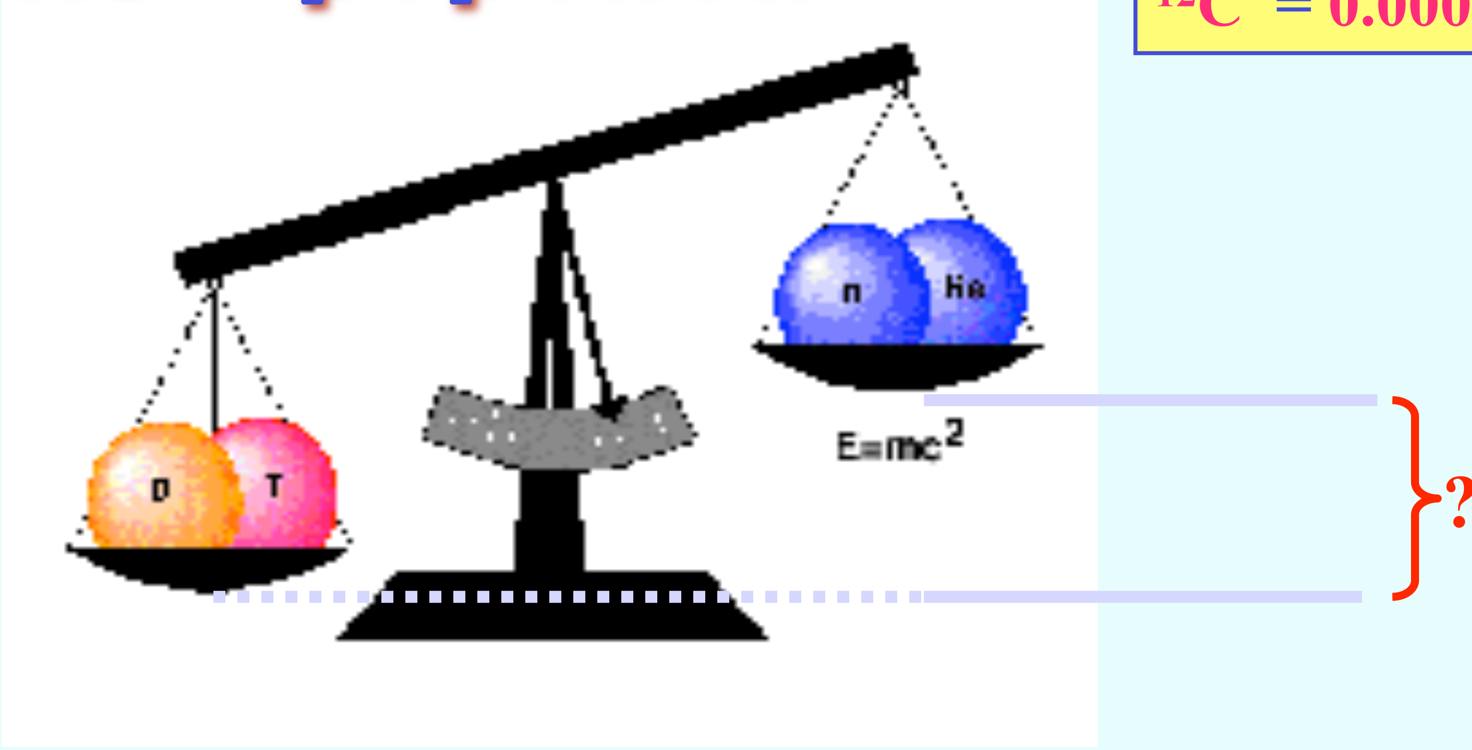


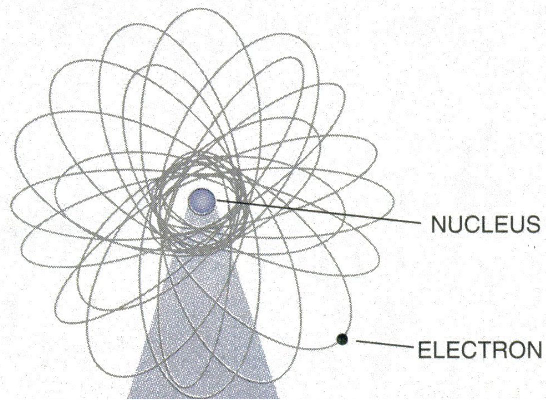
Fig.5 a) Método usado para estudar um potencial dispersor a partir duma distribuição angular.
b) Exemplo: Resultado de experiências de Rutherford realizadas com partículas α de 5,5 MeV sobre um alvo de ouro (Geiger e Marsden, 1913)

massa dos nucleos

d = p+n
t = p+n+n
4He = p+p+n+n

n =1.00866 u.m.a.
p = 1.0079 u.m.a.
d = 2.01410 u.m.a.
t = 3.01860 u.m.a.
^{4}He = 4. 00260 u.m.a.
^{6}Li = 6. 01512 u.m.a.
^{12}C = 0.00000 u.m.a.





$M_{(proton)} = M_p = 938.27$ MeV

$M_{(neutron)} = M_n = 939.56$ MeV

$M_{(eletron)} = M_e = 0.511$ MeV

$M_{(\text{átomo de Hidrogênio})} = M_H = 938.58$ MeV

$M_{(16O)} = 14$ 899.17 MeV

$M_p + M_e = 938.27 + 0.511 =$ 938.78 MeV

$8M_p + 8M_n = 15$ 022.64 MeV

$\Delta M =$ 123.47 MeV

$M_H =$ 938.58 MeV

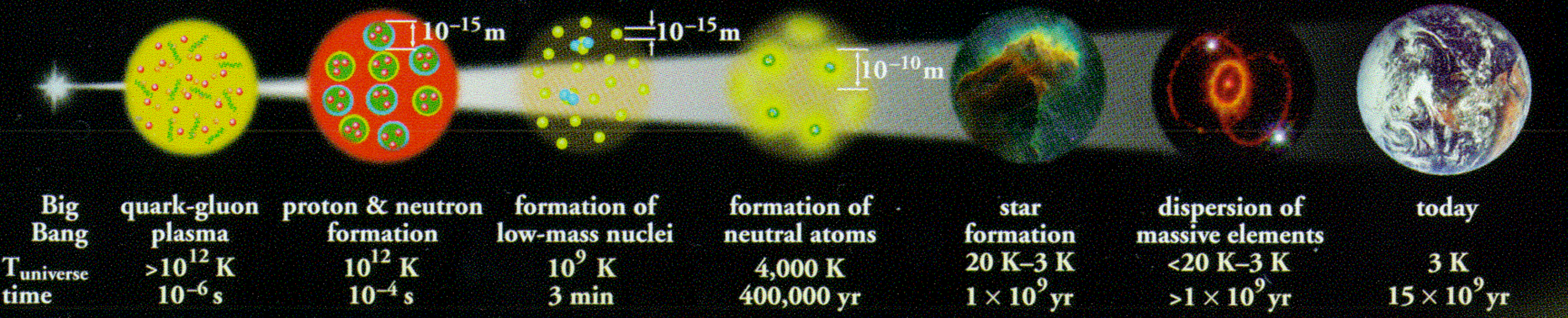
$p = (u + u + d)$

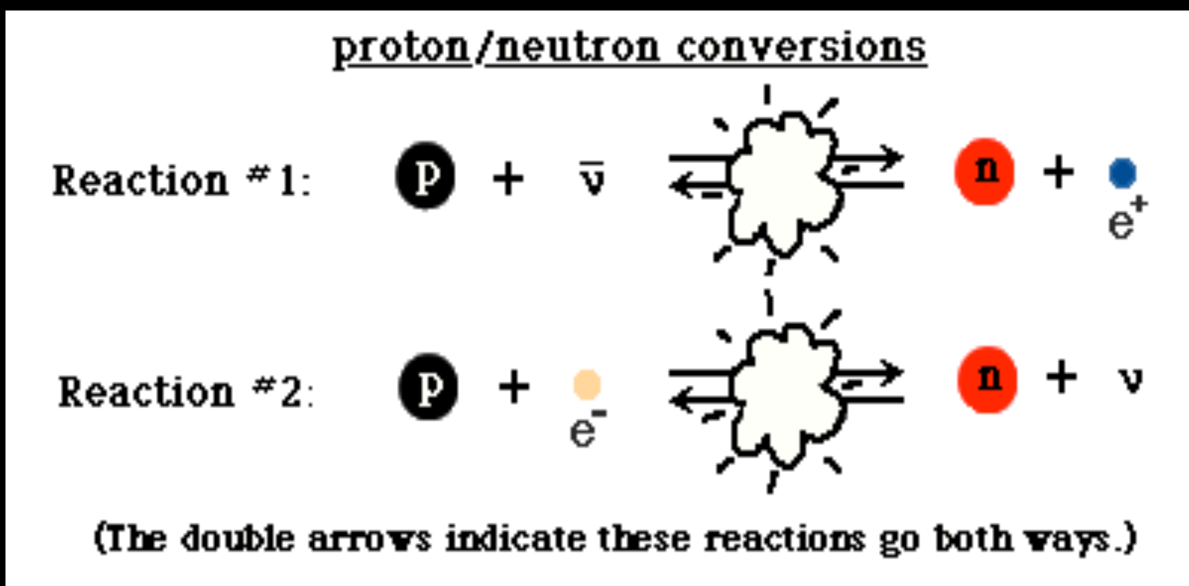
$M_{(u)} \approx 3$ MeV

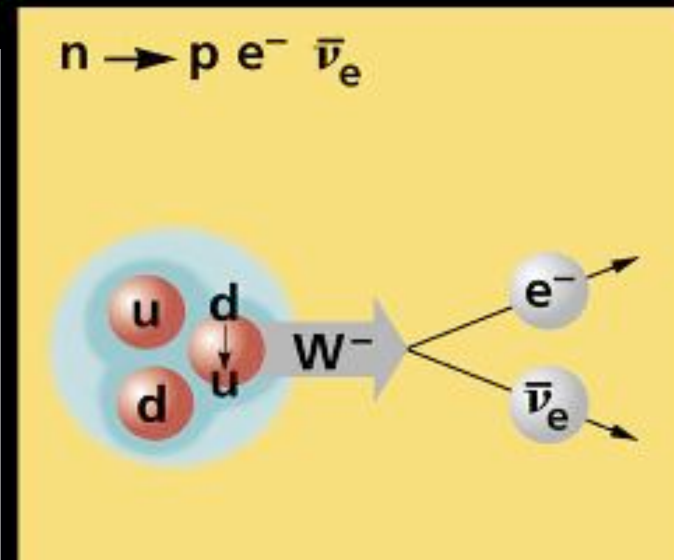
$M_{(d)} \approx 6$ MeV

Expansion of the Universe

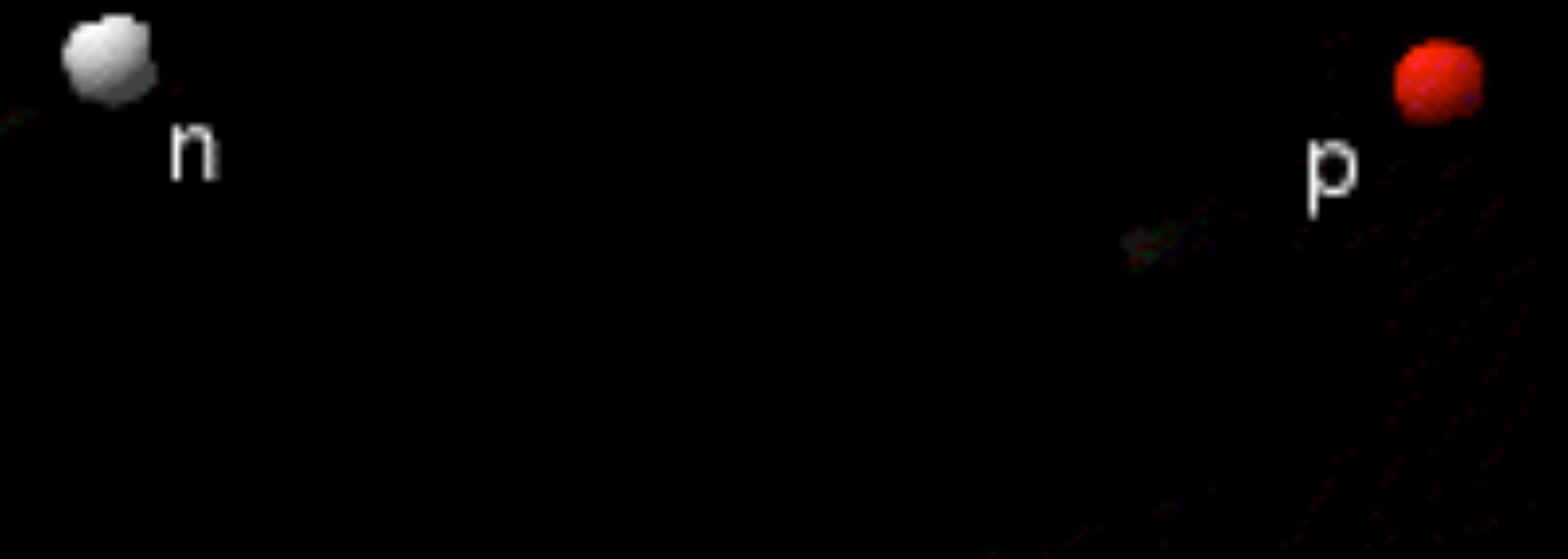
After the Big Bang, the universe expanded and cooled. At about 10^{-6} second, the universe consisted of a soup of quarks, gluons, electrons, and neutrinos. When the temperature of the Universe, $T_{universe}$, cooled to about 10^{12} K, this soup coalesced into protons, neutrons, and electrons. As time progressed, some of the protons and neutrons formed deuterium, helium, and lithium nuclei. Still later, electrons combined with protons and these low-mass nuclei to form neutral atoms. Due to gravity, clouds of atoms contracted into stars, where hydrogen and helium fused into more massive chemical elements. Exploding stars (supernovae) form the most massive elements and disperse them into space. Our earth was formed from supernova debris.



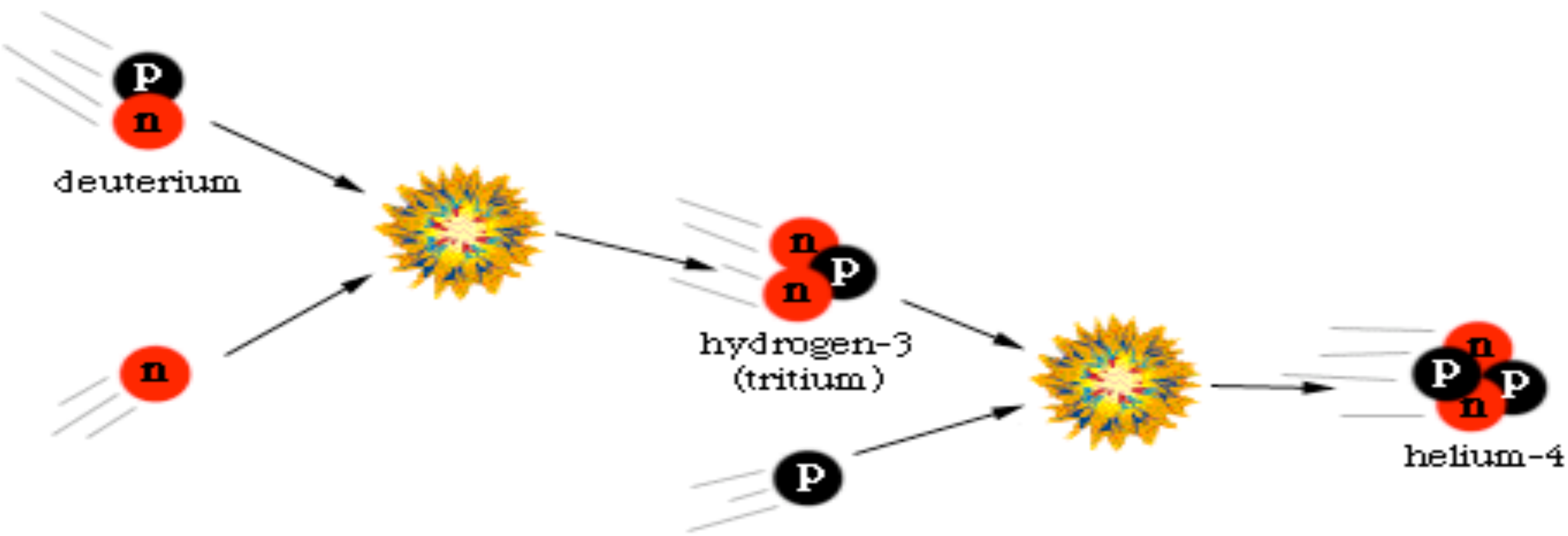




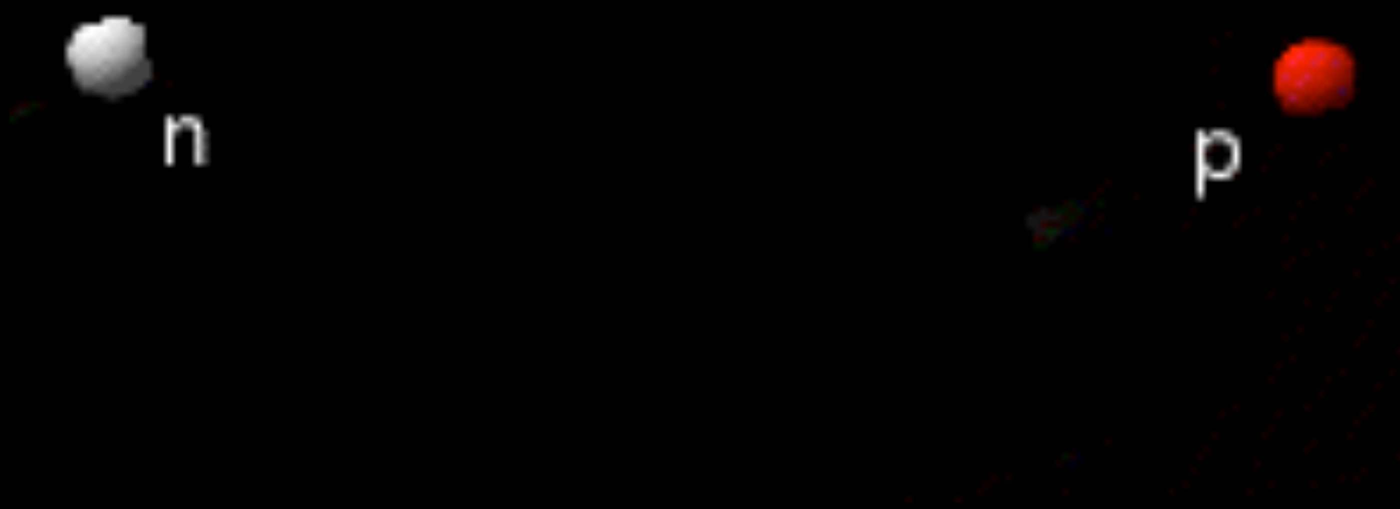
A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.


n
p

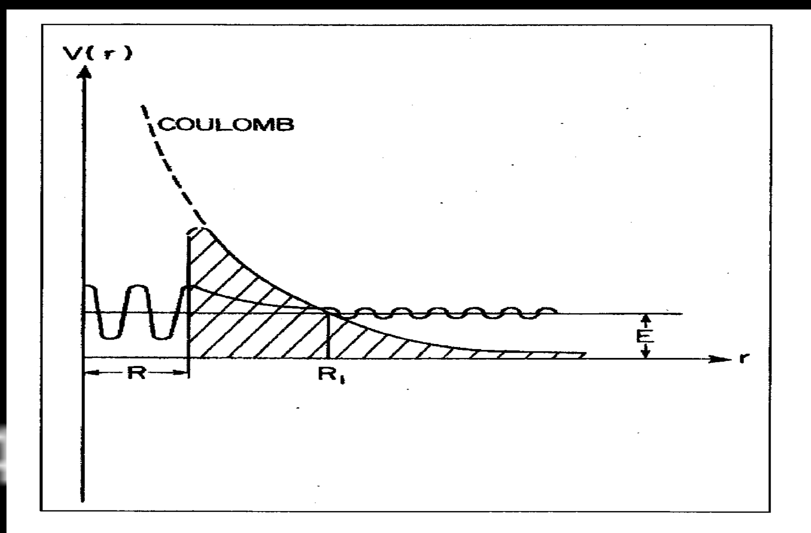

p
2H

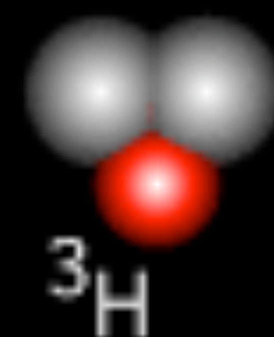

p
n
deuterium
n
n
p
n
hydrogen-3
(tritium)
p
n
p
p
n
helium-4

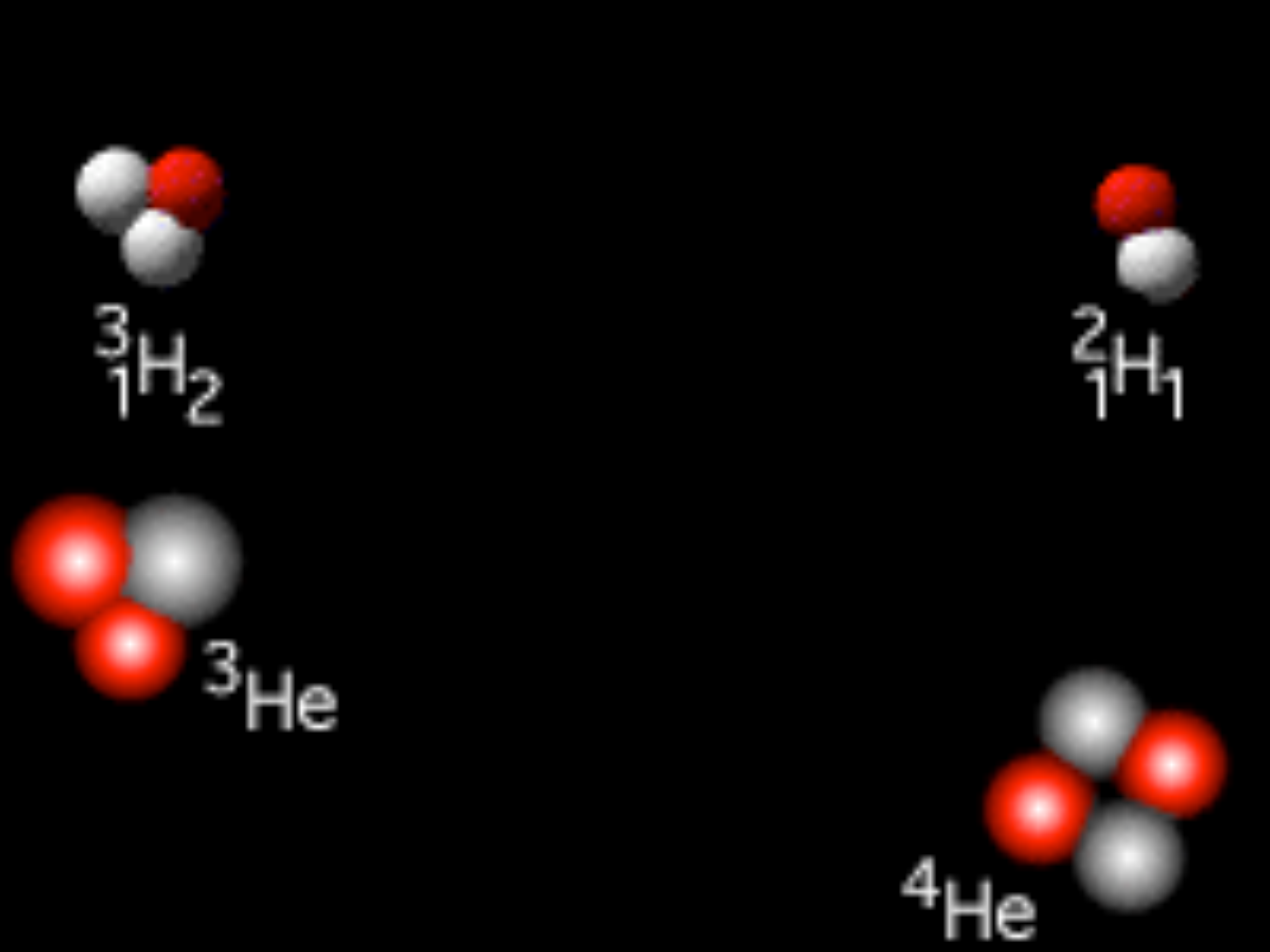
ESPALHAMENTO



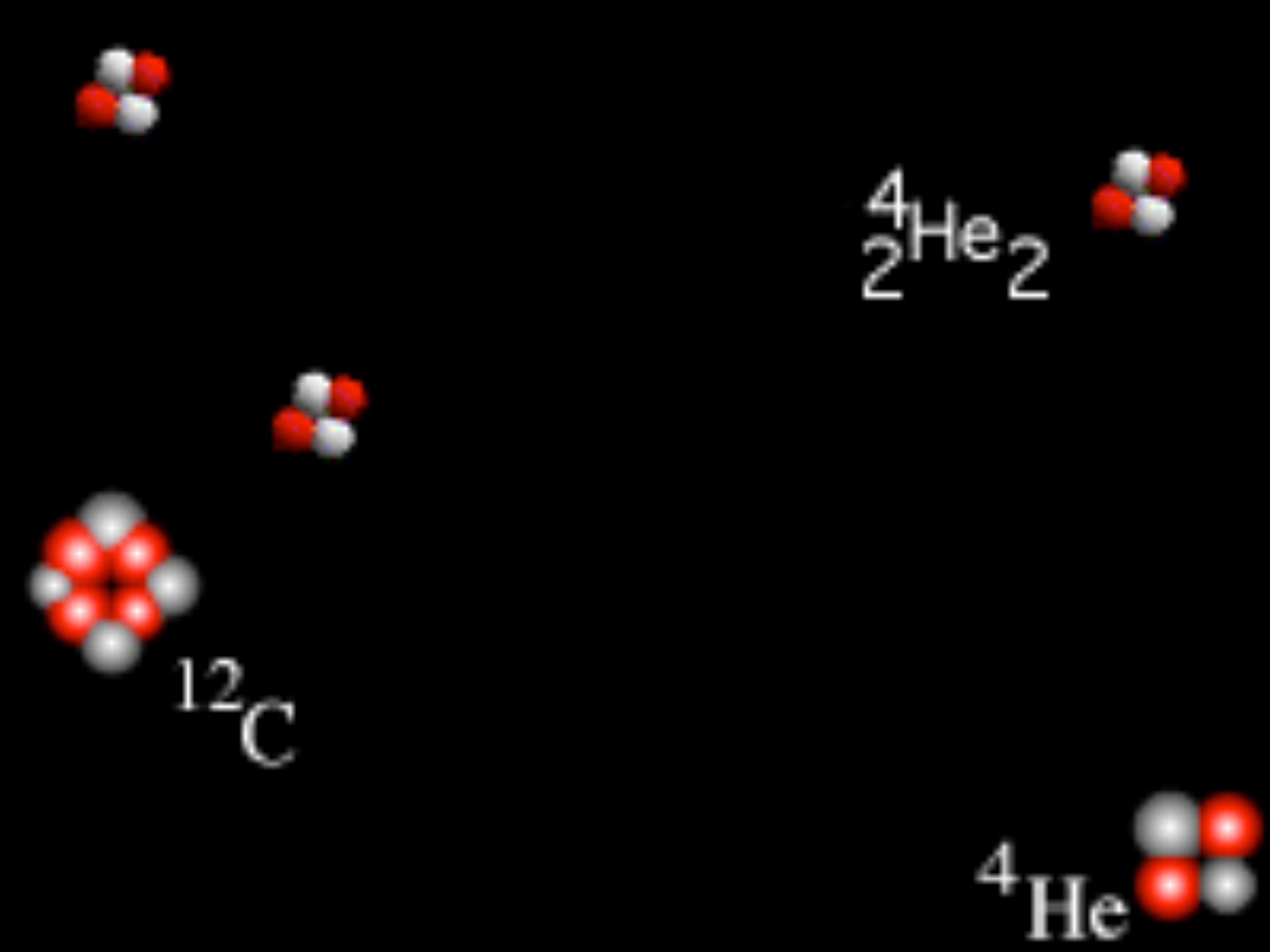
TUNELAMENTO

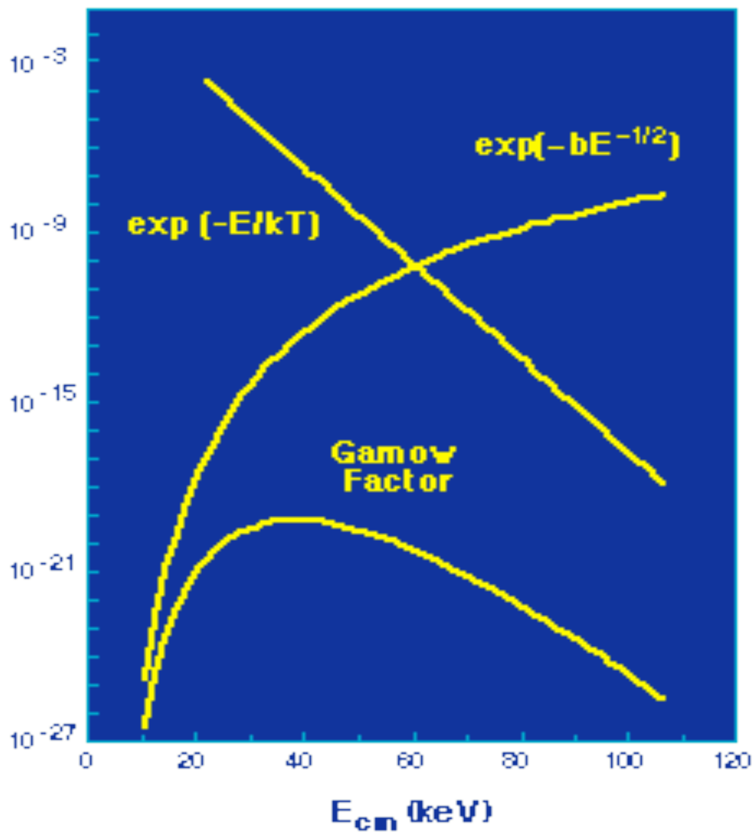




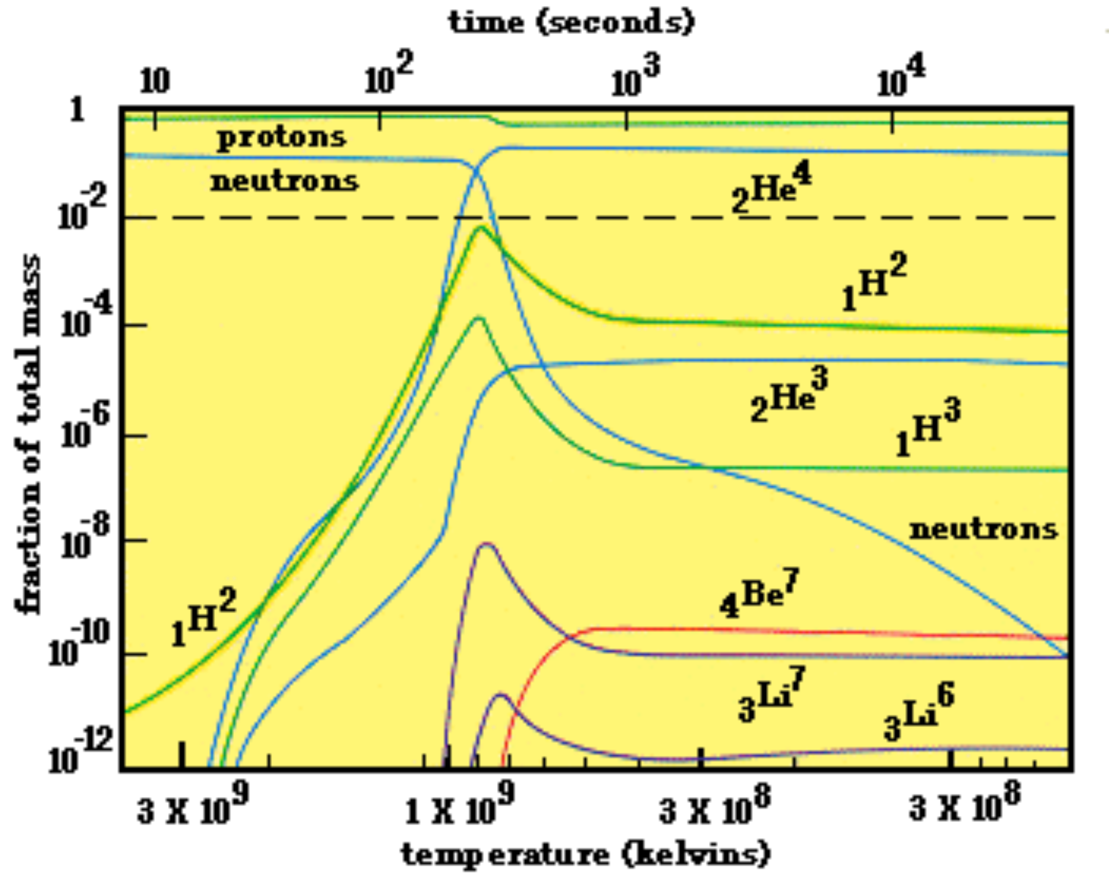


$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$
^{3}He
^{4}He

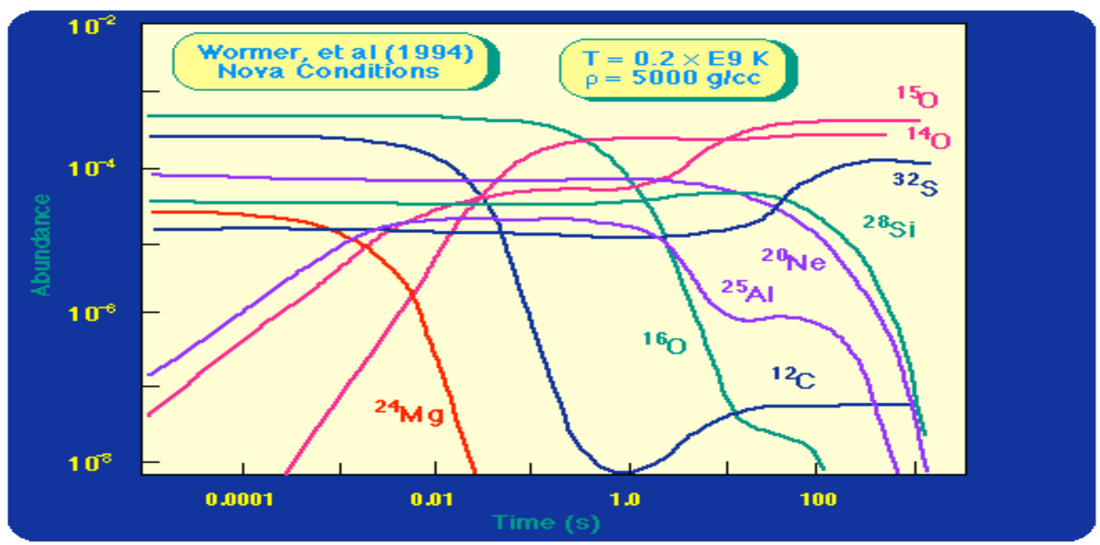

$^{4}_{2}He_{2}$
^{12}C
^{4}He



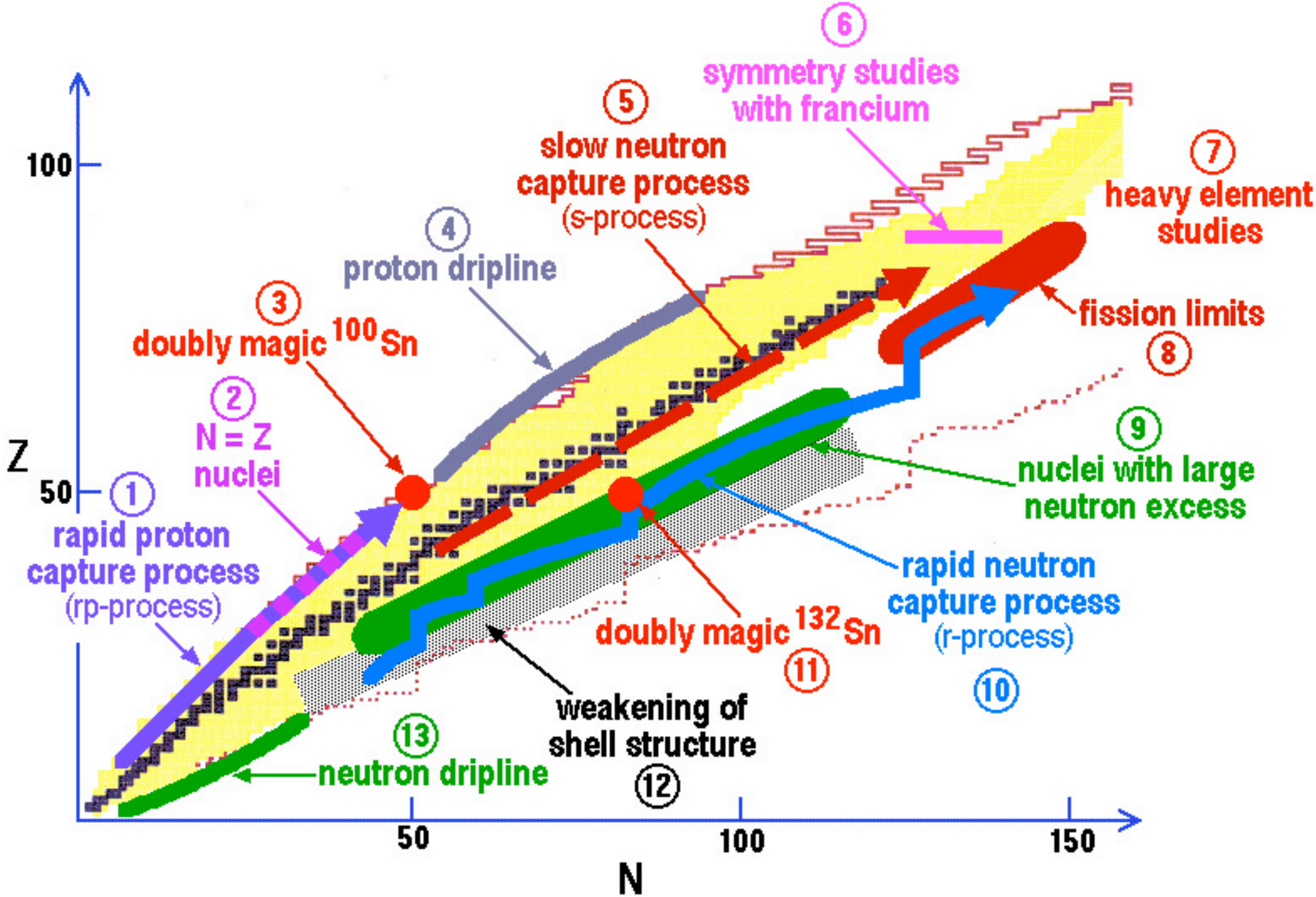
10 -3
10 -9
10 -15
10 -21
10 -27
exp(-bE-1/2)
exp (-E/kT)
Gamow Factor
0
20
40
60
80
100
120
Ecm (keV)



time (seconds)
10
10^2
10^3
10^4
1
10^-2
10^-4
10^-6
10^-8
10^-10
10^-12
fraction of total mass
protons
neutrons
2He4
1H2
2He3
1H3
neutrons
4Be7
1H2
3Li7
3Li6
3 X 10^9
1 X 10^9
3 X 10^8
3 X 10^8
temperature (kelvins)



Wormer, et al (1994)
Nova Conditions
T = 0.2 × E9 K
ρ = 5000 g/cc
10^-2
10^-4
10^-6
10^-8
Abundance
15O
14O
32S
28Si
20Ne
25Al
16O
12C
24Mg
0.0001
0.01
1.0
100
Time (s)



Z
100
50
N
50
100
150
1
rapid proton capture process (rp-process)
2
N = Z nuclei
3
doubly magic ^{100}Sn
4
proton dripline
5
slow neutron capture process (s-process)
6
symmetry studies with francium
7
heavy element studies
fission limits
8
9
nuclei with large neutron excess
rapid neutron capture process (r-process)
10
doubly magic ^{132}Sn
11
weakening of shell structure
12
13
neutron dripline


n
235U
92
143

d = p+n

t = p+n+n

4He = p+p+n+n

n =1.00866 u.m.a.

p = 1.0079 u.m.a.

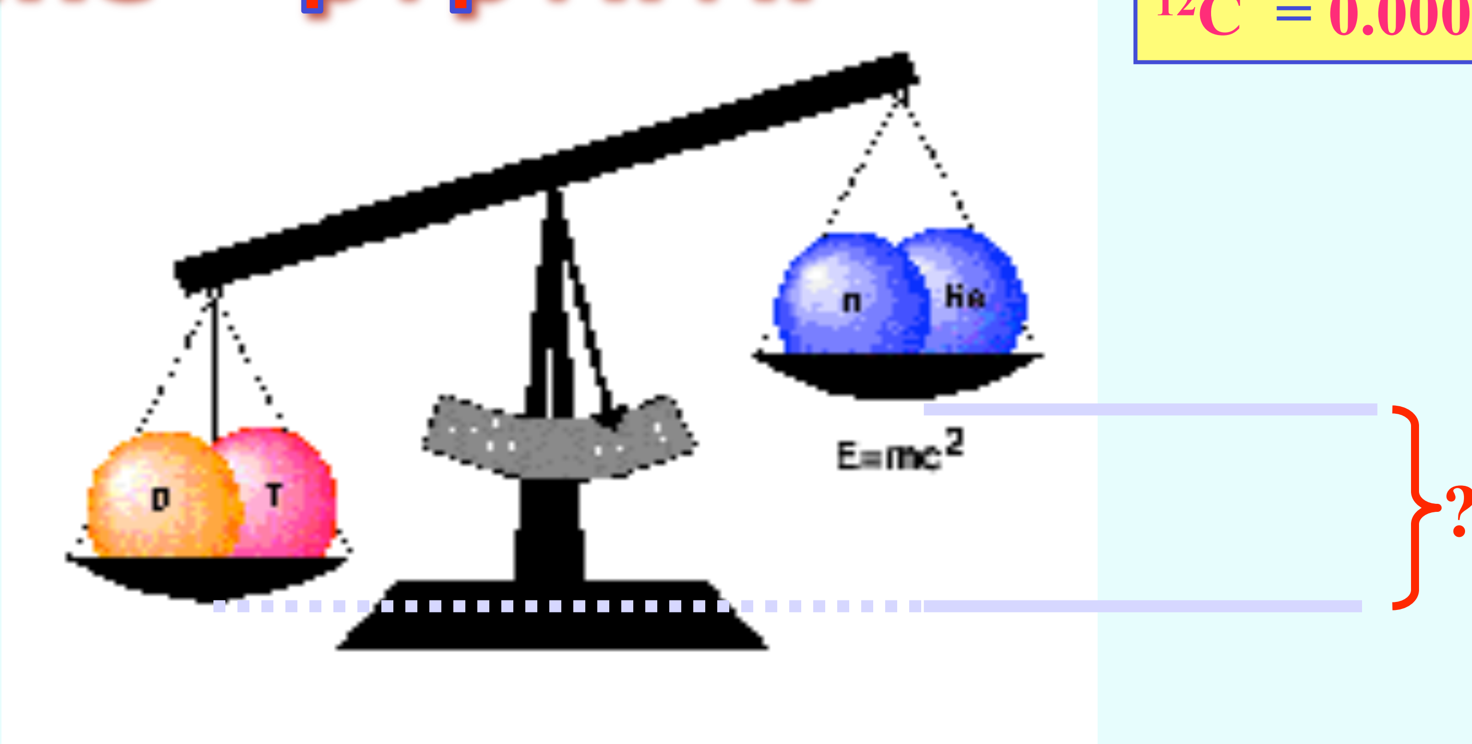
d = 2.01410 u.m.a.

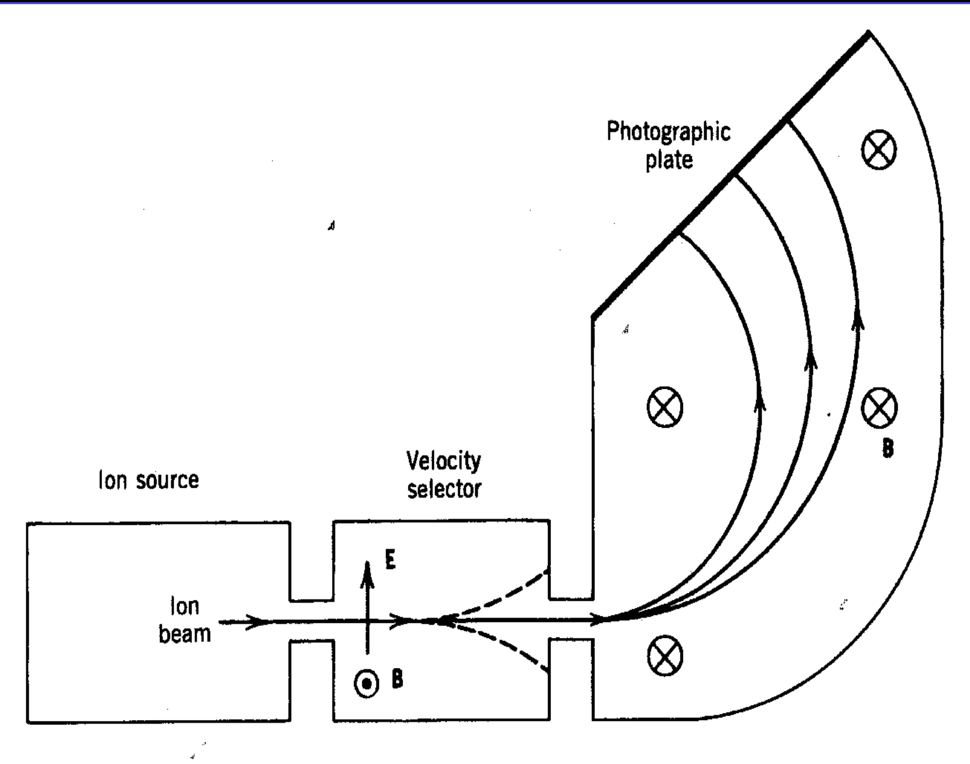
t = 3.01860 u.m.a.

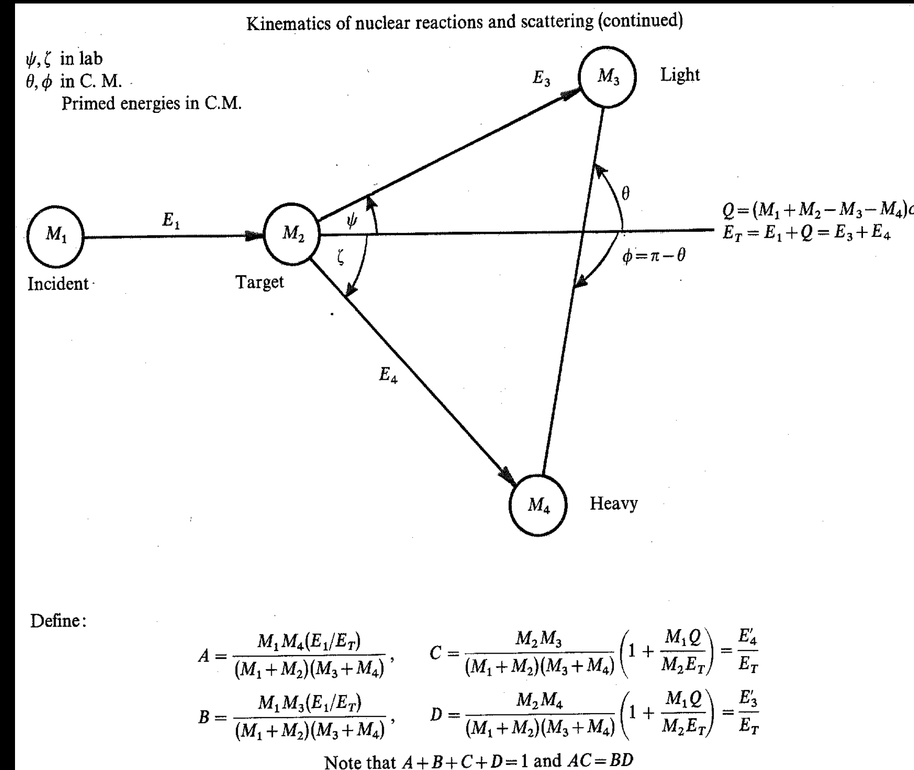
^{4}He = 4. 00260 u.m.a.

^{6}Li = 6. 01512 u.m.a.

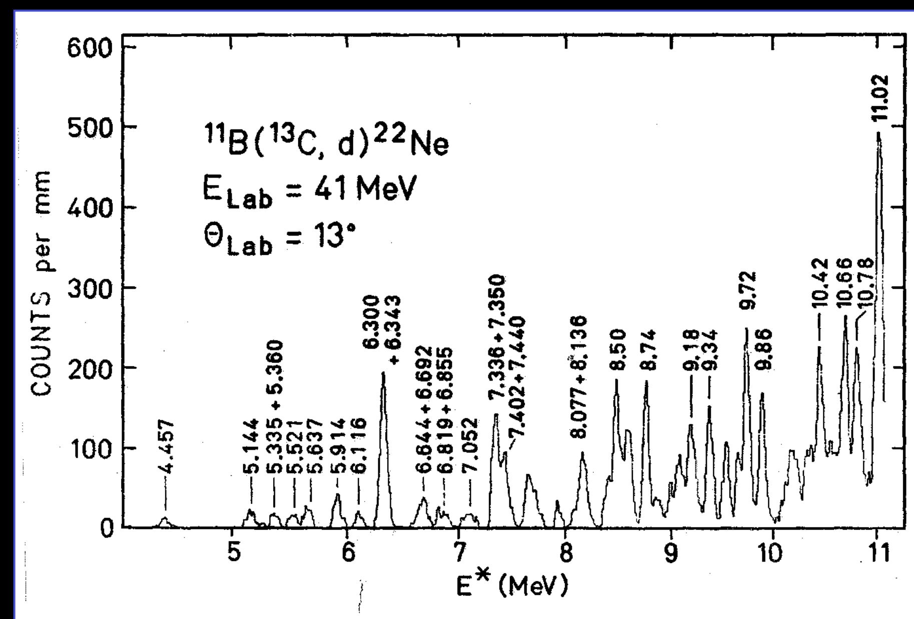
^{12}C = 0.00000 u.m.a.







$$\frac{mv^2}{r} = qvB \Rightarrow m = \frac{qrB^2}{E}$$



$(m_{12C}) = 12.0000\ u$ → $u = m_u = (m_{12C})/12$

$1u = 1.66056 \times 10^{-24}\ g$

$\Rightarrow m_u c^2 = 931.50\ MeV/c^2$

m_e	0,511 MeV
m_n	939,566 MeV
m_p	938,272 MeV
m_d	1875,613 MeV
$m\ (^3He)$	2808,350 MeV
m_α	3727,323 MeV
u	931,494 MeV

$Z+N = A$

$m(Z,N) = Zm_H + Nm_n - B(Z,N)/c^2$

$B(Z,N) = [\ Zm_H + Nm_n - m(Z,N)]\ c^2$

$B(Z,N) = [\ \Delta m\ ]\ c^2$

Excesso de massa (mass excess)
Δ (MeV)

$$\Delta_A \text{ (MeV)} = (m_A - A)u \,.\, c^2$$

$$\Delta(^{12}C) = 0$$

dada a reação: A(B,C)D

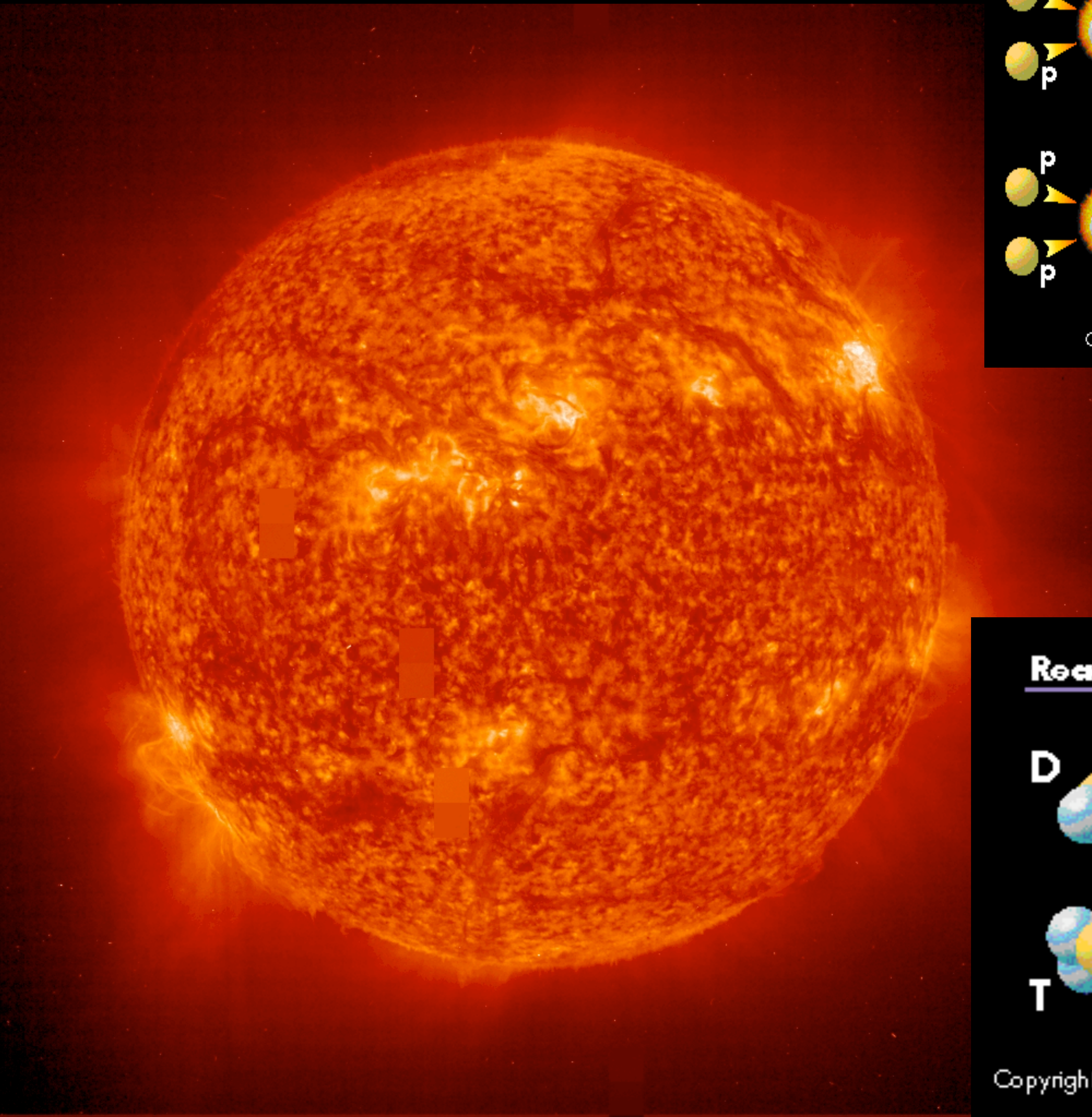
$$A\,(Z_A,N_A) + B\,(Z_B,N_B) \rightarrow C\,(Z_C,N_C) + D\,(Z_D,N_D)$$

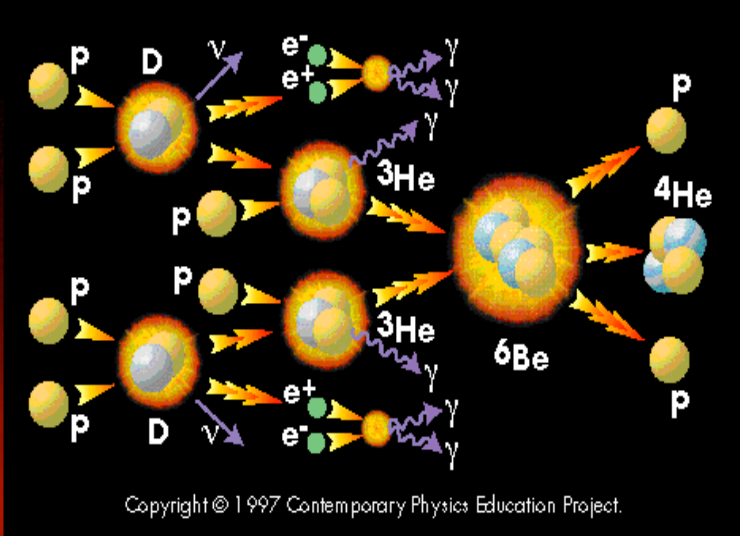
Definimos o valor de "Q" de uma reação

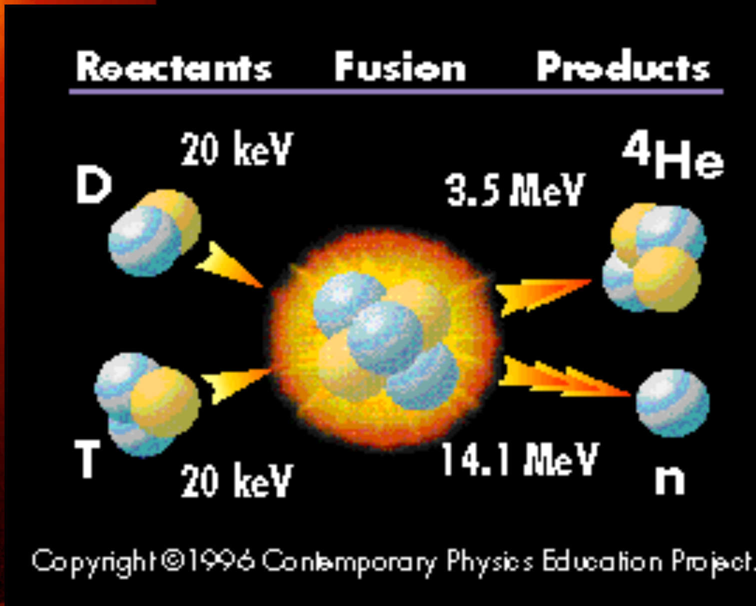
$$(M_A + M_B) = (M_C + M_D) + Q(A+B \rightarrow C+D)$$

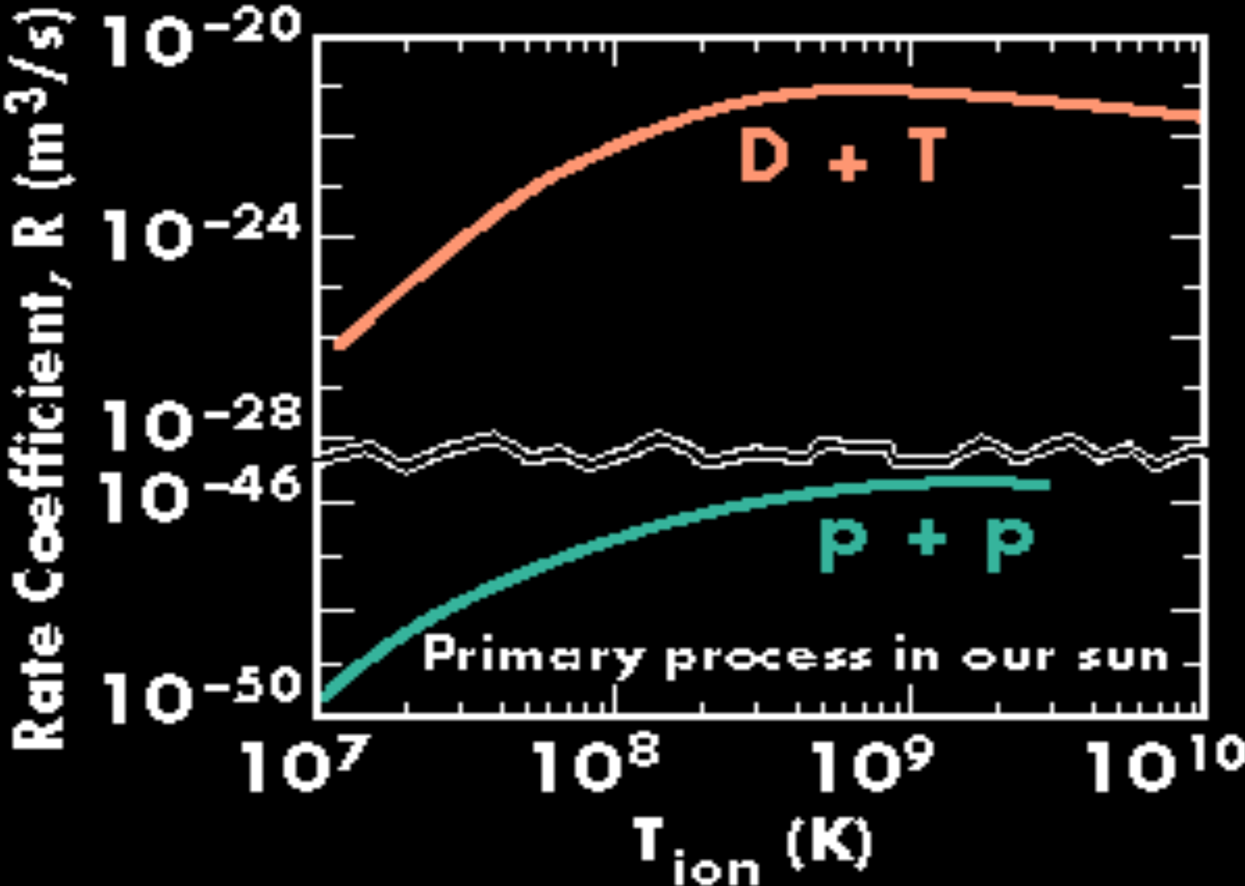
$$Q_{(A+B \rightarrow C+D)} = (B_C + B_D) - (B_A + B_B)$$

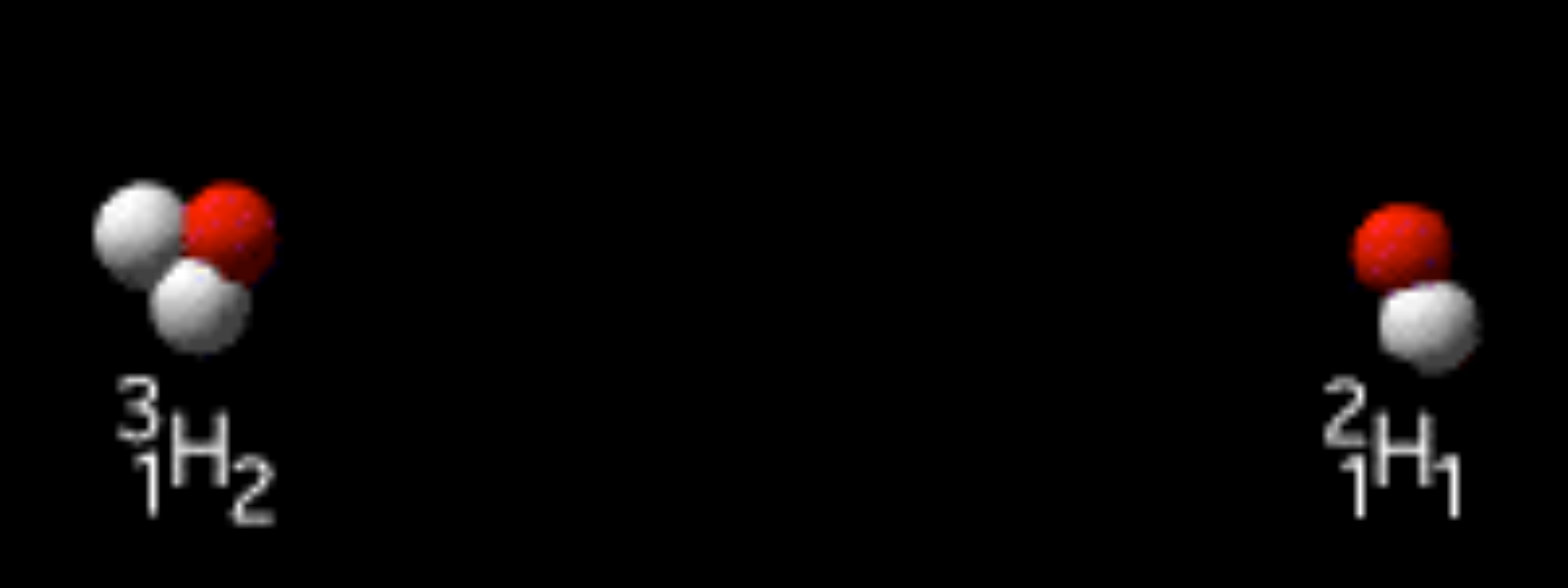
$$Q_{(A+B \rightarrow C+D)} = (\Delta_C + \Delta_D) - (\Delta_A + \Delta_B)$$

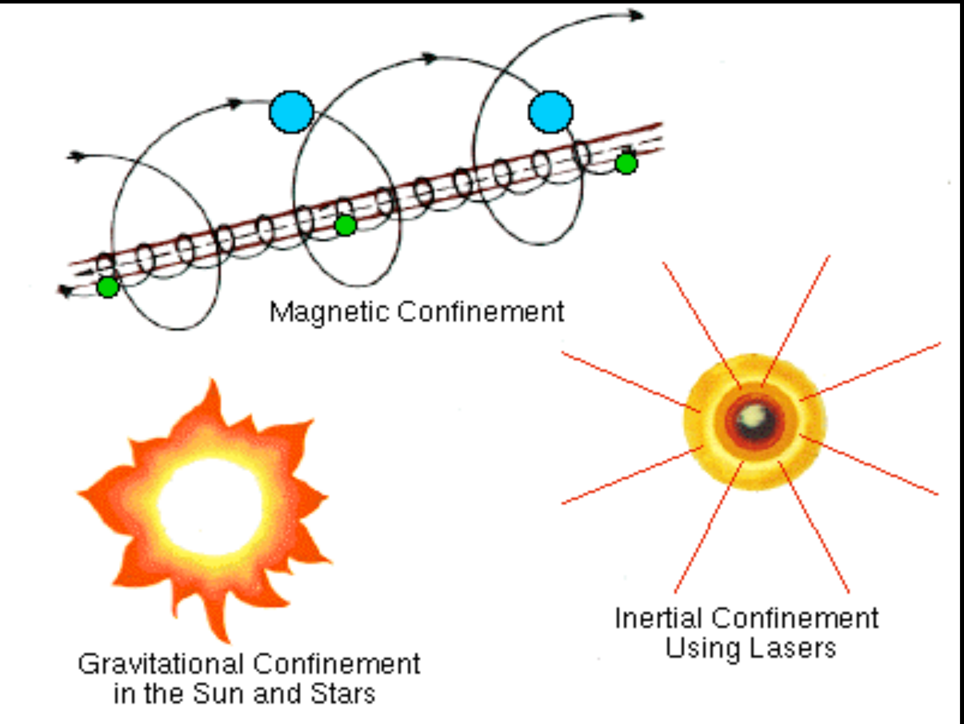


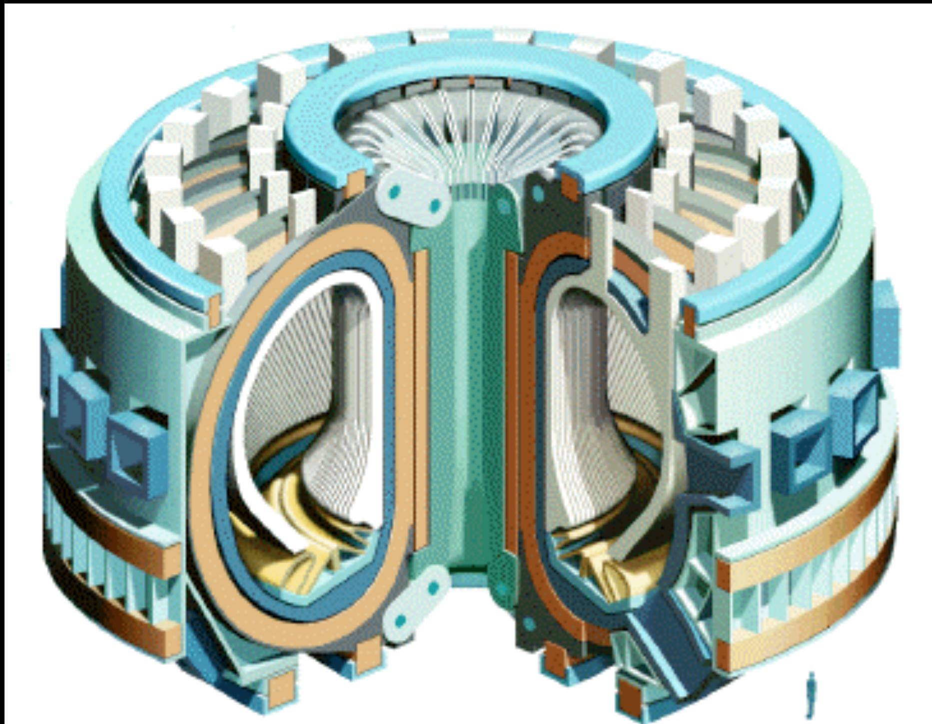

p
D
ν
e-
e+
γ
γ
p
γ
3He
p
p
4He
p
p
3He
6Be
γ
p
e+
γ
D
ν
e-
γ
p
p
Copyright © 1997 Contemporary Physics Education Project.


Reactants
Fusion
Products
20 keV
D
3.5 MeV
4He
T
20 keV
14.1 MeV
n
Copyright ©1996 Contemporary Physics Education Project.


Rate Coefficient, R (m3/s)
10-20
10-24
10-28
10-46
10-50
D + T
p + p
Primary process in our sun
107
108
109
1010
Tion (K)


$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$


Magnetic Confinement
Inertial Confinement Using Lasers
Gravitational Confinement in the Sun and Stars

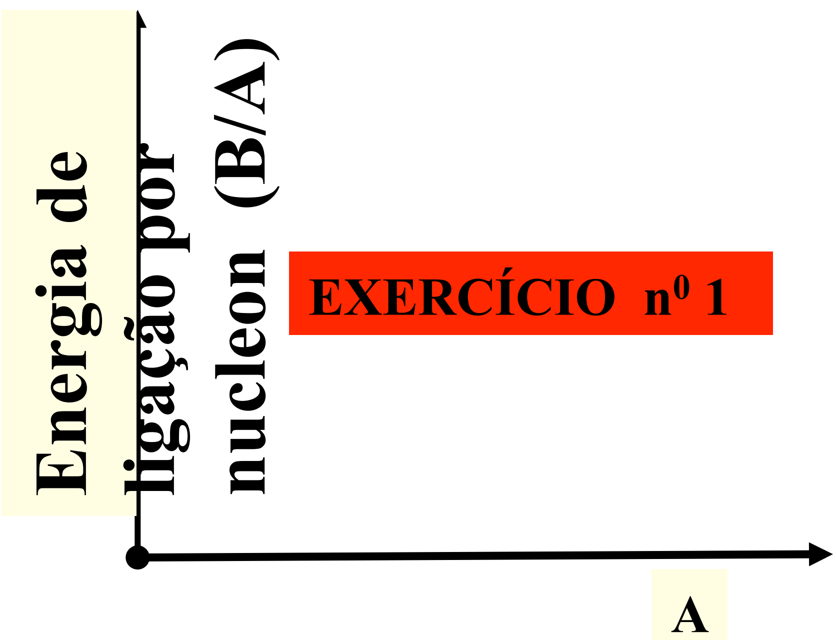


N	Z	A	EL	O	MASS EXCESS (KEV)		B/A	BINDING ENERGY (KEV)		ATOMIC MASS (U)	
1	0	1	N		8071.69	0.10		0.0	0.0	1.00866522	0.00000006
0	1		H		7289.22	0.09		0.0	0.0	1.00782522	0.00000004
1	1	2	H		13136.27	0.16	1.11	2224.64	0.04	2.01410222	0.00000007
2	1	3	H		14950.38	0.22	2.42	8482.22	0.15	3.01604972	0.00000016
1	2		HE		14931.73	0.22	2.57	7718.40	0.14	3.01602970	0.00000016
3	1	4	H	-N	25920	500		5580	500	4.02783	0.00054
2	2		HE		2424.94	0.25	7.07	28296.9	0.4	4.00260326	0.00000027
1	3		LI	+NN	25130	300		4810	300	4.02697	0.00032
4	1	5	H	+	33790	800		5790	800	5.03627	0.00086
3	2		HE	-N	11390	50		27410	50	5.01222	0.00005
2	3		LI	-P	11680	50		26330	50	5.01254	0.00005
4	2	6	HE		17597.3	3.6		29267.9	3.6	6.0188913	0.0000039
3	3		LI		14087.5	0.7	5.33	31995.2	0.8	6.0151234	0.0000008
2	4		BE	-	18375	5		26926	5	6.019726	0.000006
5	2	7	HE	+	26111	30		28826	30	7.028031	0.000032
4	3		LI		14908.6	0.8	5.60	39245.9	0.9	7.0160048	0.0000008
3	4		BE		15770.3	0.8		37601.6	0.9	7.0169299	0.0000008
2	5		B	-	27940	100		24650	100	7.02999	0.00011
6	2	8	HE	+	31650	120		31360	120	8.03397	0.00013
5	3		LI	-N	20947.5	1.0		41278.6	1.2	8.0224879	0.0000011
4	4		BE		4941.8	0.5	7.06	56501.9	0.8	8.0053052	0.0000005
3	5		B	-PP	22922.3	1.2		37738.8	1.3	8.0246079	0.0000013
6	3	9	LI	+	24966	5		45331	5	9.026802	0.000005
5	4		BE		11348.4	0.6	6.46	58167.0	0.9	9.0121828	0.0000006
4	5		B	-	12415.7	0.9		56317.1	1.1	9.0133287	0.0000010
3	6		C		28912	5		39038	5	9.031038	0.000006
7	3	10	LI	-N	35340	SYST		43030	SYST	10.03794	SYST
6	4		BE		12608.1	0.7		64978.9	1.0	10.0135352	0.0000008
5	5		B		12052.3	0.4	6.47	64752.3	0.9	10.0129385	0.0000004
4	6		C		15702.7	1.8		60319.4	2.0	10.0168573	0.0000020
8	3	11	LI	-N	43310	SYST		43130	SYST	11.04649	SYST
7	4		BE		20177	6		65482	6	11.021660	0.000007
6	5		B		8667.95	0.29	6.93	76208.3	1.0	11.00930533	0.00000030
5	6		C	-	10650.2	1.1		73443.6	1.4	11.0114333	0.0000011
4	7		N	-	25450	SYST		57860	SYST	11.02732	SYST
8	4	12	BE	-N	24950	SYST		68780	SYST	12.02678	SYST
[illegible]	[illegible]		[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]
6	6		C		0.0	0.0	7.68	92165.5	1.1	12.00000000	0.0
[illegible]	[illegible]		N		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]

http://ie.lbl.gov/toimass.html

http://ie.lbl.gov/mass/

http://www.phy.ornl.gov/hribf/calculators/mass-diff.shtml



EXERCÍCIO nº 1

ELSEVIER

Available online at www.sciencedirect.com
SCIENCE DIRECT

Nuclear Physics A 729 (2003) 337–676

NUCLEAR PHYSICS A

www.elsevier.com/locate/npe

The AME2003 atomic mass evaluation *

(II). Tables, graphs and references

G. Audi[a,§], A.H. Wapstra[b] and C. Thibault[a]

[a] *Centre de Spectrometrie Nucleaire et de Spectrometrie de Masse, CSNSM, IN2P3-CNRS&UPS, Batiment 108, F-91405 Orsay Campus, France*

[b] *National Institute of Nuclear Physics and High-Energy Physics, NIKHEF, PO Box 41882, 1009DB Amsterdam, The Netherlands*

Abstract

This paper is the second part of the new evaluation of atomic masses AME2003. From the results of a least-squares calculation described in Part I for all accepted experimental data, we derive here tables and graphs to replace those of 1993. The first table lists atomic masses. It is followed by a table of the influences of data on primary nuclides, a table of separation energies and reaction energies, and finally, a series of graphs of separation and decay energies. The last section in this paper lists all references to the input data used in Part I of this AME2003 and also to the data entering the NUBASE2003 evaluation (first paper in this volume).

AMDC: http://csnwww.in2p3.fr/AMDC/

1. Introduction

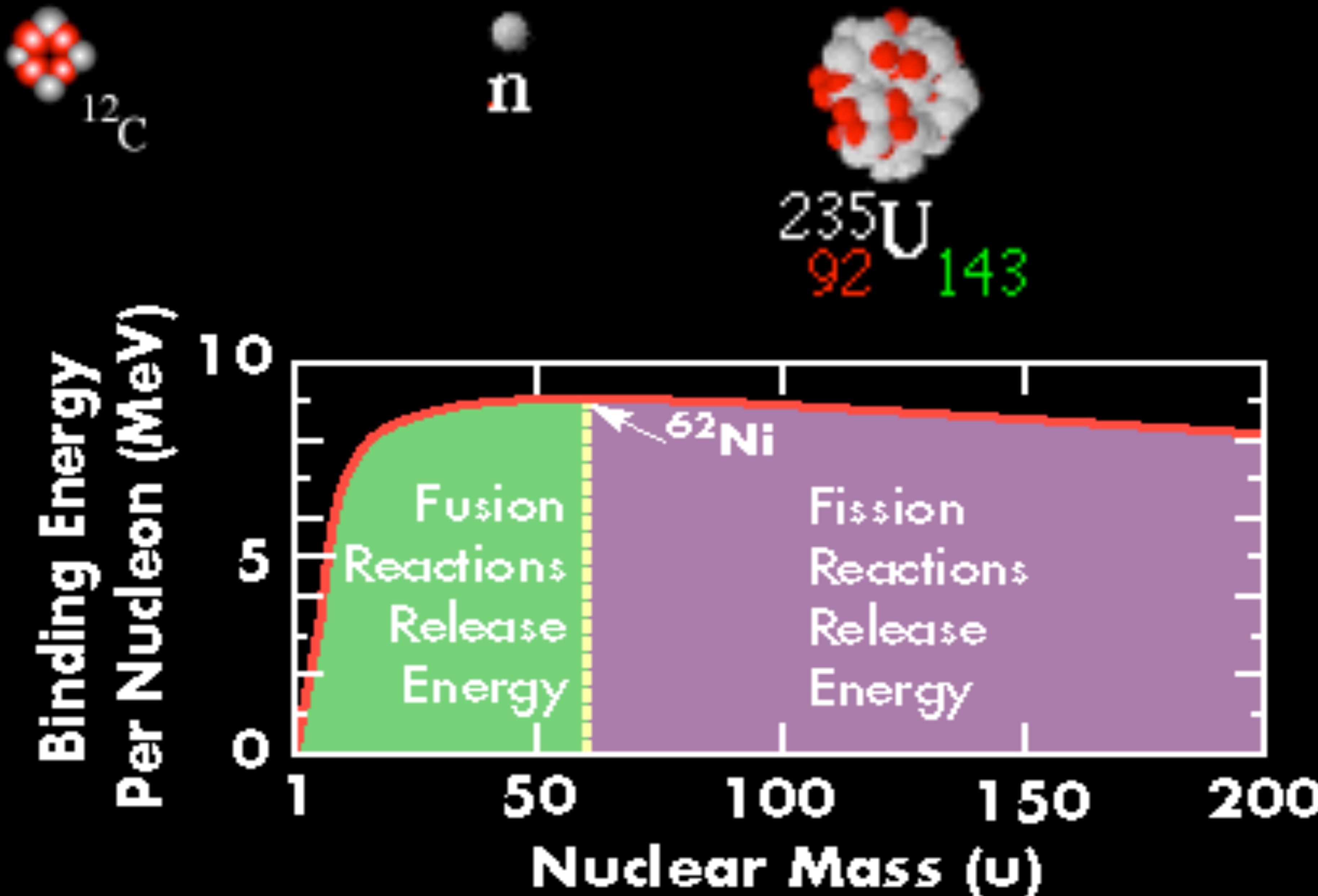
The description of the general procedures and policies are given in Part I of this series of two papers, where the input data used in the evaluation are presented. In this paper we give tables and graphs derived from the evaluation of the input data in Part I.

Firstly, we present the table of atomic masses (Table I) expressed as mass excesses in energy units, together with the binding energy per nucleon, the beta-decay energy and the full atomic mass in mass units.

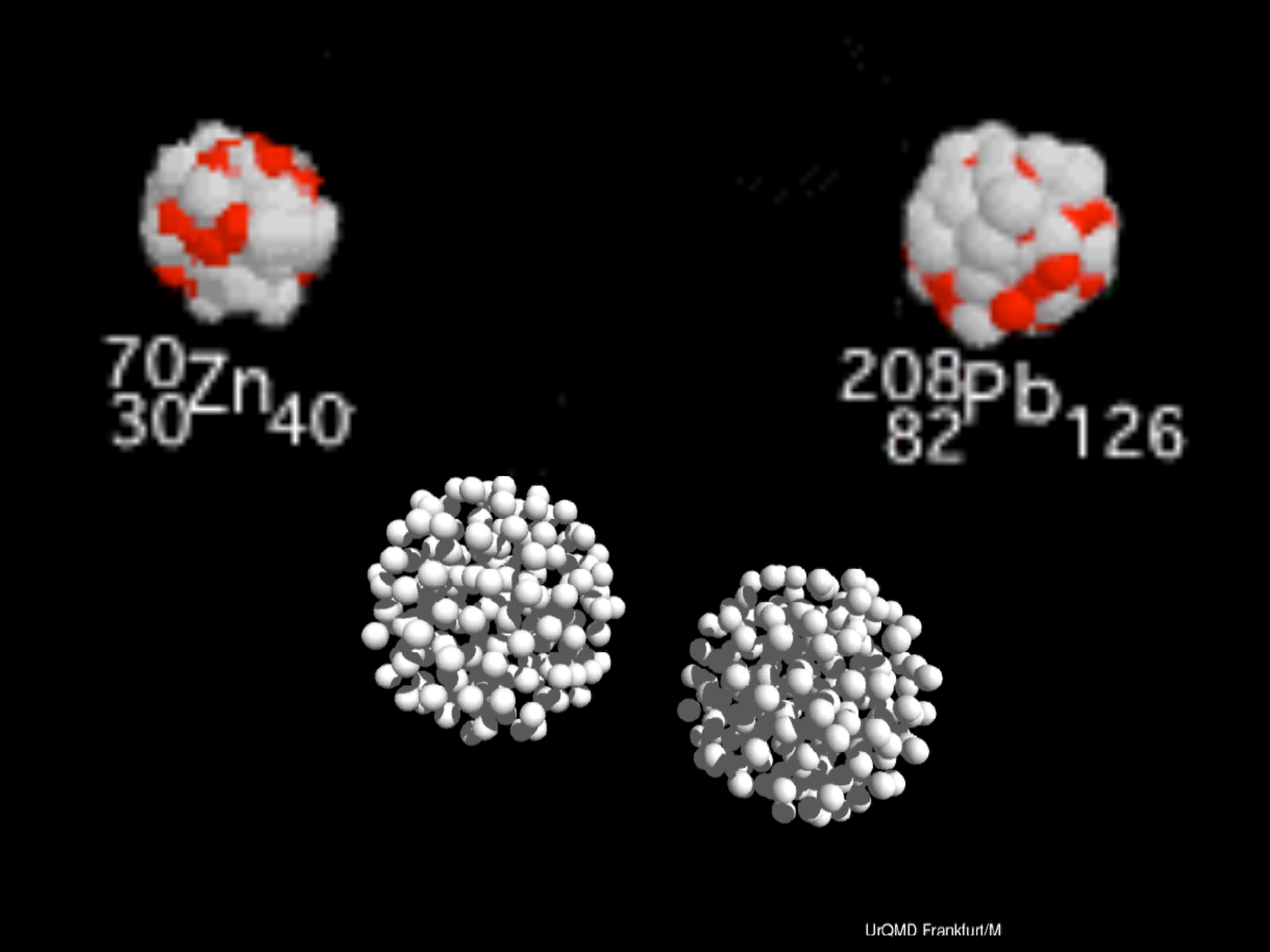
* This work has been undertaken with the encouragement of the IUPAP Commission on Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants (SUN-AMCO).

§ Corresponding author. *E-mail address:* audi@csnsm.in2p3.fr (G. Audi).

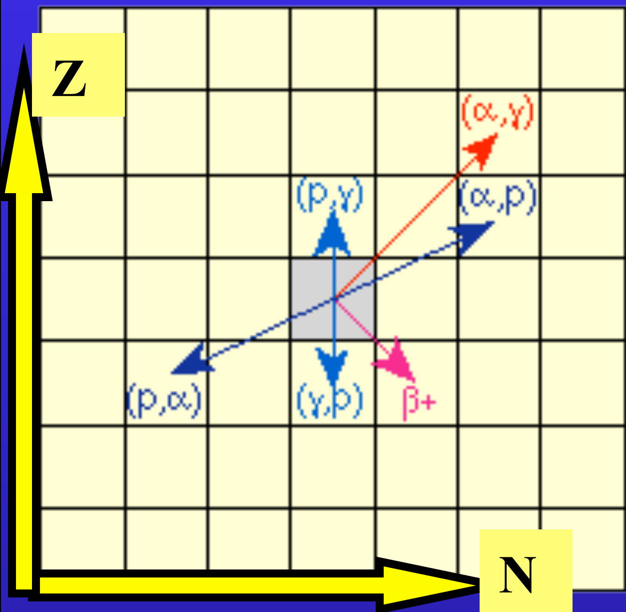
doi:10.1016/j.nuclphysa.2003.11.003



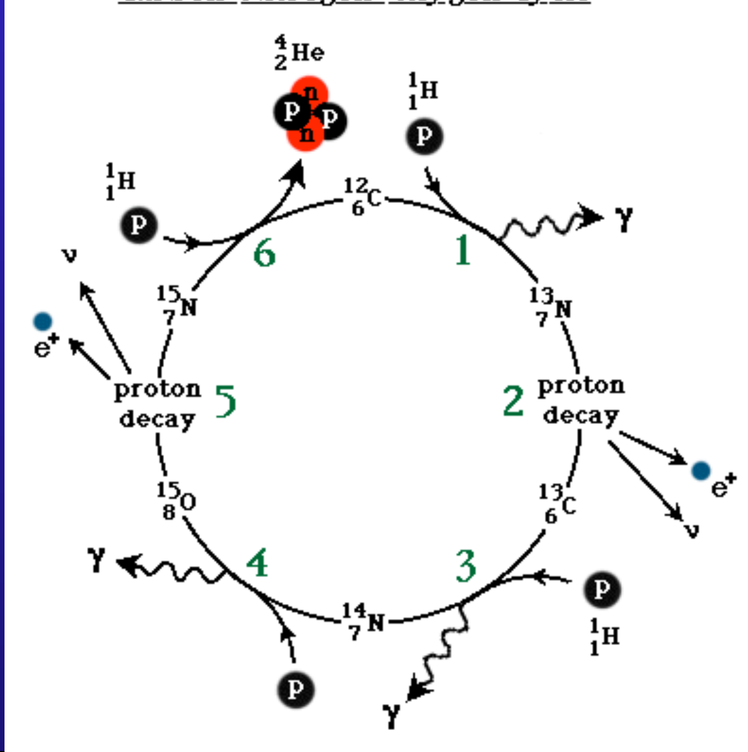
12C
n
235U
92
143
Binding Energy
Per Nucleon (MeV)
10
5
0
62Ni
Fusion
Reactions
Release
Energy
Fission
Reactions
Release
Energy
1
50
100
150
200
Nuclear Mass (u)

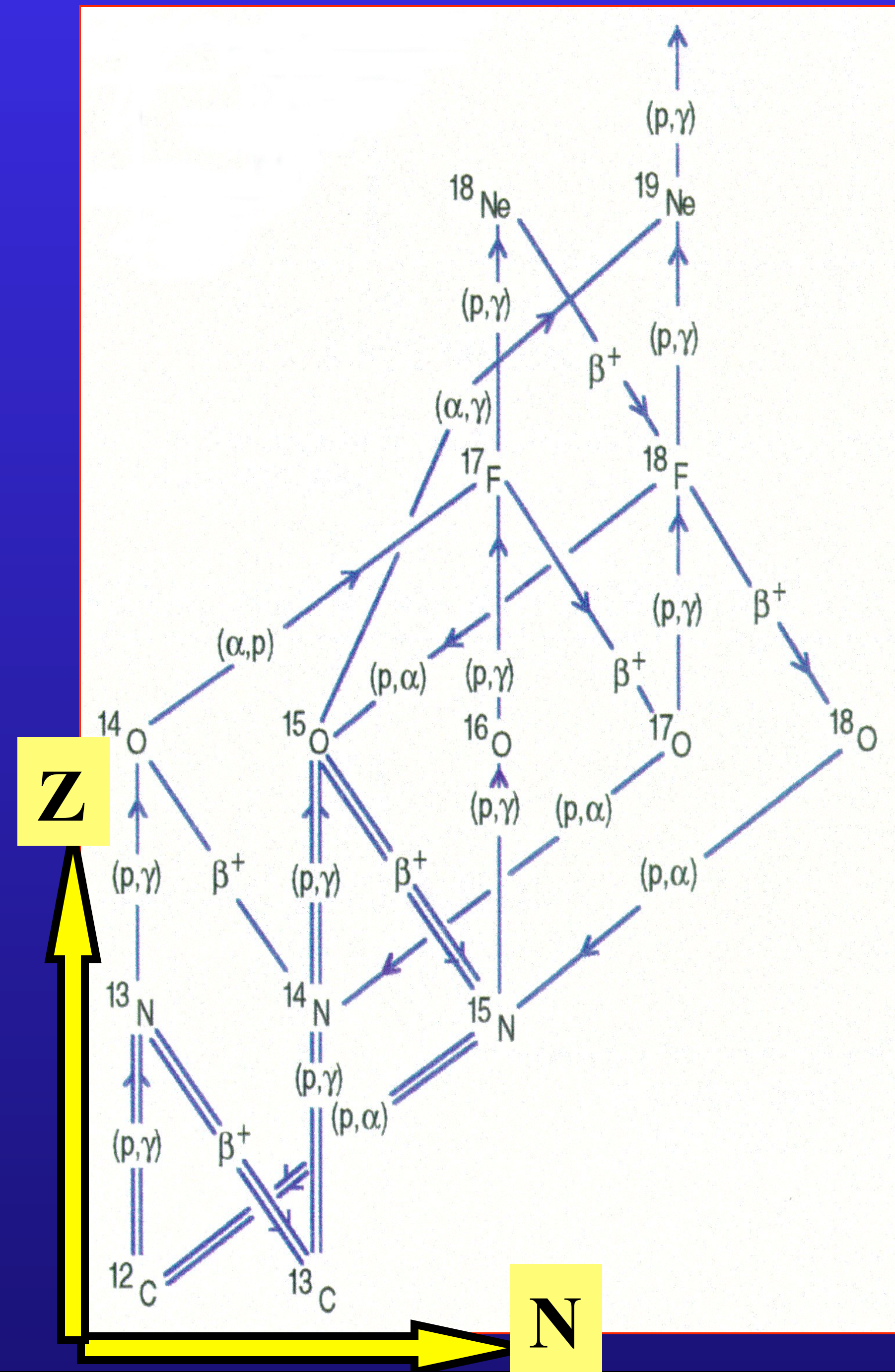


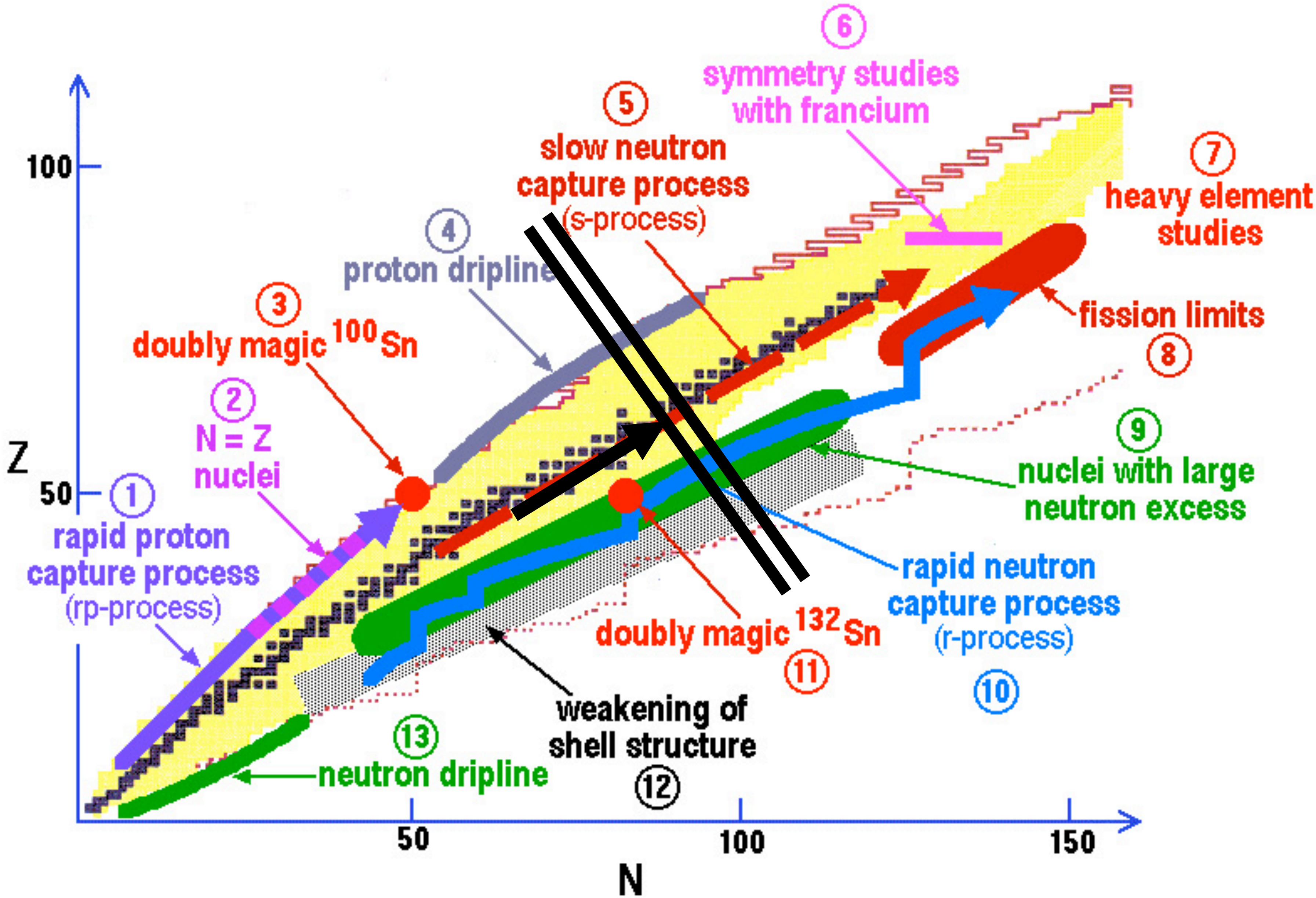
$^{70}_{30}Zn_{40}$
$^{208}_{82}Pb_{126}$
UrQMD Frankfurt/M



Carbon-Nitrogen-Oxygen Cycle

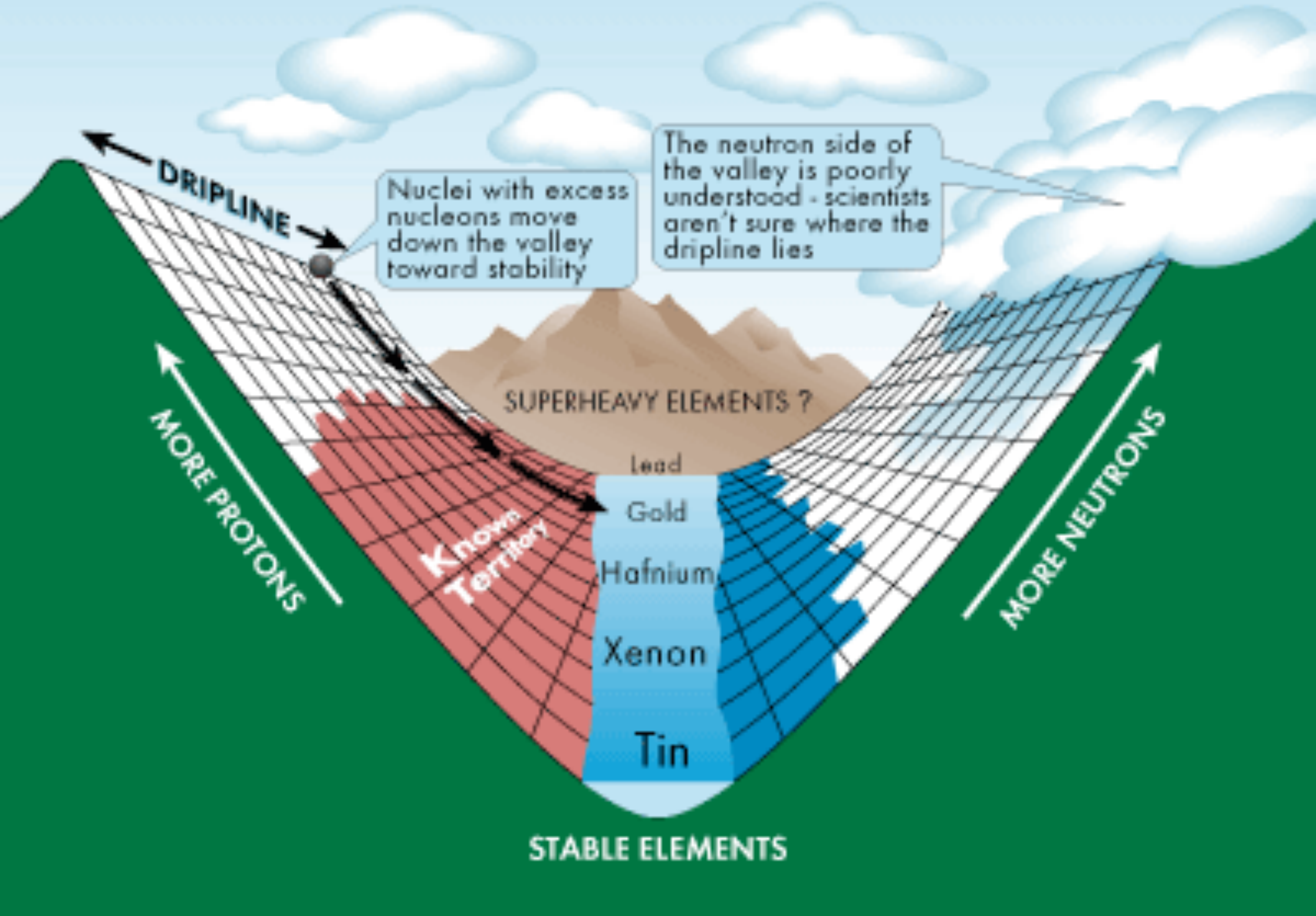


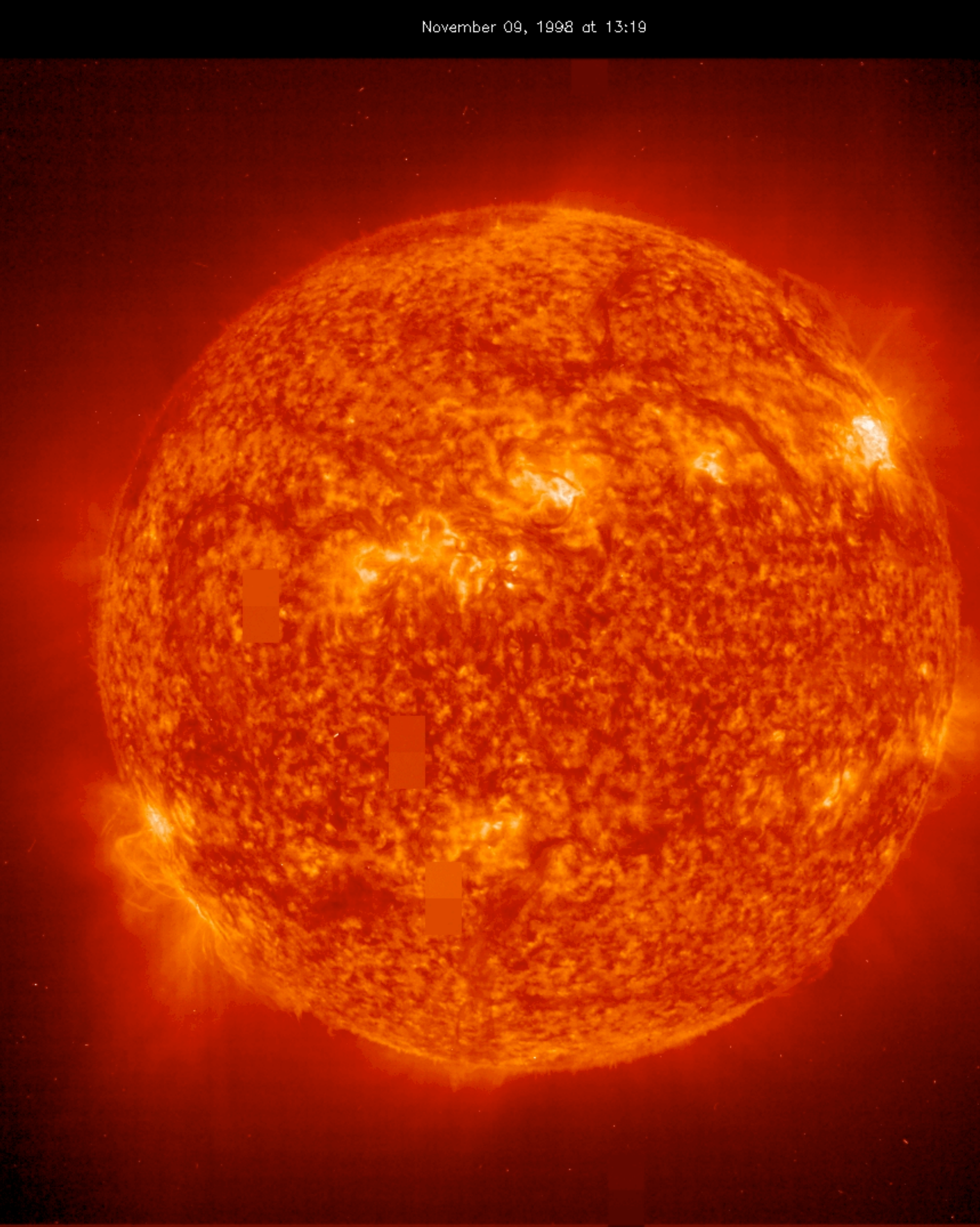


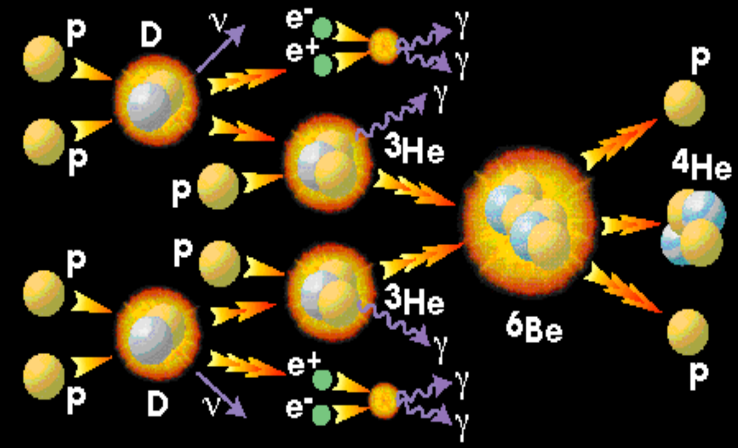


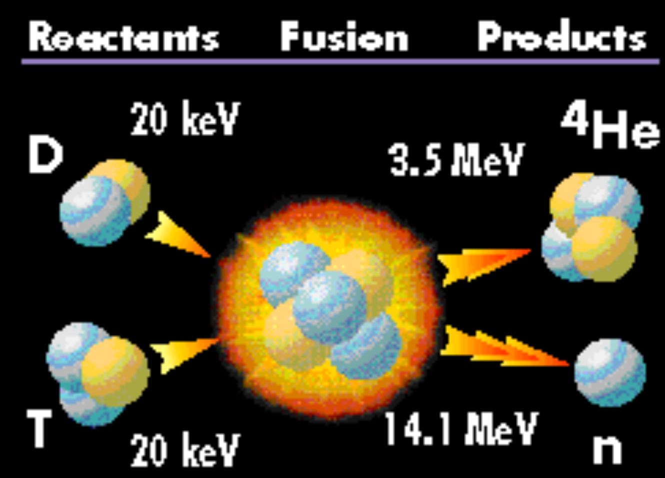
Z
100
50
N
50
100
150
1
rapid proton capture process (rp-process)
2
N = Z nuclei
3
doubly magic 100Sn
4
proton dripline
5
slow neutron capture process (s-process)
6
symmetry studies with francium
7
heavy element studies
fission limits
8
9
nuclei with large neutron excess
rapid neutron capture process (r-process)
10
doubly magic 132Sn
11
weakening of shell structure
12
13
neutron dripline

THE VALLEY OF STABILITY



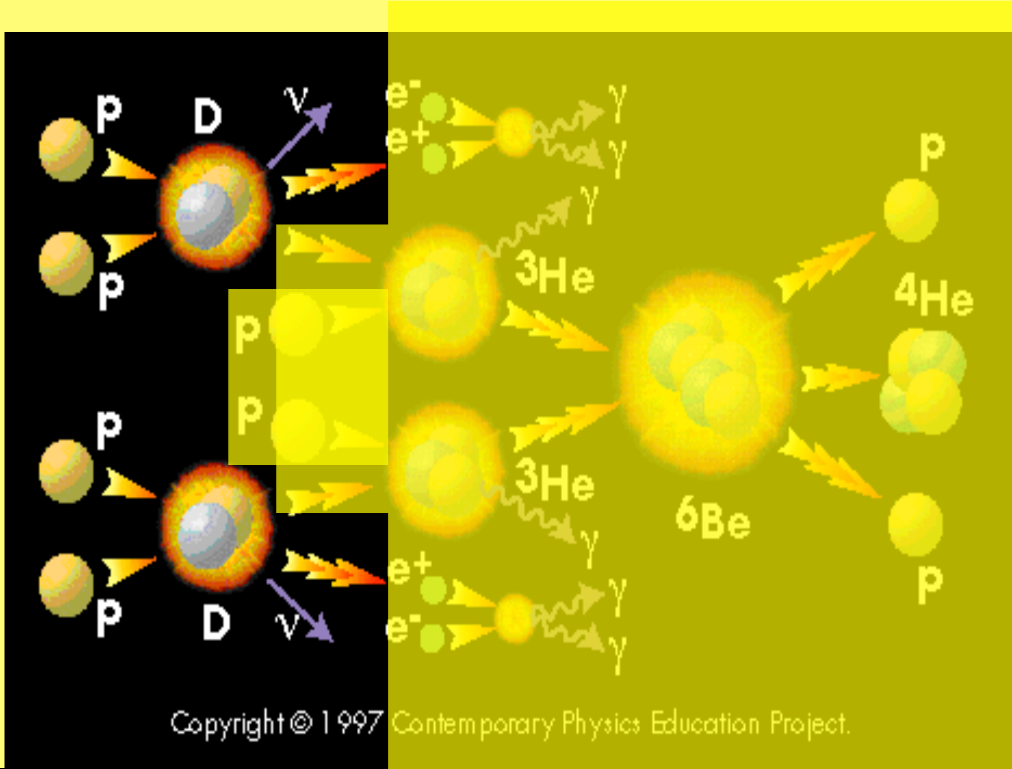

November 09, 1998 at 13:19


p
p
D
ν
e-
e+
γ
γ
γ
3He
p
4He
p
p
p
3He
6Be
γ
e+
e-
γ
γ
ν
D
p
p


Reactants
Fusion
Products
20 keV
D
3.5 MeV
4He
T
20 keV
14.1 MeV
n

$$p + p \rightarrow d + e^+ + \nu$$

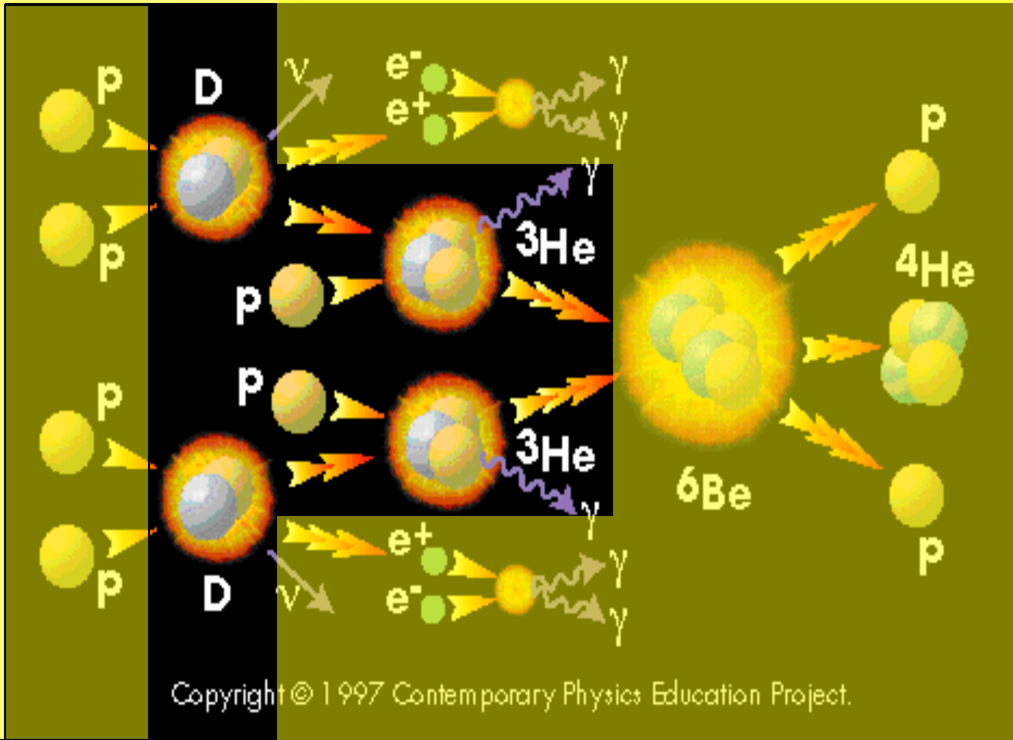
n =	8.071
p =	7.289
d =	13.136
t =	14.950
^3He=	14.931
^4He=	2.425
^6Be=	18.375



$p + p \rightarrow d + e^+ + \nu$

$p + d \rightarrow {}^3He + \gamma$

n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He=	14.931
^{4}He=	2.425
^{6}Be=	18.375



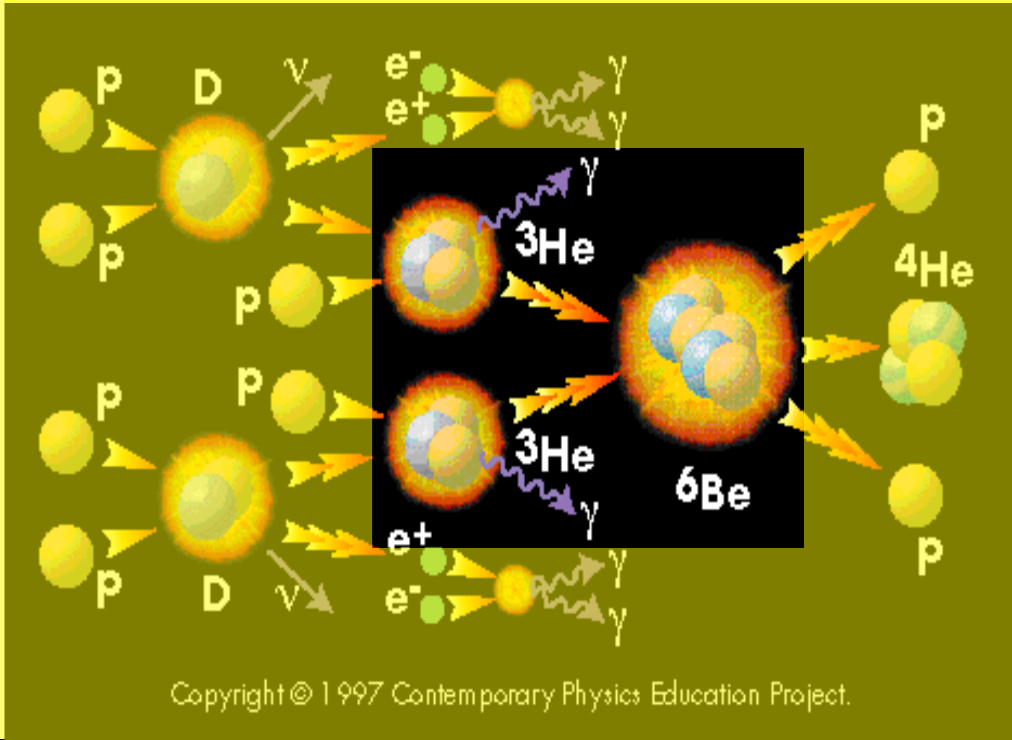
Copyright © 1997 Contemporary Physics Education Project.

$$p + p \rightarrow d + e^+ + \nu$$

$$p + d \rightarrow {}^3He + \gamma$$

$${}^3He + {}^3He \rightarrow {}^6Be + \gamma$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He= 14.931
^{4}He= 2.425
^{6}Be= 18.375



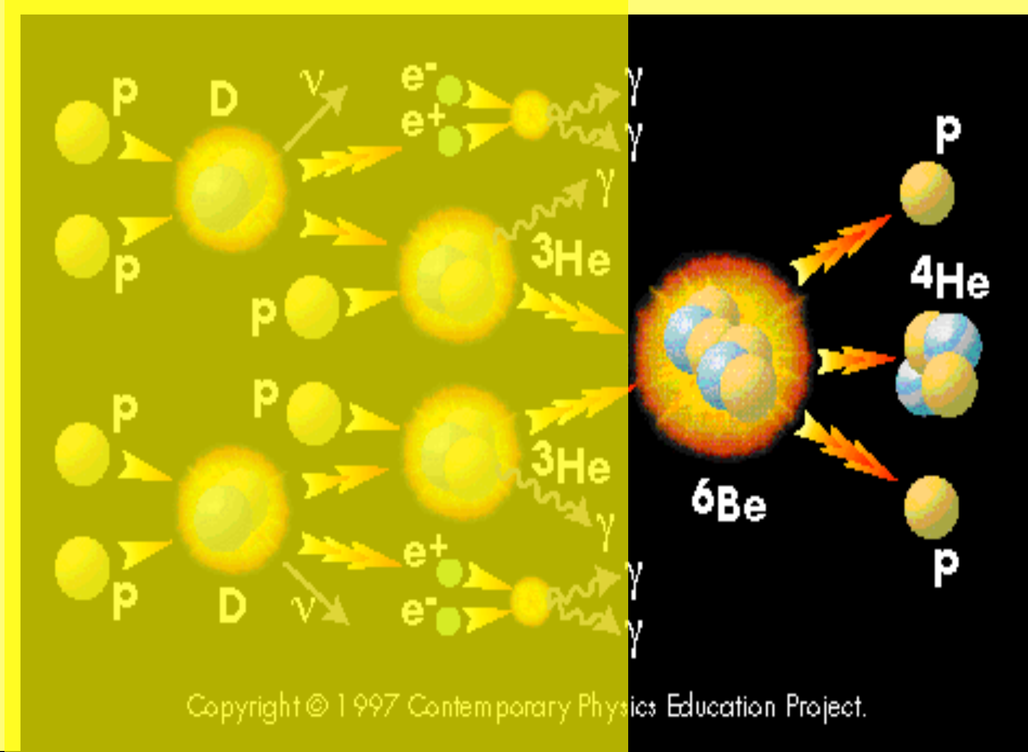
$$p + p \rightarrow d + e^+ + \nu$$

$$p + d \rightarrow {}^3He + \gamma$$

$${}^3He + {}^3He \rightarrow {}^6Be + \gamma$$

$${}^6Be \rightarrow {}^4He + p + p$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He= 14.931
^{4}He= 2.425
^{6}Be= 18.375



$$p + p \rightarrow d + e^+ + \nu$$

$$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931 \text{ MeV}$$

$$p + d \rightarrow {}^3He + \gamma$$

$$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494 \text{ MeV}$$

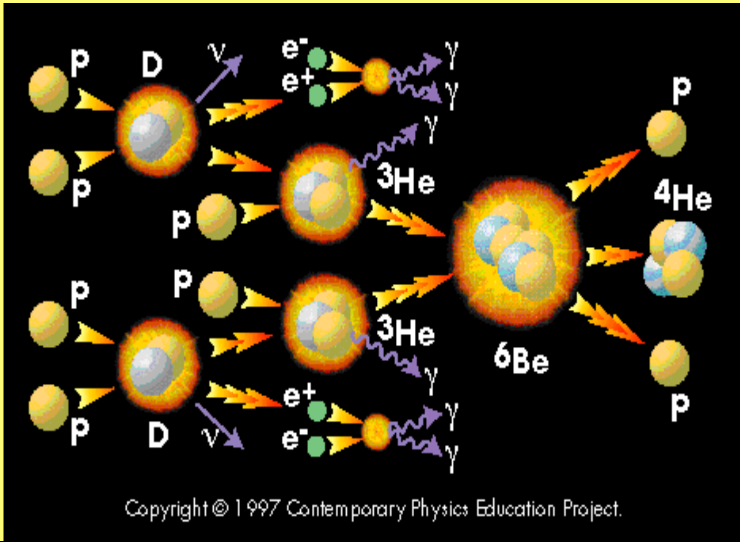
$${}^3He + {}^3He \rightarrow {}^6Be + \gamma$$

$$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487 \text{ MeV}$$

$${}^6Be \rightarrow {}^4He + p + p$$

$$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372 \text{ MeV}$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He = 14.931
^{4}He = 2.425
^{6}Be = 18.375



$p + p \rightarrow d + e^+ + \nu$

$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931$ MeV

n	=	8.071
p	=	7.289
d	=	13.136
t	=	14.950
^{3}He	=	14.931
^{4}He	=	2.425
^{6}Be	=	18.375

$p + d \rightarrow {}^3He + \gamma$

$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494$ MeV

${}^3He + {}^3He \rightarrow {}^6Be + \gamma$

$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487$ MeV

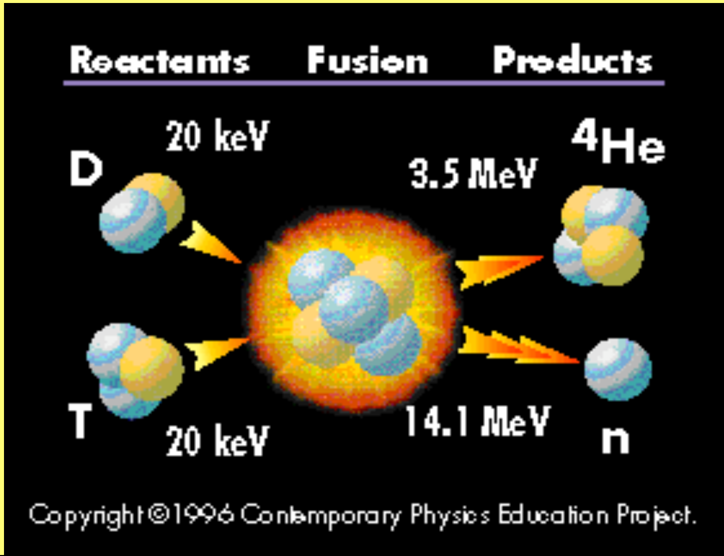
${}^6Be \rightarrow {}^4He + p + p$

$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372$ MeV

$d + t \rightarrow {}^4He + n$

$13.136 + 14.950 \rightarrow 2.425 + 8.071 + Q$

$Q = 17.59$ MeV



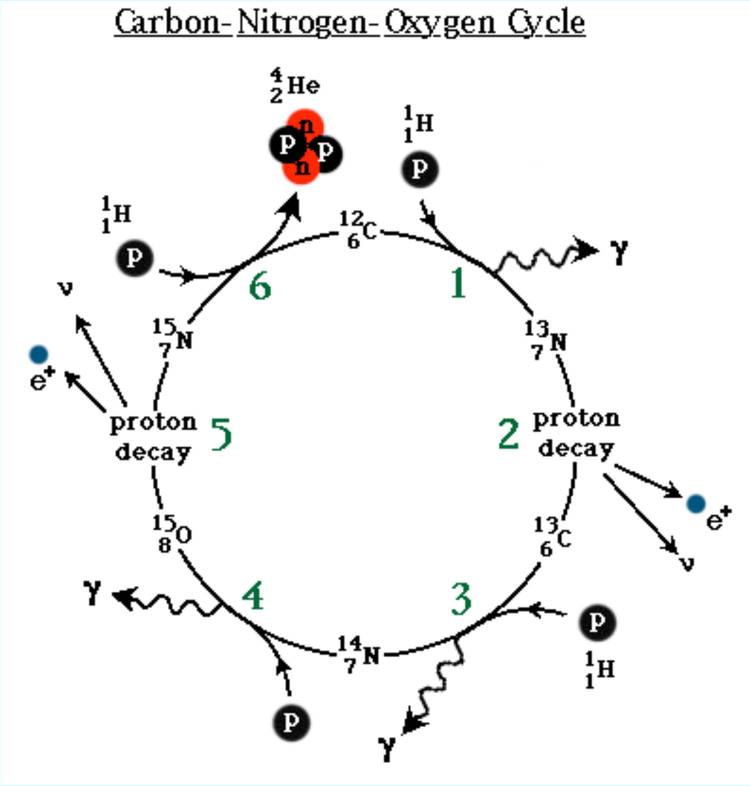
EXERCÍCIOS

CALCULAR O BALANÇO ENERGÉTICO NAS SEGUINTES REAÇÕES

$\Delta = (M - A)c^2$ (MeV)

Exemplo== $^{12}C + p \Rightarrow {}^{13}N + \gamma$

n = 8.071	^{12}C = 0.00
p = 7.289	^{13}C = 3.125
d = 13.136	^{13}N = 5.345
t = 14.950	^{14}N = 2.863
^{3}He = 14.931	^{15}N = 0.011
^{4}He = 2.425	^{15}O = 2.855
^{6}Li = 14.086	^{16}O = -4.737
^{7}Li = 14.908	^{17}O = -0.809
^{6}Be = 18.375	^{18}O = -0.782



$^{12}C + p \rightarrow {}^{13}N + \gamma$

$0 + 7.289 \rightarrow 5.345 + Q \Rightarrow Q = 1.944$ MeV

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$

$5.345 \rightarrow 3.125 + 0.511 + Q \Rightarrow Q = 1.709$ MeV

$^{13}C + p \rightarrow {}^{14}N + \gamma$

$3.125 + 7.289 \rightarrow 2.863 + Q \Rightarrow Q = 7.551$ MeV

$^{14}N + p \rightarrow {}^{15}O + \gamma$

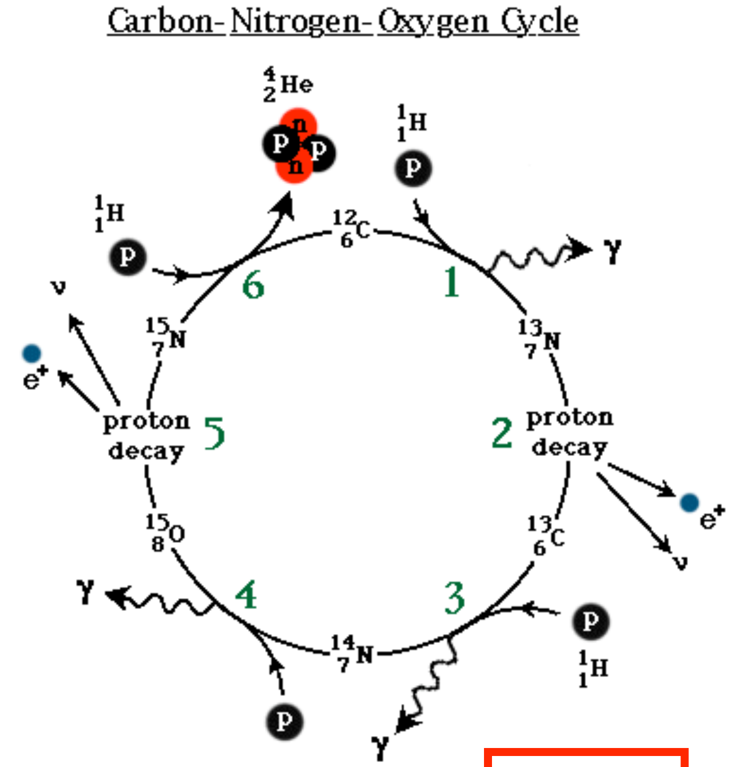
$2.863 + 7.289 \rightarrow 2.855 + Q \Rightarrow Q = 7.297$ MeV

$^{15}O \rightarrow {}^{15}N + e^+ + \nu$

$2.855 \rightarrow 0.101 + 0.511 + Q \Rightarrow Q = 2.243$ MeV

$^{15}N + p \rightarrow {}^{12}C + \alpha + \gamma$

$0.101 + 7.289 \rightarrow 0 + 2.425 + Q \Rightarrow Q = 4.965$ MeV



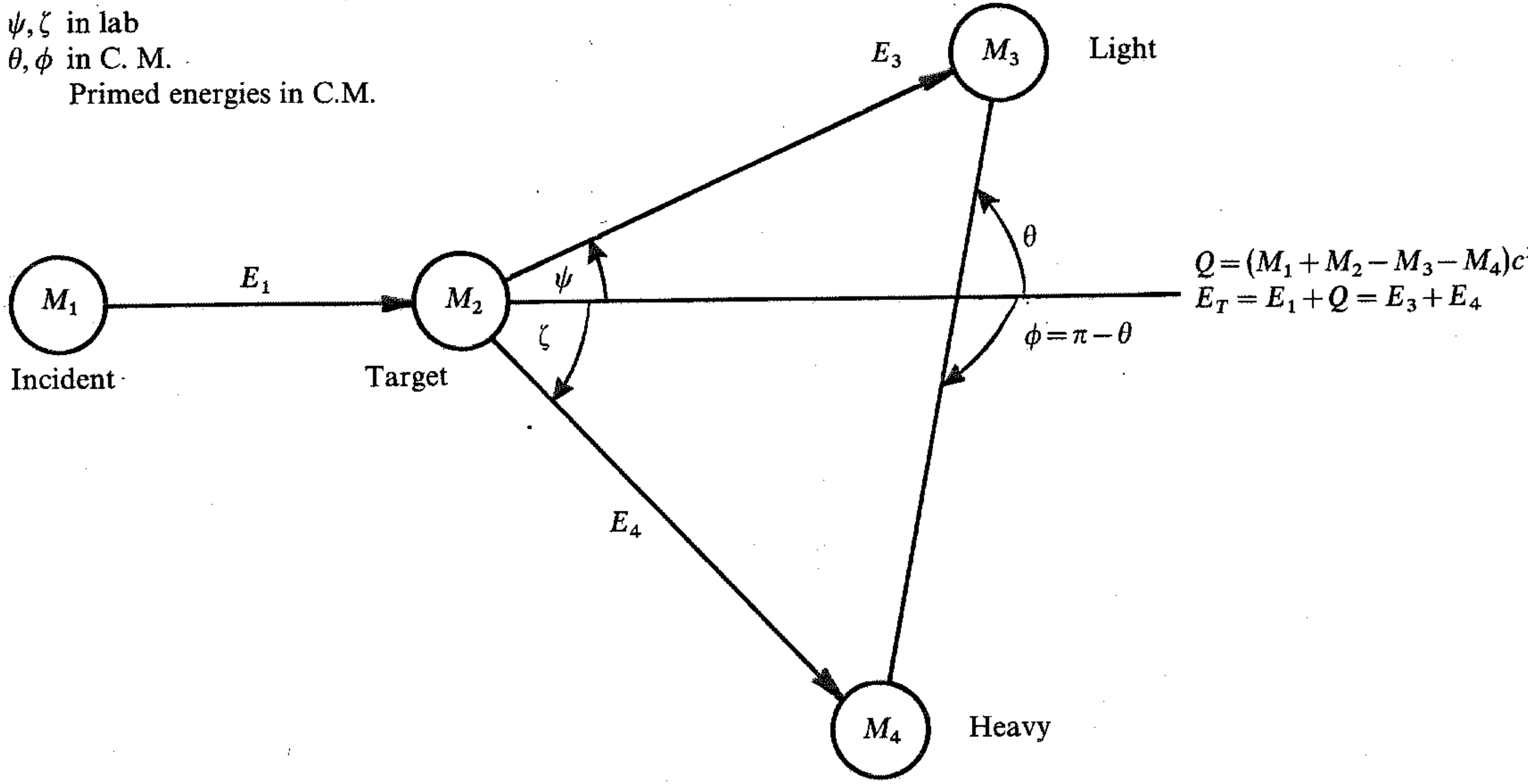
$$\begin{array}{r} 1.944 \\ +1.709 \\ +7.551 \\ +7.297 \\ +2.243 \\ +4.965 \\ \hline 25.709 \end{array}$$

$^{12}C + p + p + p + p \rightarrow {}^{12}C + {}^4He + 2e^+ + 2\nu$

$0 + 4x\,(7.289) \rightarrow 0 + 2.425 + 1.022 + Q$

Q = 25.709 MeV

reações nucleares (tipos)
cinematica de reação



Define:

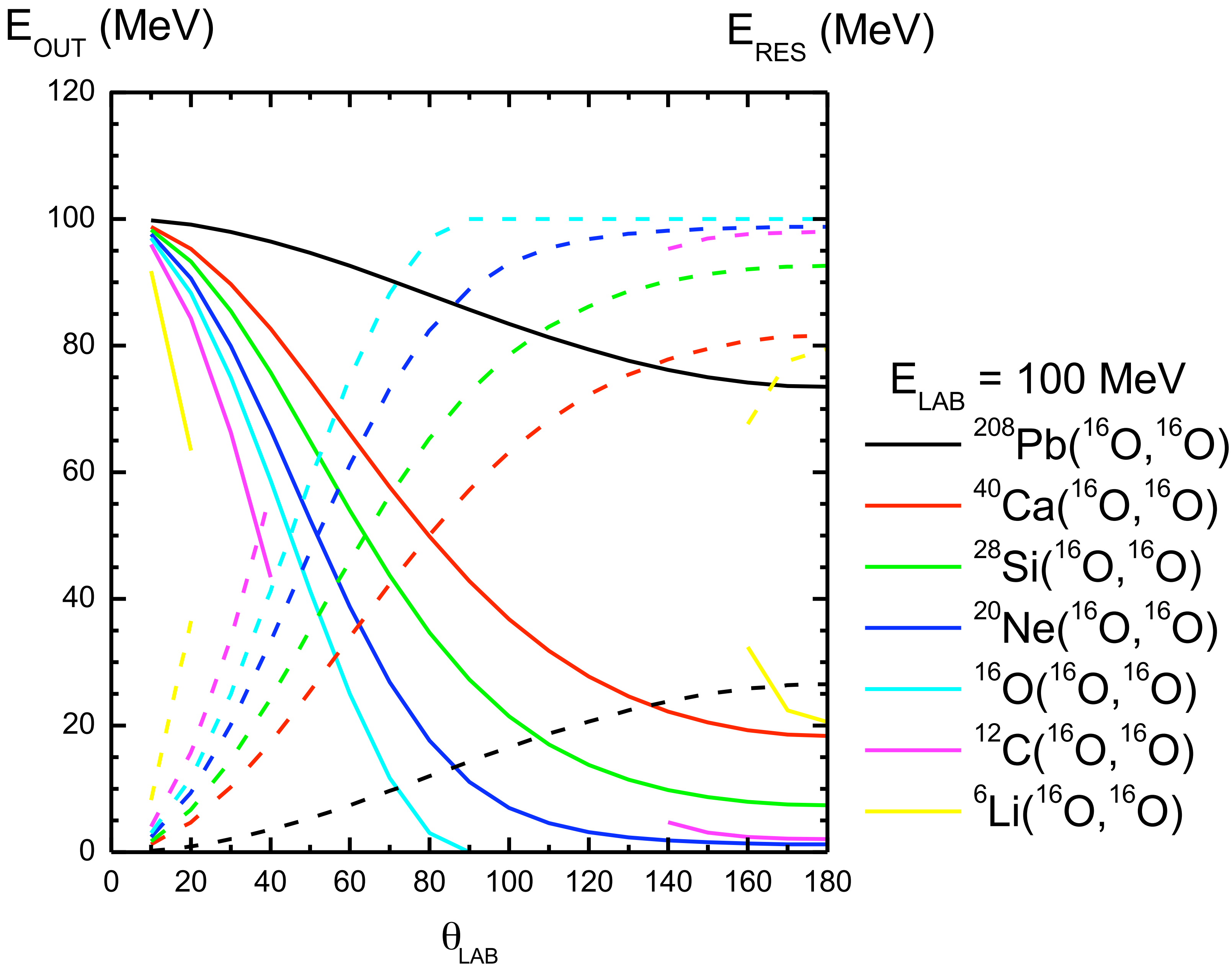
$$A = \frac{M_1 M_4 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad C = \frac{M_2 M_3}{(M_1+M_2)(M_3+M_4)}\left(1 + \frac{M_1 Q}{M_2 E_T}\right) = \frac{E_4'}{E_T}$$

$$B = \frac{M_1 M_3 (E_1/E_T)}{(M_1+M_2)(M_3+M_4)}, \qquad D = \frac{M_2 M_4}{(M_1+M_2)(M_3+M_4)}\left(1 + \frac{M_1 Q}{M_2 E_T}\right) = \frac{E_3'}{E_T}$$

Note that $A+B+C+D=1$ and $AC=BD$

Note that $A+B+C+D=1$ and $AC=BD$

Lab energy of light product:	$\frac{E_3}{E_T}=B+D+2(AC)^{\frac{1}{2}}\cos\theta$ $=B[\cos\psi\pm(D/B-\sin^2\psi)^{\frac{1}{2}}]^2$		Use only plus sign unless $B>D$, in which case $\psi_{\max}=\sin^{-1}(D/B)^{\frac{1}{2}}$
Lab energy of heavy product:	$\frac{E_4}{E_T}=A+C+2(AC)^{\frac{1}{2}}\cos\phi$ $=A[\cos\zeta\pm(C/A-\sin^2\zeta)^{\frac{1}{2}}]^2$		Use only plus sign unless $A>C$, in which case $\zeta_{\max}=\sin^{-1}(C/A)^{\frac{1}{2}}$
Lab angle of heavy product:	$\sin\zeta=\left(\frac{M_3E_3}{M_4E_4}\right)^{\frac{1}{2}}\sin\psi$	C.M. angle of light product:	$\sin\theta=\left(\frac{E_3/E_T}{D}\right)\sin\psi$
Intensity or solid-angle ratio for light product:	$\frac{\sigma(\theta)}{\sigma(\psi)}=\frac{I(\theta)}{I(\psi)}=\frac{\sin\psi\,d\psi}{\sin\theta\,d\theta}=\frac{\sin^2\psi}{\sin^2\theta}\cos(\theta-\psi)=\frac{(AC)^{\frac{1}{2}}(D/B-\sin^2\psi)^{\frac{1}{2}}}{E_3/E_T}$		
Intensity or solid-angle ratio for heavy product:	$\frac{\sigma(\phi)}{\sigma(\zeta)}=\frac{I(\phi)}{I(\zeta)}=\frac{\sin\zeta\,d\zeta}{\sin\phi\,d\phi}=\frac{\sin^2\zeta}{\sin^2\phi}\cos(\phi-\zeta)=\frac{(AC)^{\frac{1}{2}}(C/A-\sin^2\zeta)^{\frac{1}{2}}}{E_4/E_T}$		
Intensity or solid-angle ratio for associated particles in the lab system:	$\frac{\sigma(\zeta)}{\sigma(\psi)}=\frac{I(\zeta)}{I(\psi)}=\frac{\sin\psi\,d\psi}{\sin\zeta\,d\zeta}=\frac{\sin^2\psi\cos(\theta-\psi)}{\sin^2\zeta\cos(\phi-\zeta)}$		



E_OUT (MeV)
E_RES (MeV)
120
100
80
60
40
20
0
0 20 40 60 80 100 120 140 160 180
θ_LAB
E_LAB = 100 MeV
208Pb(16O,16O)
40Ca(16O,16O)
28Si(16O,16O)
20Ne(16O,16O)
16O(16O,16O)
12C(16O,16O)
6Li(16O,16O)

EXERCÍCIO

CINEMÁTICA DE REAÇÕES: **CALCULAR**

E_3 , E_4 vs θ ou E_{out} , E_{res} vs θ

E_3 , E_4 vs E_1

Ψ vs χ

θ vs φ

ESPALHAMENTO ELASTICO

REAÇÕES DIRETAS

ESPALHAMENTO INELASTICO

TRANSFERENCIA DE NUCLEONS
- "STRIPPING"
- "PICK-UP"

"KNOCK-OUT"

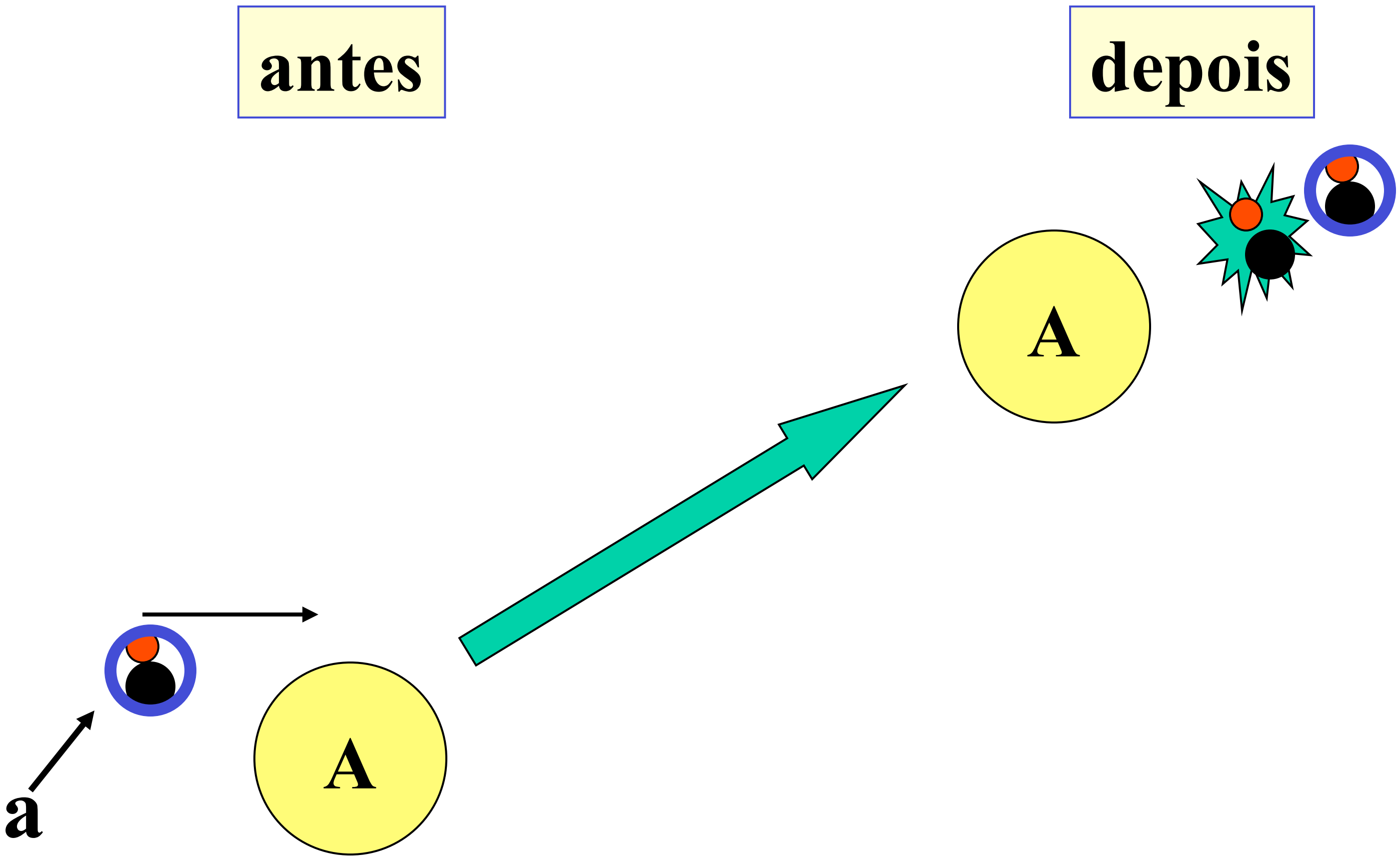
QUEBRA NUCLEAR ("BREAK-UP")

PRÉ-EQUILIBRIO

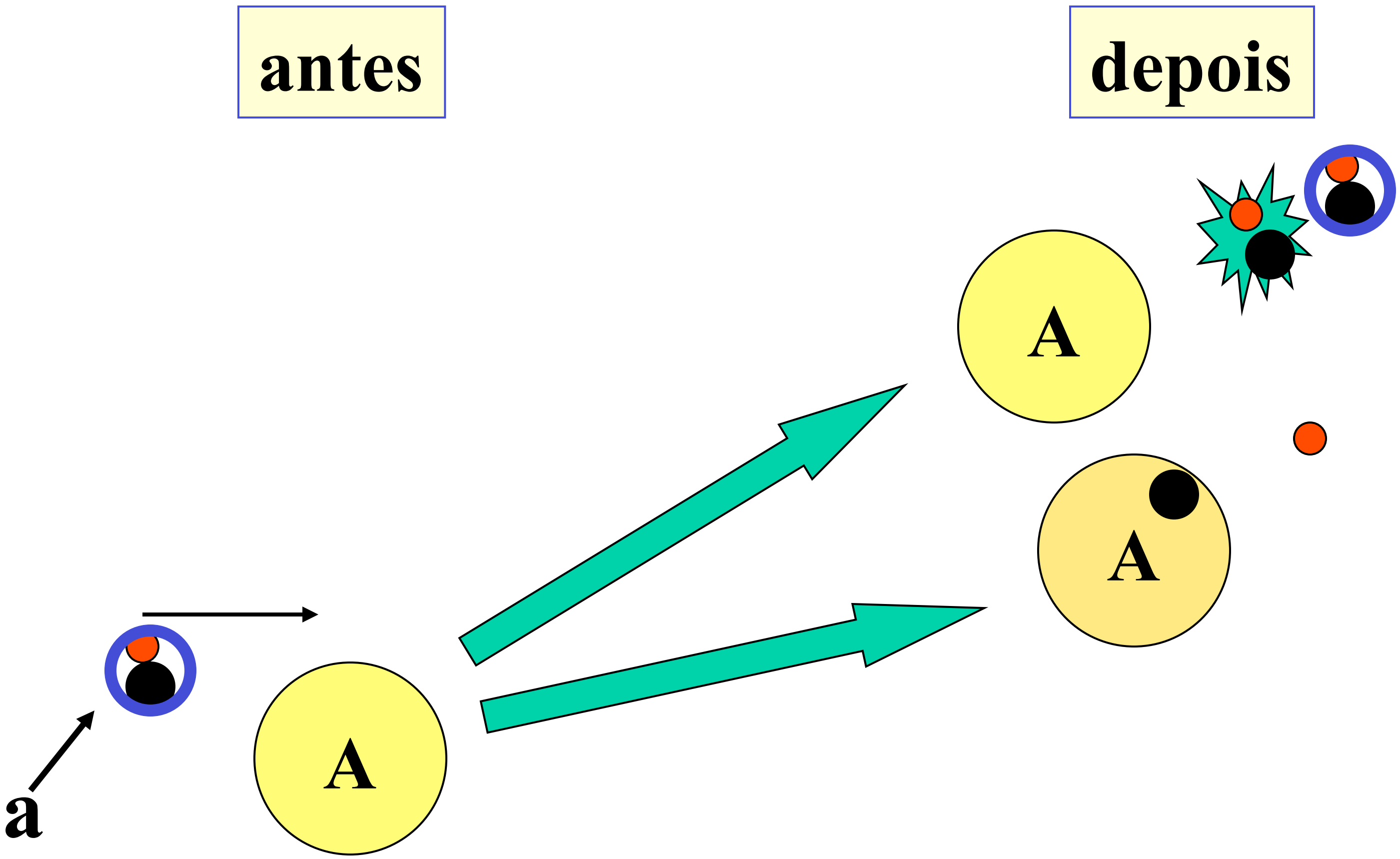
NUCLEO COMPOSTO

FUSÃO
- FUSÃO COMPLETA
- FUSÃO INCOMPLETA

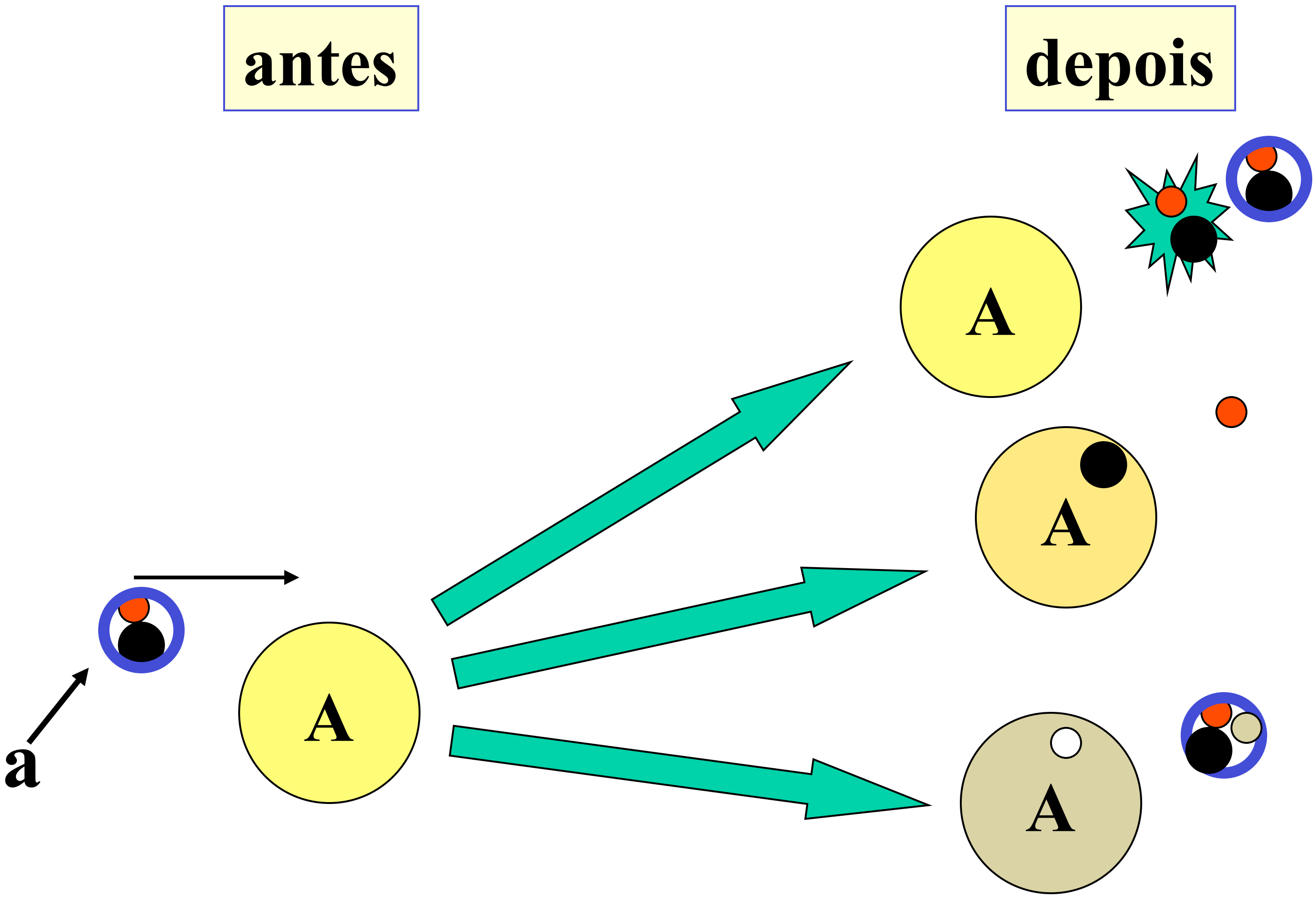
FISSÃO



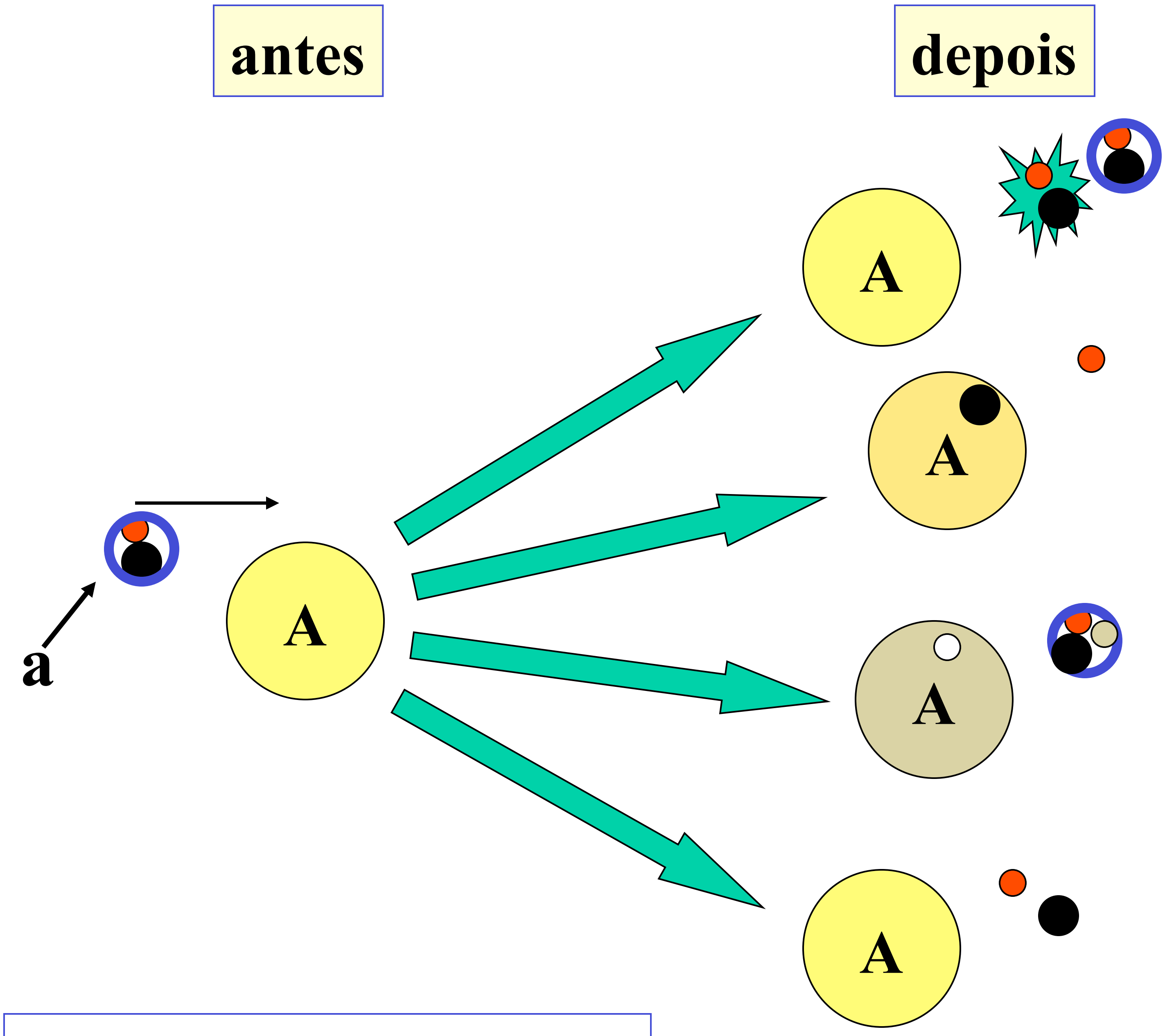
Reações diretas (rápidas)



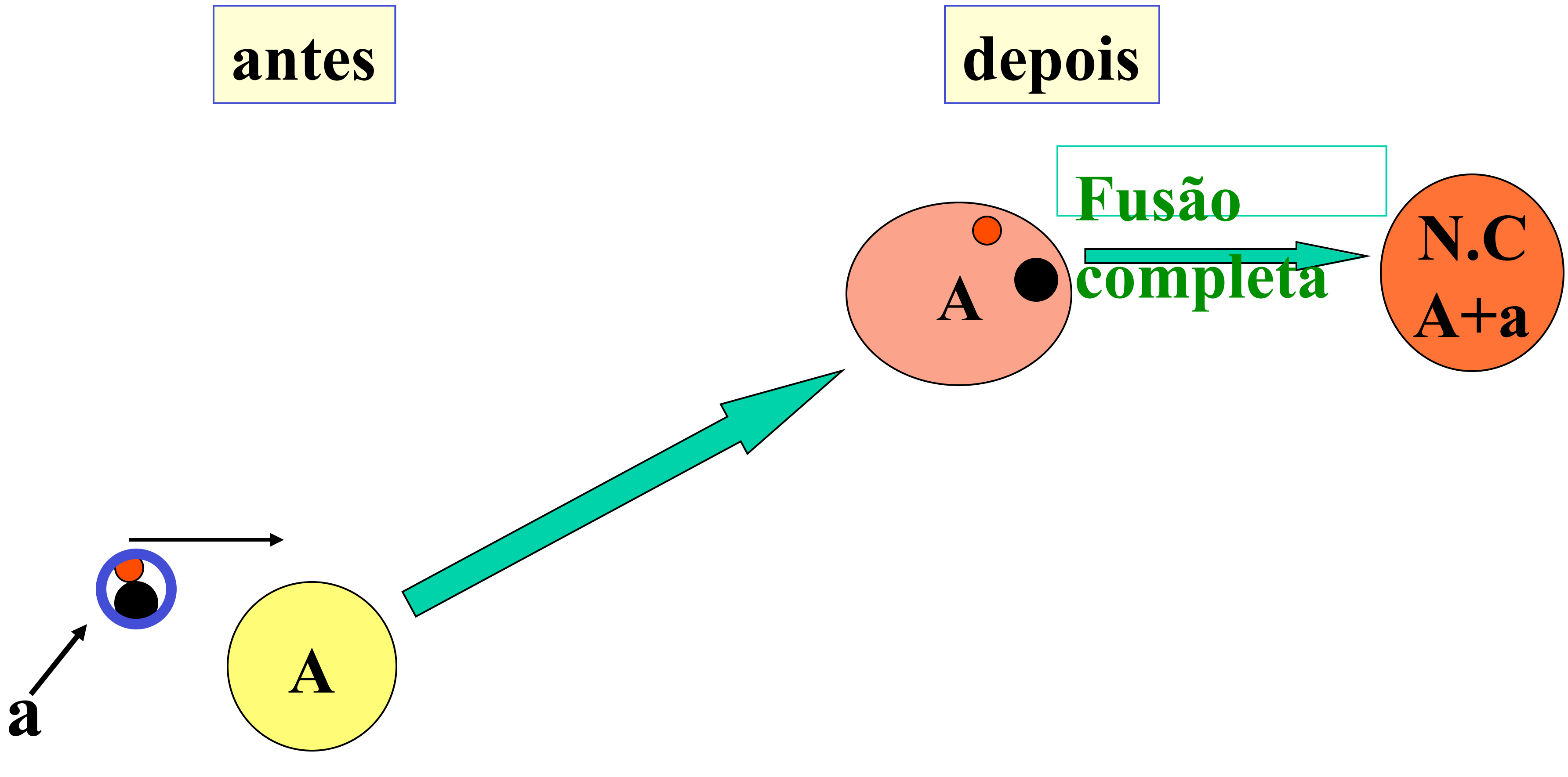
Reações diretas (rapidas)



Reações diretas (rapidas)

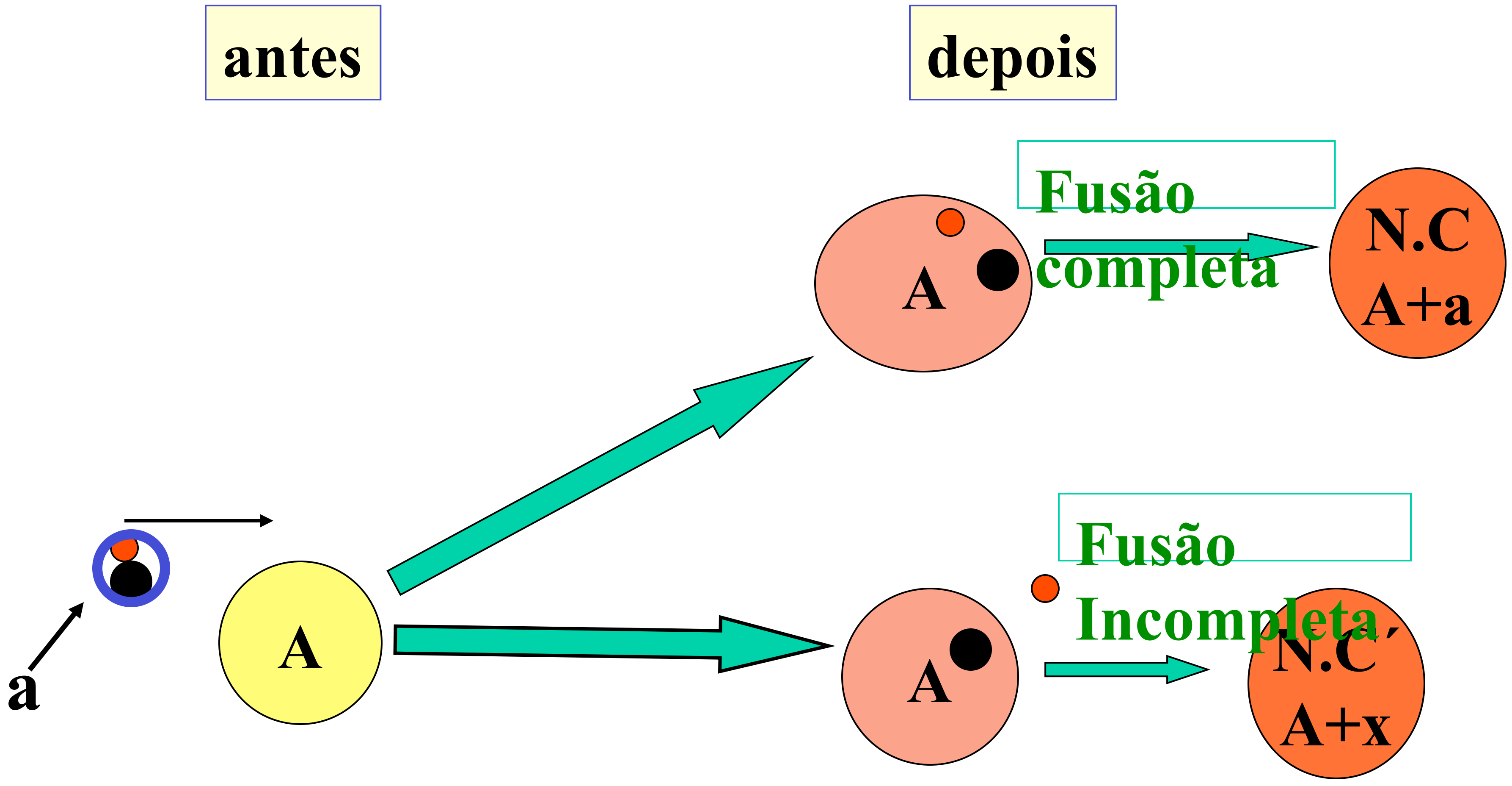


Reações diretas (rapidas)

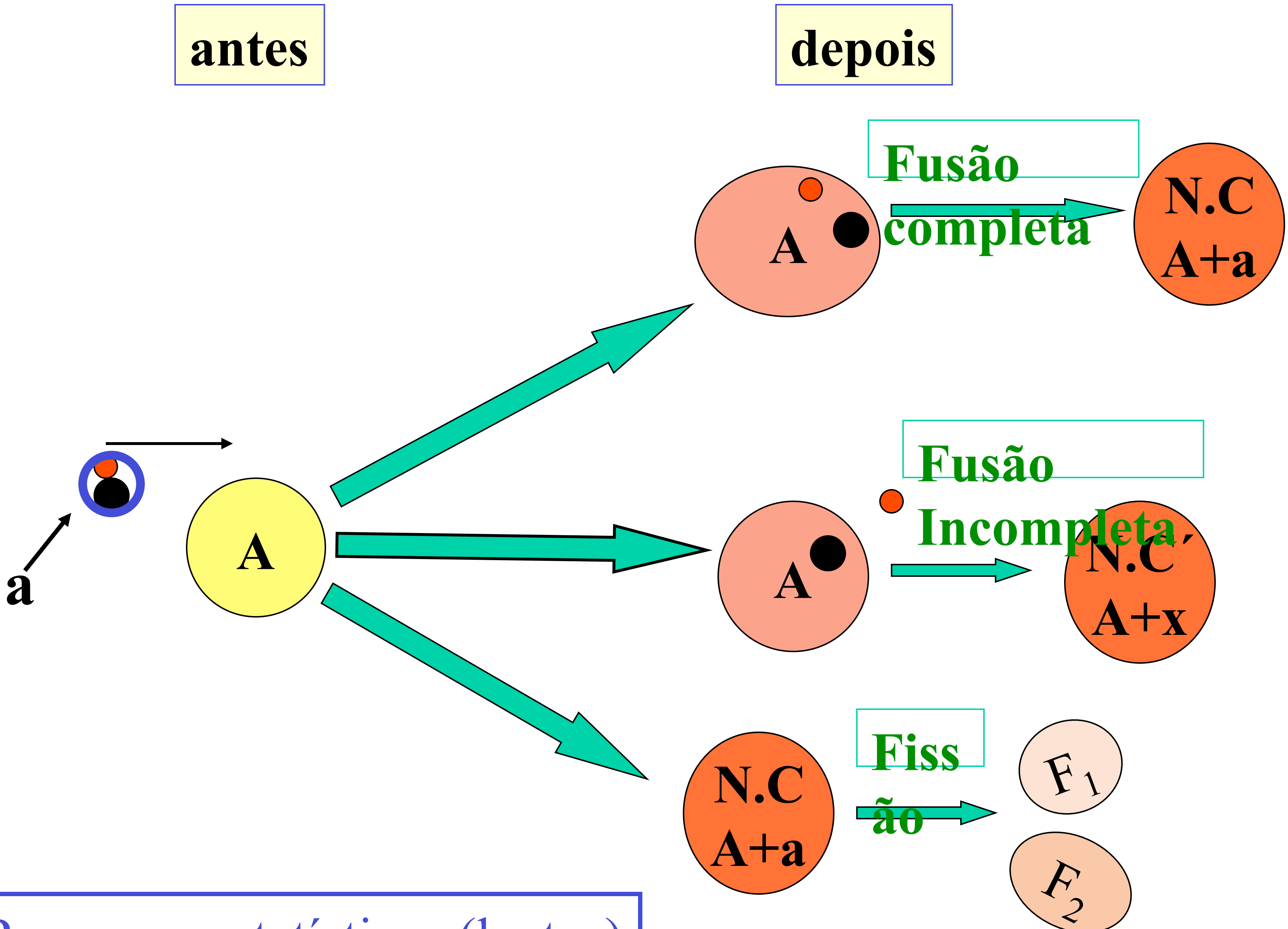


Processos estatísticos (lentos)

via **Núcleo Composto (N.C.)**

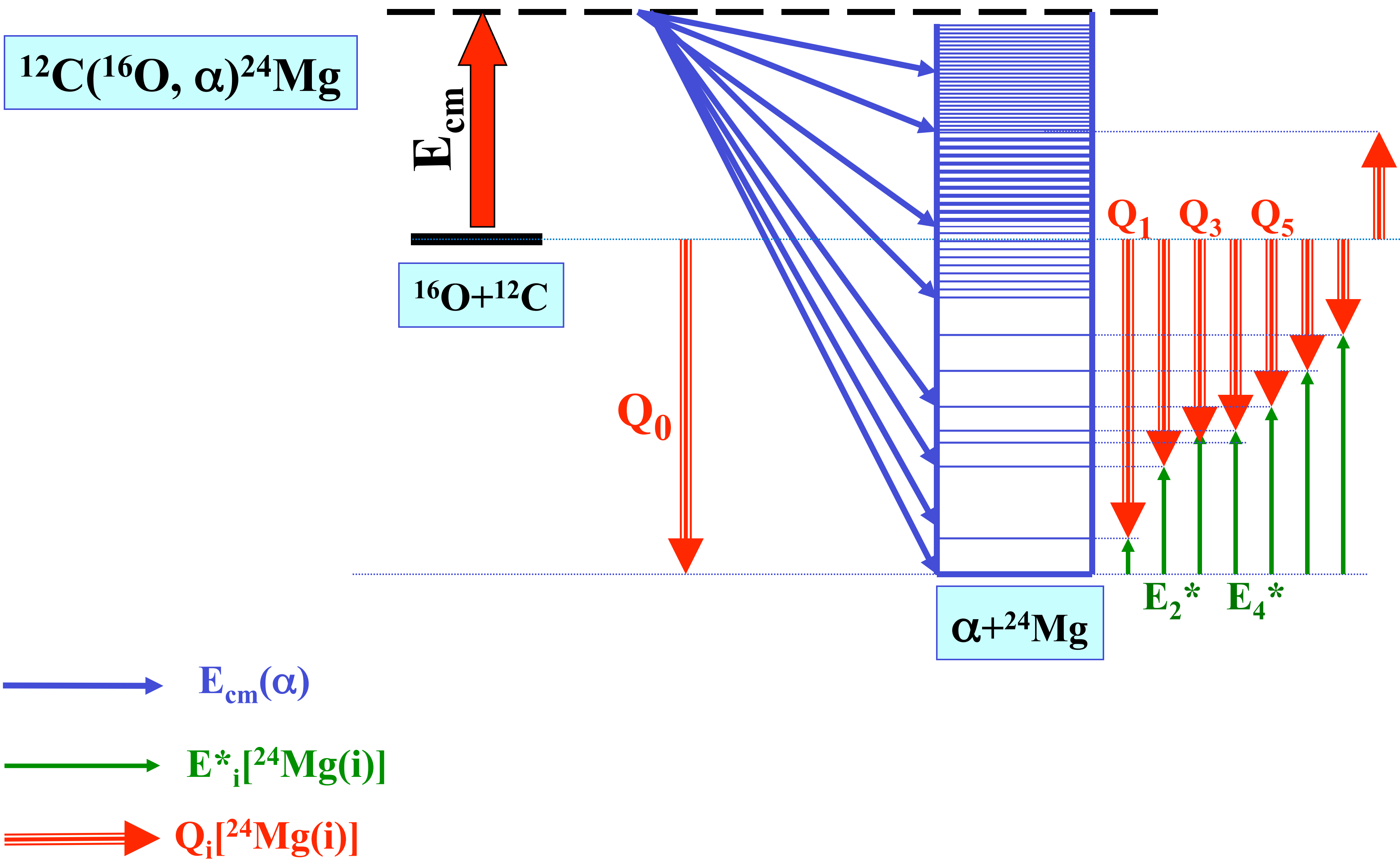


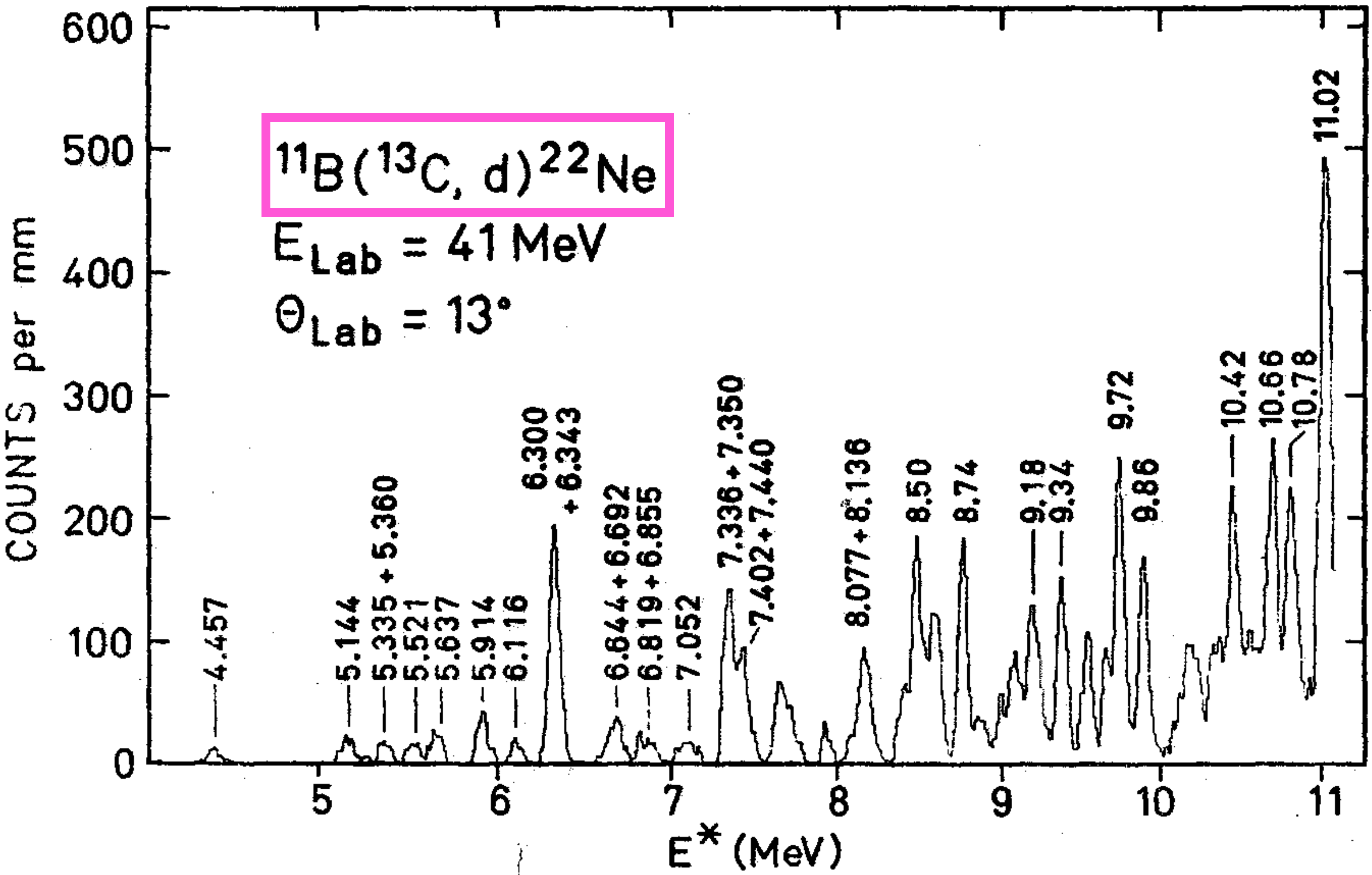
Processos estatísticos (lentos)



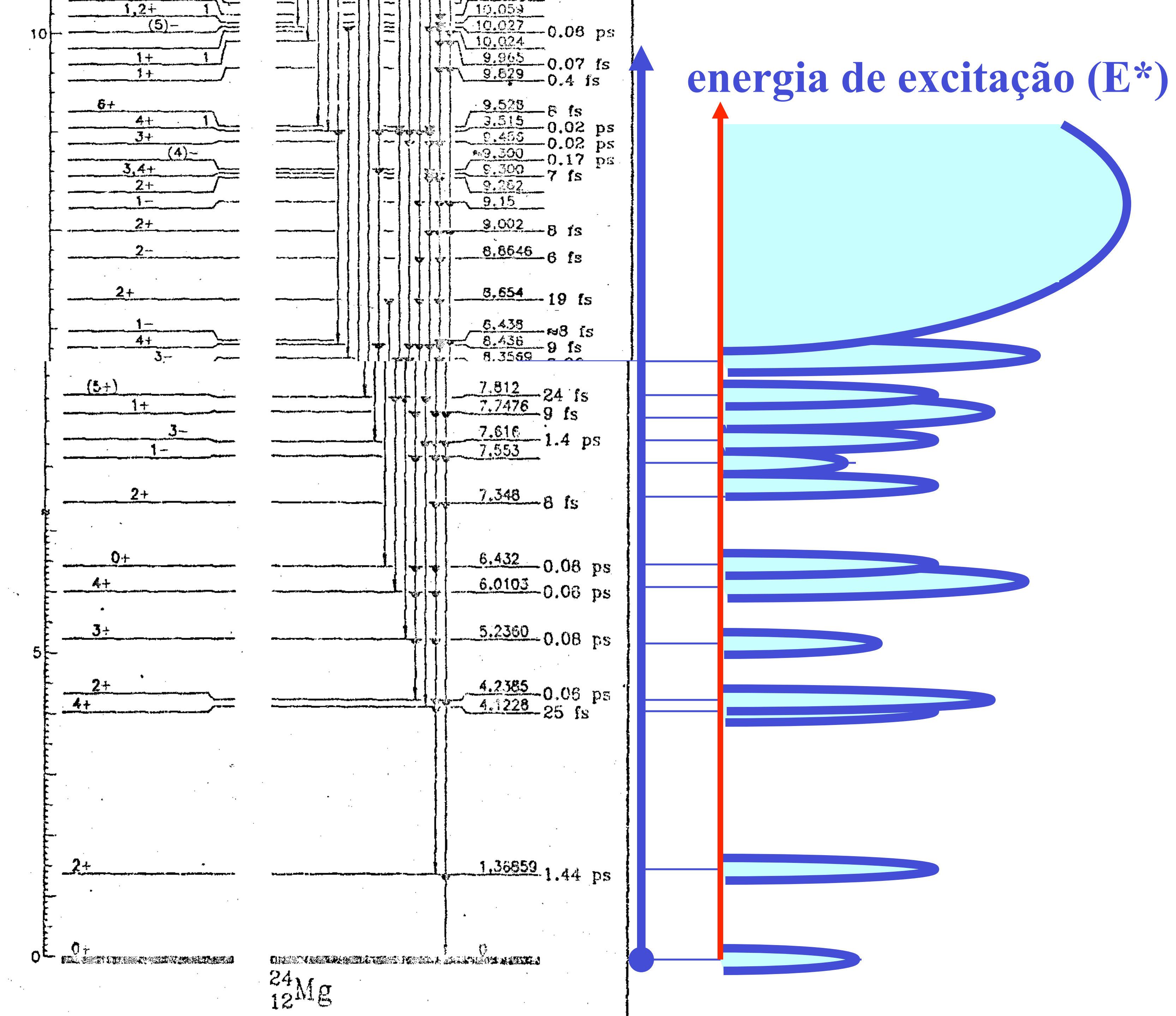
Processos estatísticos (lentos)

REAÇÃO DIRETA

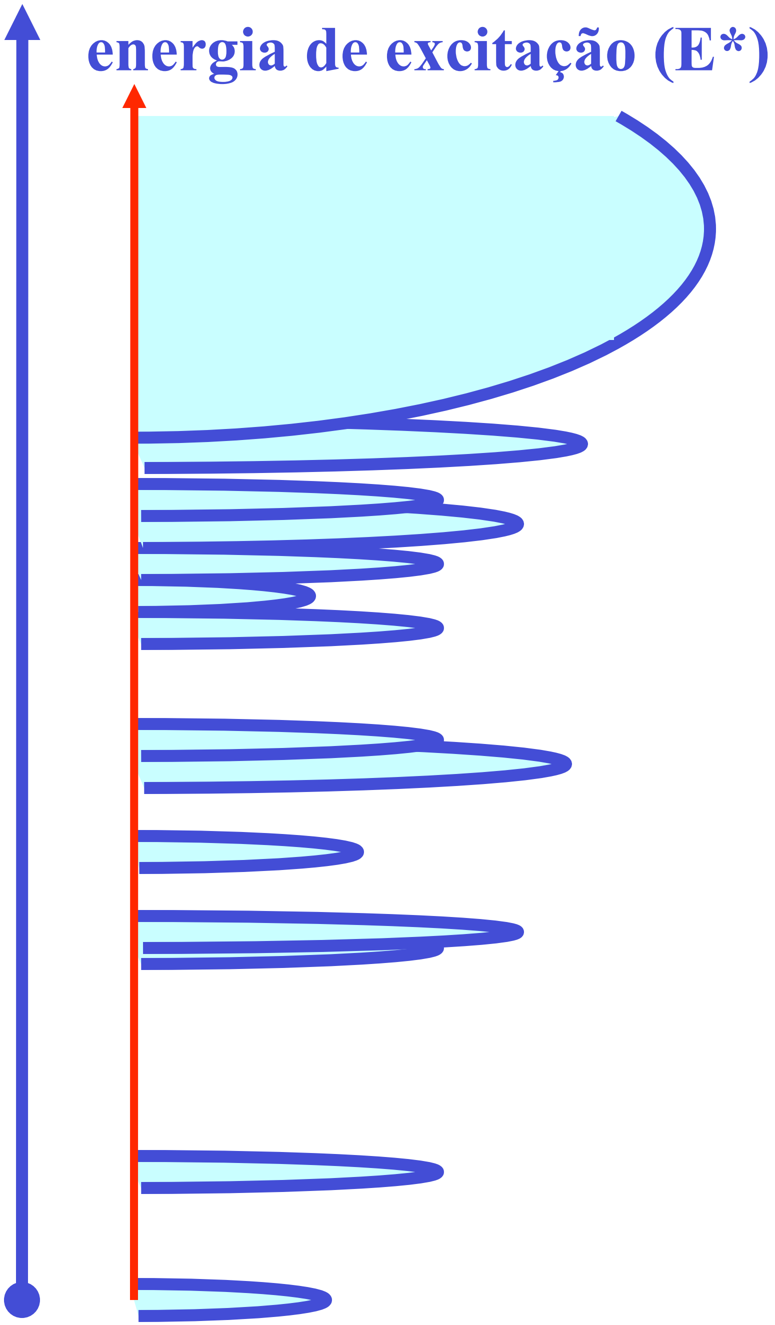




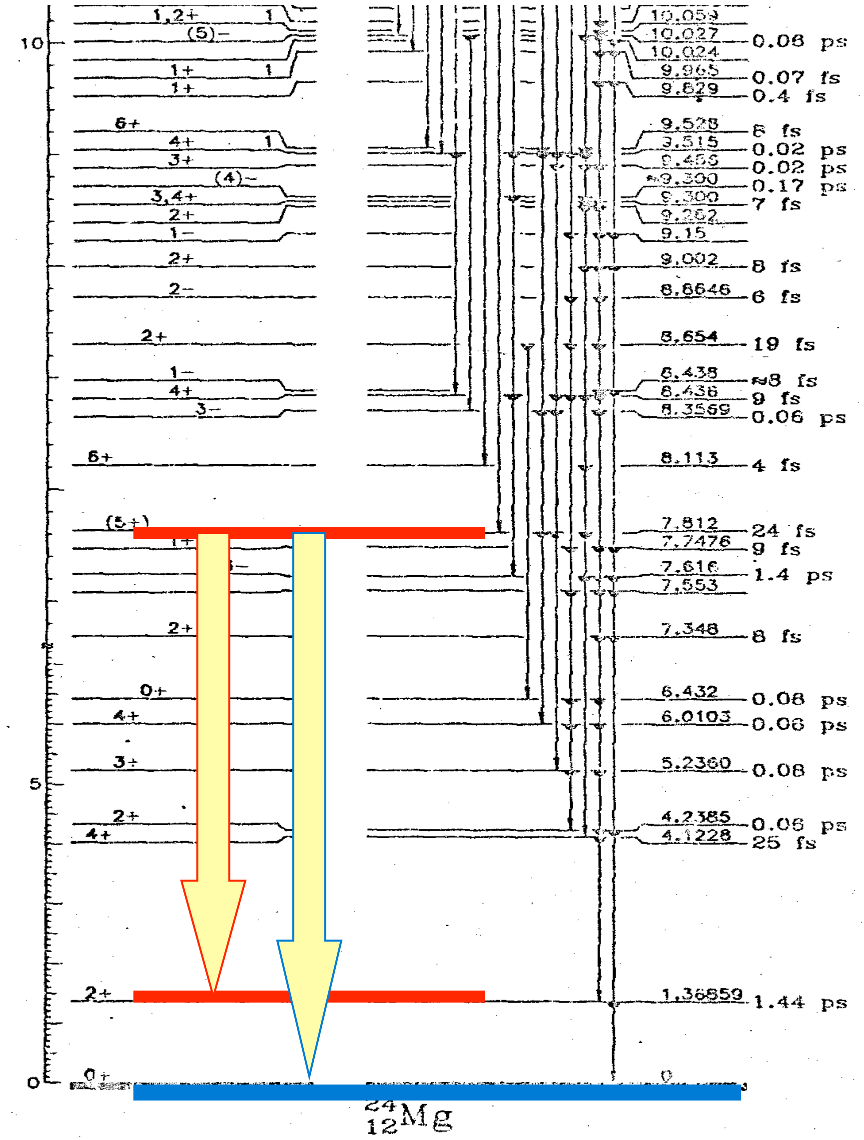
$^{11}B(^{13}C, d)^{22}Ne$
E_{Lab} = 41 MeV
Θ_{Lab} = 13°
COUNTS per mm
E* (MeV)
600
500
400
300
200
100
0
5
6
7
8
9
10
11
4.457
5.144
5.335 + 5.360
5.521
5.637
5.914
6.116
6.300 + 6.343
6.644 + 6.692
6.819 + 6.855
7.052
7.336 + 7.350
7.402 + 7.440
8.077 + 8.136
8.50
8.74
9.18
9.34
9.72
9.86
10.42
10.66
10.78
11.02

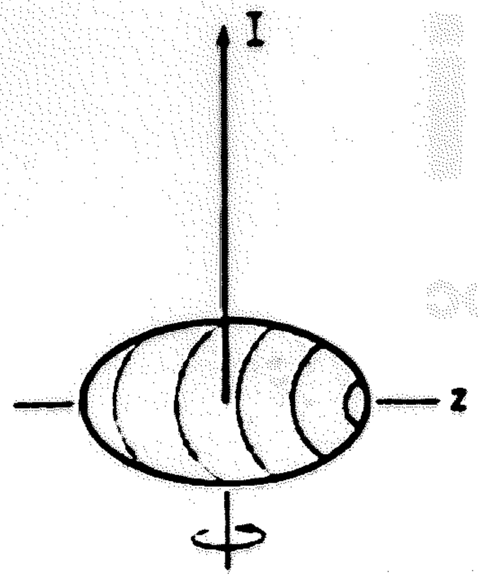


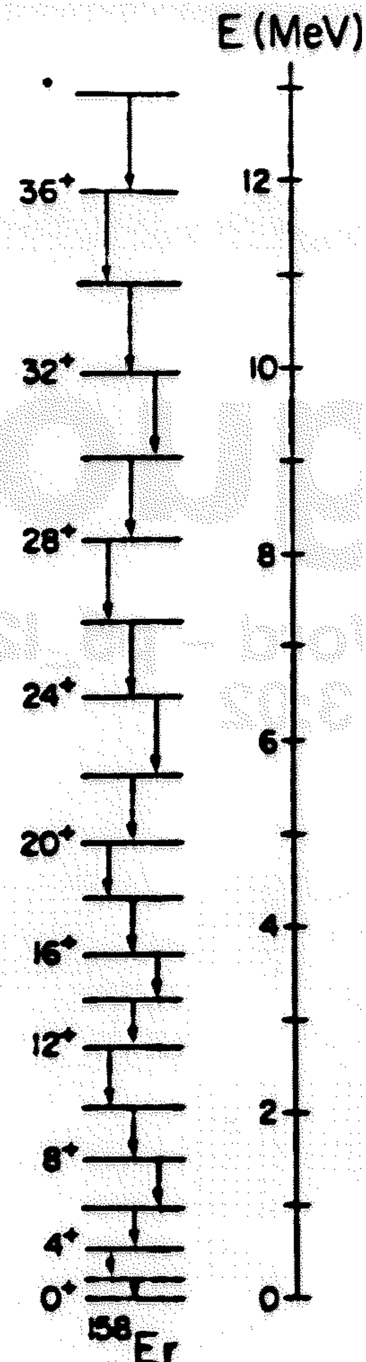
energia de excitação (E*)
10
5
0
$^{24}_{12}Mg$
10.059
10.027 0.08 ps
10.024
9.965 0.07 fs
9.829 0.4 fs
9.528 8 fs
9.515 0.02 ps
9.300 0.17 ps
9.262
9.15
9.002 8 fs
8.8646 6 fs
8.654 19 fs
8.438 ≈8 fs
8.436 9 fs
8.3569
7.812 24 fs
7.7476 9 fs
7.616 1.4 ps
7.553
7.348 8 fs
6.432 0.08 ps
6.0103 0.08 ps
5.2360 0.08 ps
4.2385 0.06 ps
4.1228 25 fs
1.36859 1.44 ps
0

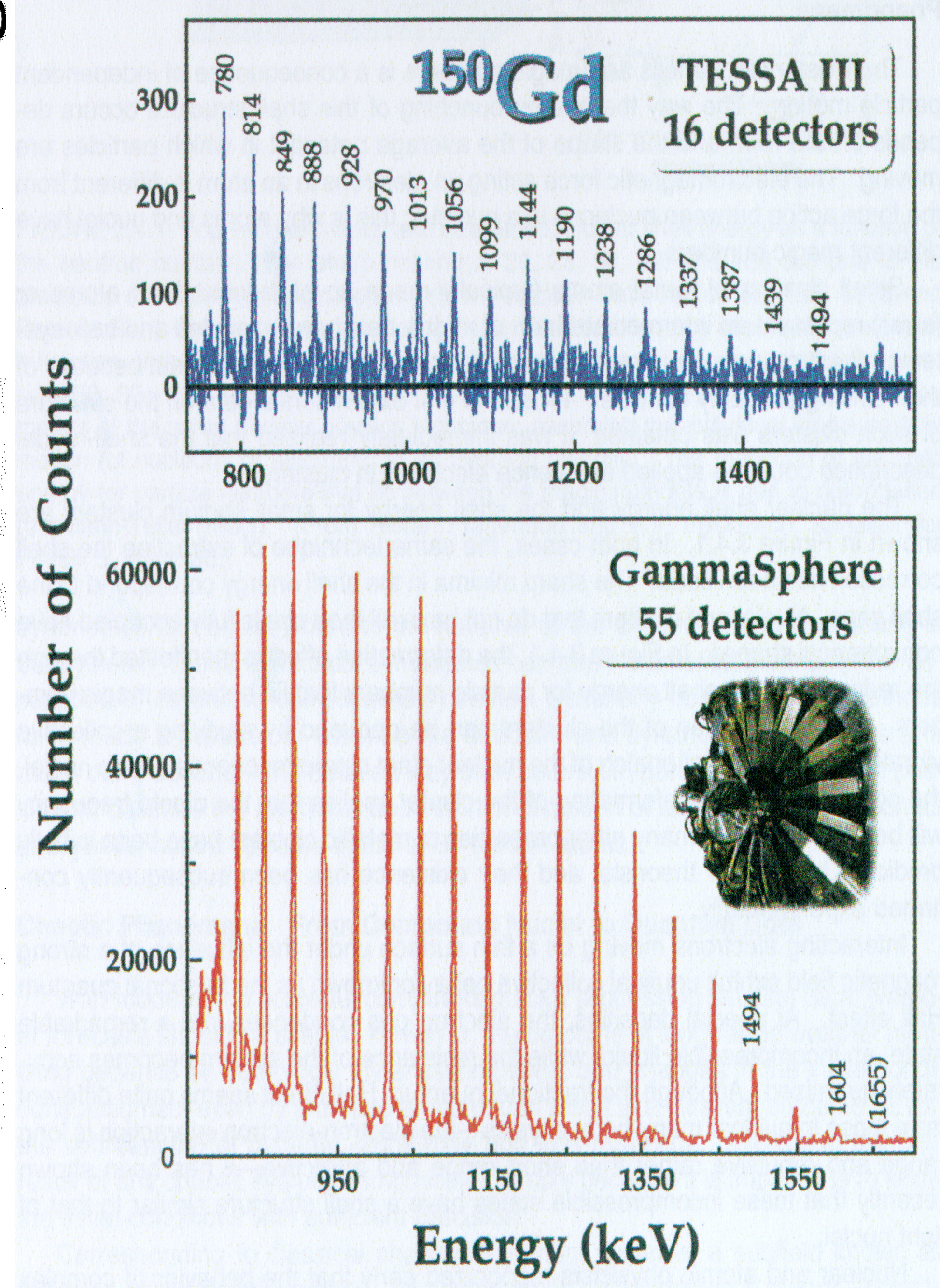

energia de excitação (E*)

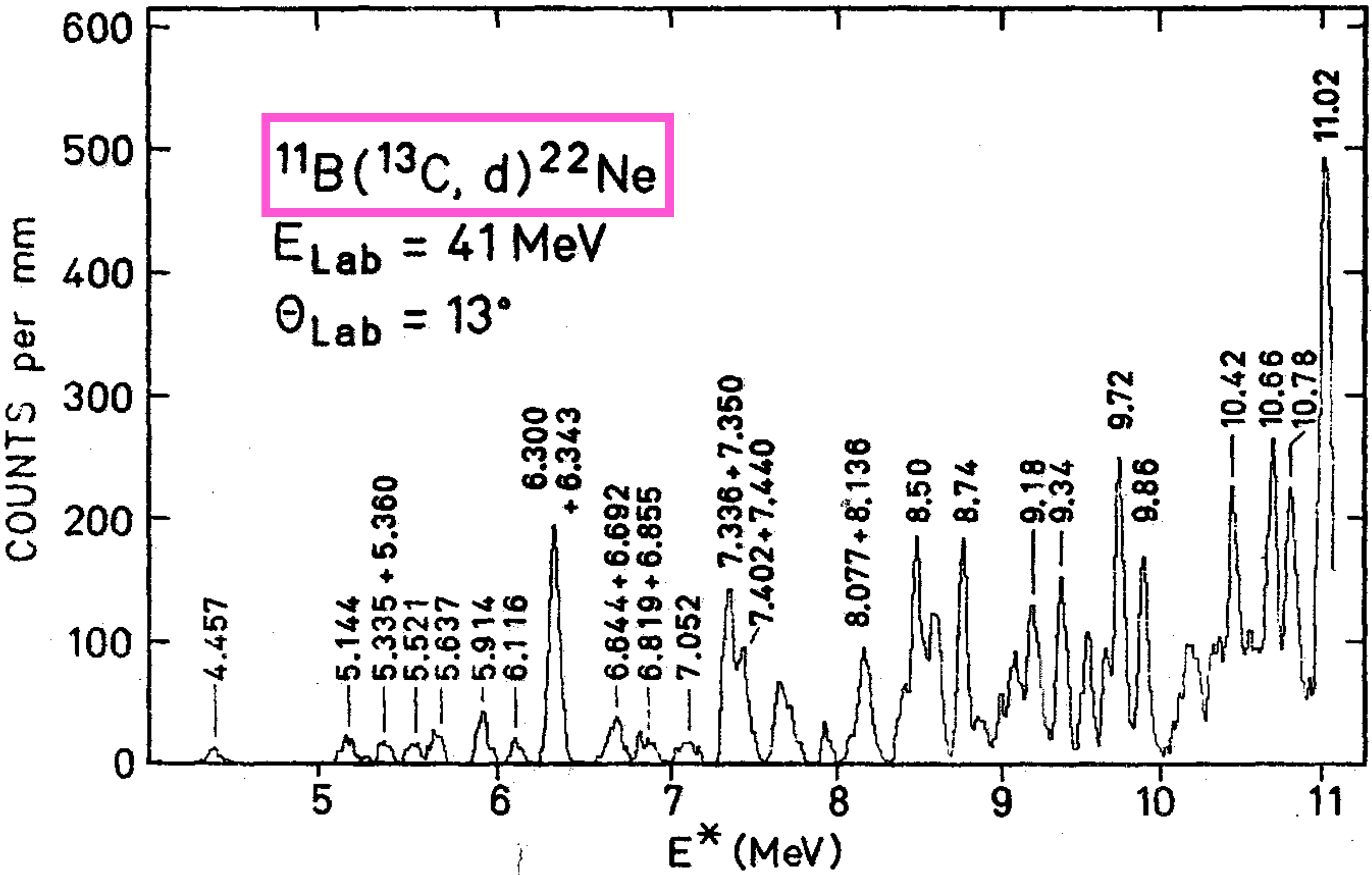
DECAIMENTO GAMMA (γ)




I
z

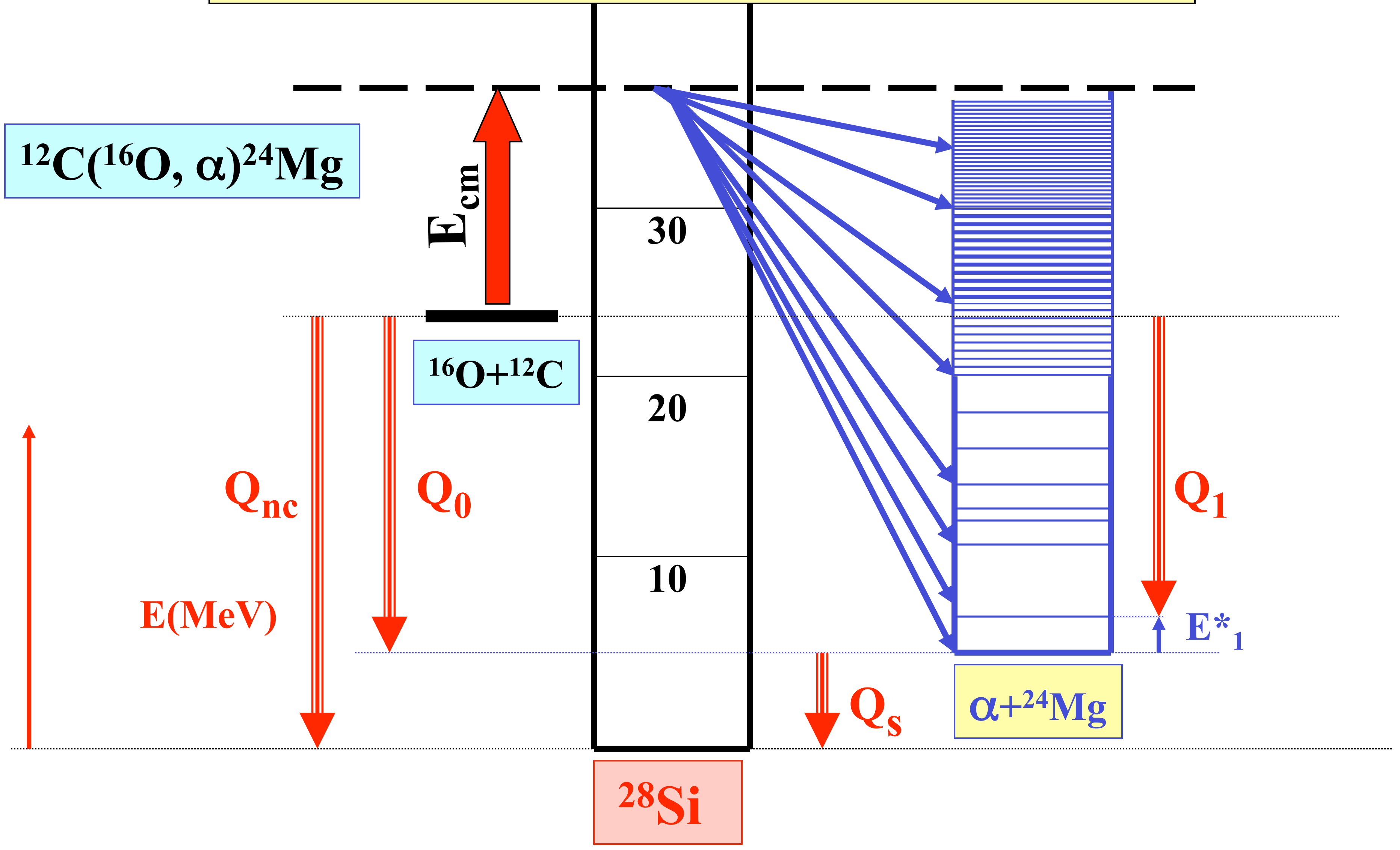

E (MeV)
36+
32+
28+
24+
20+
16+
12+
8+
4+
0+
158Er
12
10
8
6
4
2
0


150Gd
TESSA III
16 detectors
780
814
849
888
928
970
1013
1056
1099
1144
1190
1238
1286
1337
1387
1439
1494
300
200
100
0
800
1000
1200
1400
GammaSphere
55 detectors
60000
40000
20000
0
1494
1604
(1655)
950
1150
1350
1550
Number of Counts
Energy (keV)

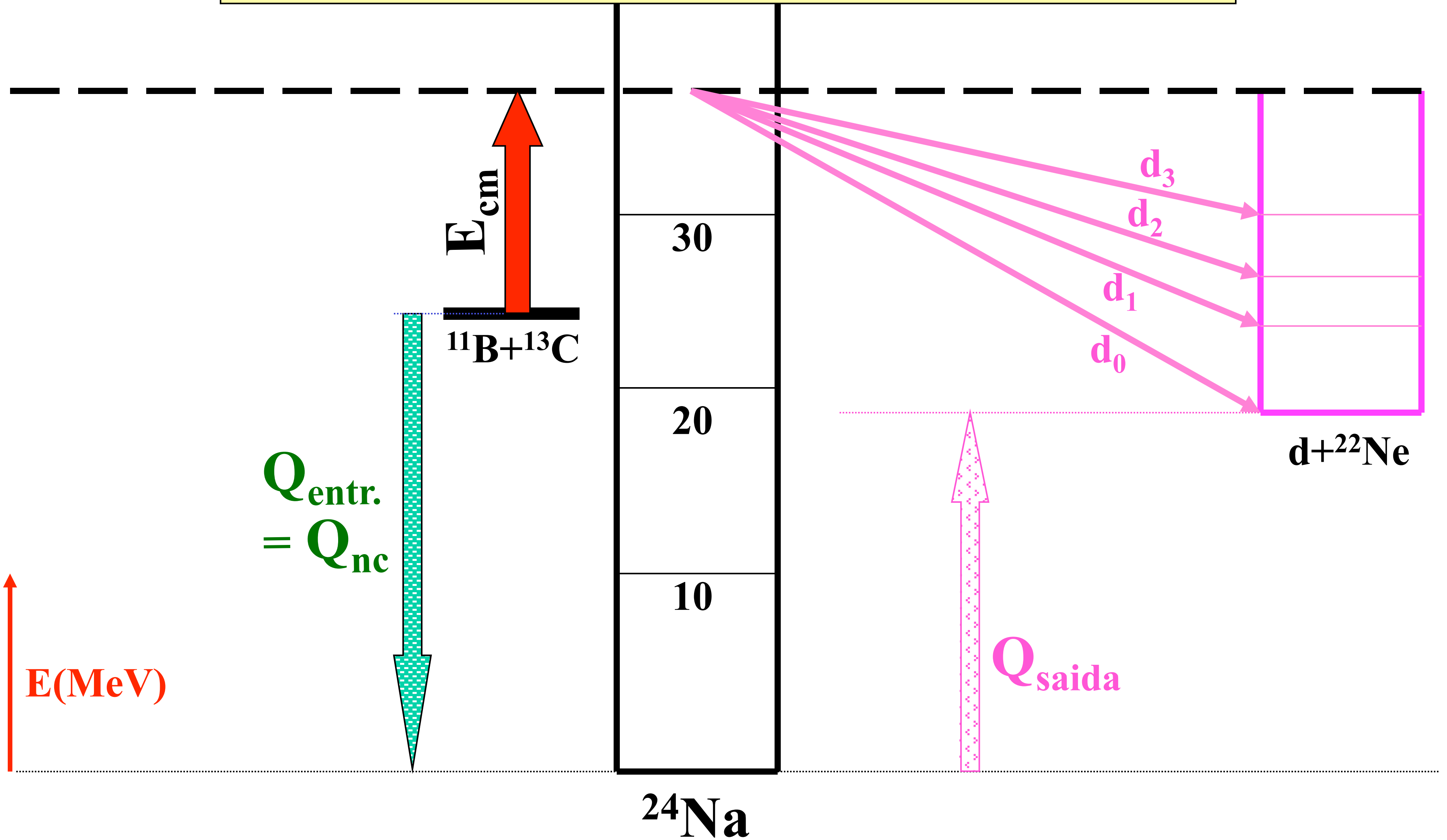


$^{11}B(^{13}C, d)^{22}Ne$
E_{Lab} = 41 MeV
Θ_{Lab} = 13°
COUNTS per mm
E* (MeV)
0
100
200
300
400
500
600
5
6
7
8
9
10
11
4.457
5.144
5.335 + 5.360
5.521
5.637
5.914
6.116
6.300
+ 6.343
6.644 + 6.692
6.819 + 6.855
7.052
7.336 + 7.350
7.402 + 7.440
8.077 + 8.136
8.50
8.74
9.18
9.34
9.72
9.86
10.42
10.66
10.78
11.02

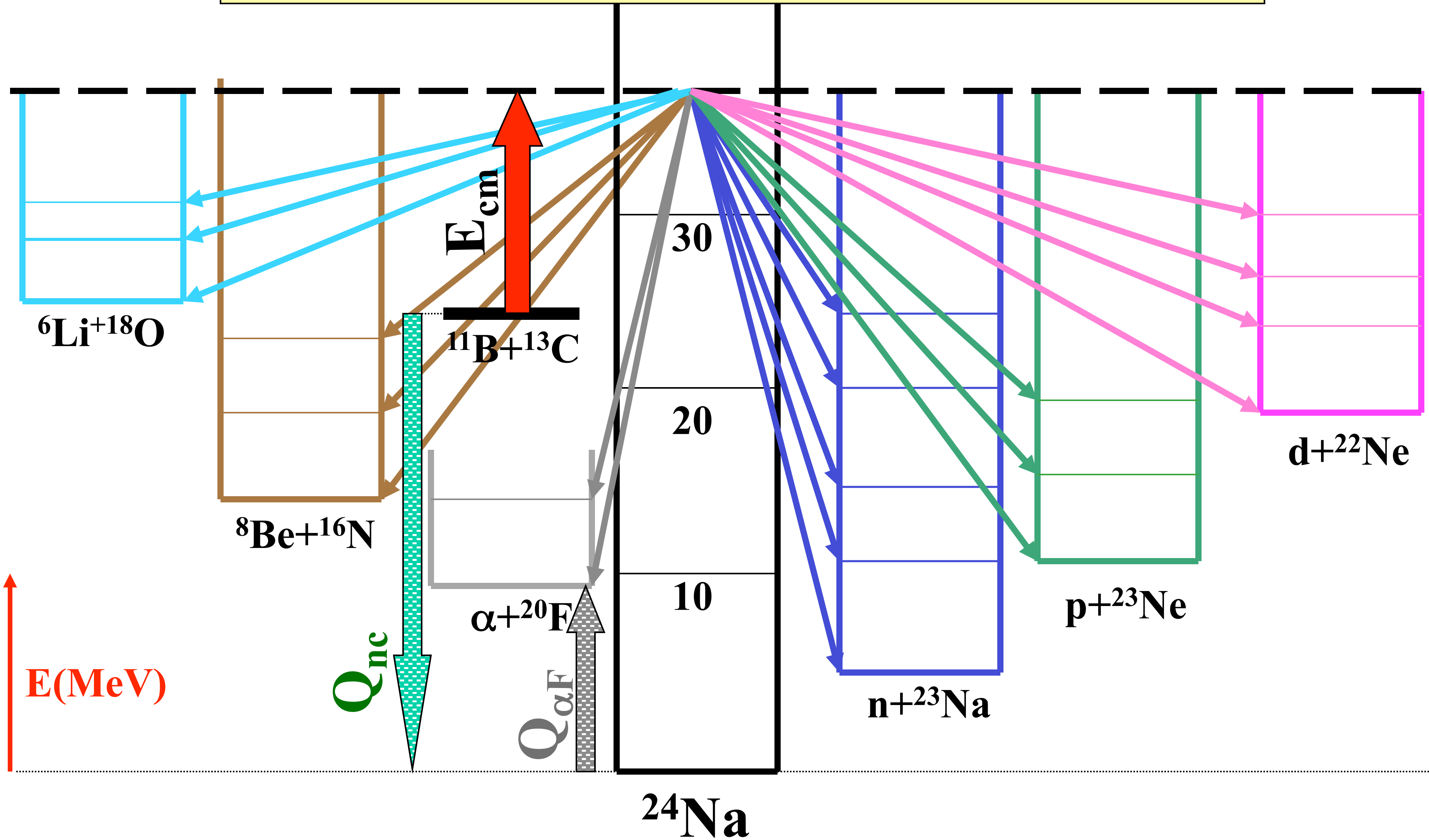
REAÇÃO VIA NÚCLEO COMPOSTO



REAÇÃO VIA NÚCLEO COMPOSTO

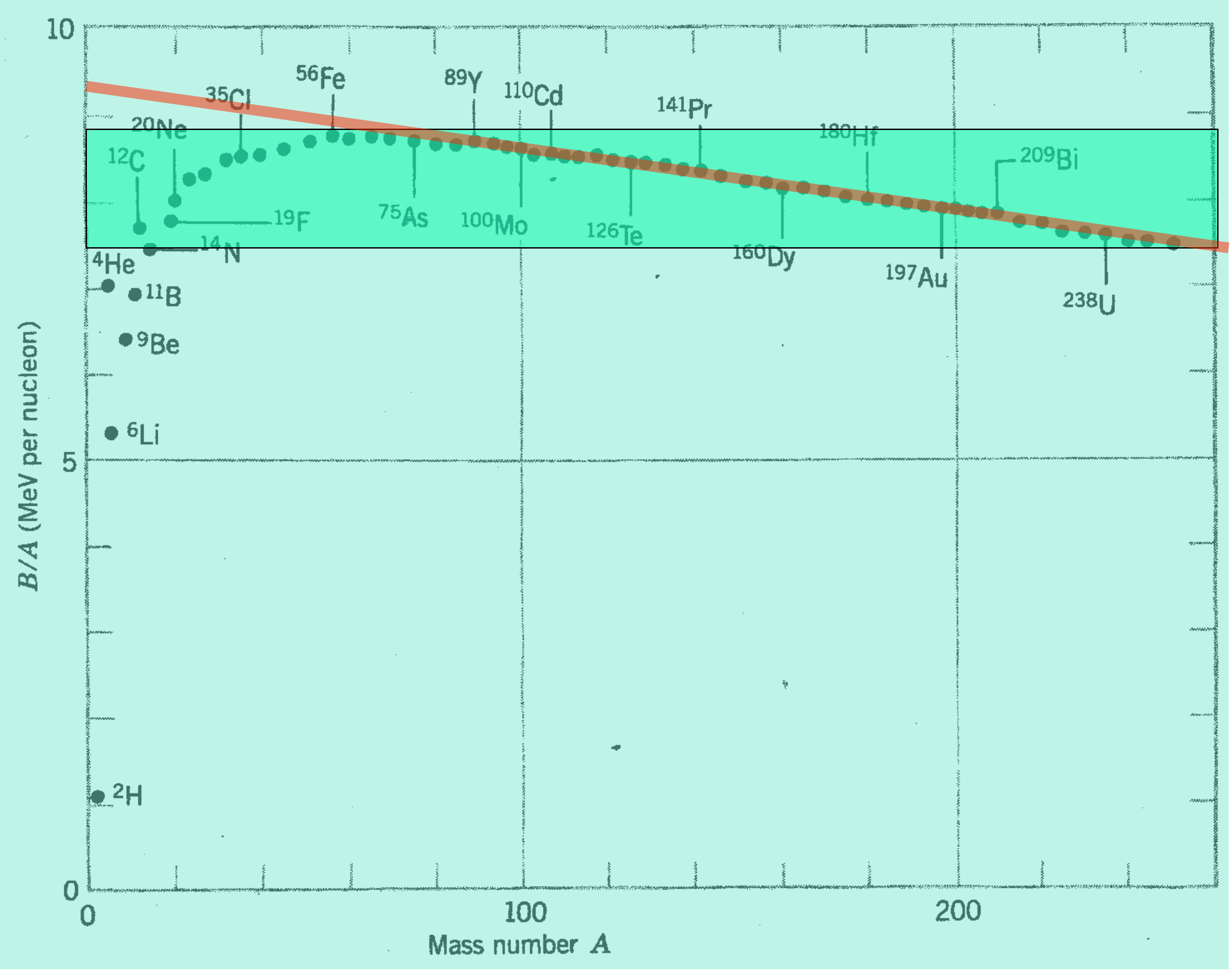


REAÇÕES VIA NÚCLEO COMPOSTO



modêlos macroscópicos

N	Z	A	EL	O	MASS EXCESS (KEV)		B/A	BINDING ENERGY (KEV)		BETA-DECAY ENERGY (KEV)		ATOMIC MASS (U)	
1	0	1	N		8071.69	0.10		0.0	0.0	782.47	0.05	1.00866522	0.00000006
0	1		H		7289.22	0.09		0.0	0.0	*		1.00782522	0.00000004
1	1	2	H		13136.27	0.16	1.11	2224.64	0.04	*		2.01410222	0.00000007
2	1	3	H		14950.38	0.22	2.42	8482.22	0.15	18.65	0.04	3.01604972	0.00000016
1	2		HE		14931.73	0.22	2.57	7718.40	0.14	*		3.01602970	0.00000016
3	1	4	H	-N	25920	500		5580	500	23500	500	4.02783	0.00054
2	2		HE		2424.94	0.25	7.07	28296.9	0.4	-22700	300	4.00260326	0.0000027
1	3		LI	+NN	25130	300		4810	300	*		4.02697	0.00032
4	1	5	H	+	33790	800		5790	800	22400	800	5.03627	0.00086
3	2		HE	-N	11390	50		27410	50	-290	70	5.01222	0.00005
2	3		LI	-P	11680	50		26330	50	*		5.01254	0.00005
4	2	6	HE		17597.3	3.6		29267.9	3.6	3509.8	3.6	6.0188913	0.0000039
3	3		LI		14087.5	0.7	5.33	31995.2	0.8	-4287	5	6.0151234	0.0000008
2	4		BE	-	18375	5		26926	5	*		6.019726	0.000006
5	2	7	HE	+	26111	30		28826	30	11203	30	7.028031	0.000032
4	3		LI		14908.6	0.8	5.60	39245.9	0.9	-861.75	0.09	7.0160048	0.0000008
3	4		BE		15770.3	0.8		37601.6	0.9	-12170	100	7.0169299	0.0000008
2	5		B	-	27940	100		24650	100	*		7.02999	0.00011
6	2	8	HE	+	31650	120		31360	120	10700	120	8.03397	0.00013
5	3		LI	-N	20947.5	1.0		41278.6	1.2	16005.8	1.1	8.0224879	0.0000011
4	4		BE		4941.8	0.5	7.06	56501.9	0.8	-17980.5	1.3	8.0053052	0.0000005
3	5		B	-PP	22922.3	1.2		37738.8	1.3	*		8.0246079	0.0000013
6	3	9	LI	+	24966	5		45331	5	13618	5	9.026802	0.000005
5	4		BE		11348.4	0.6	6.46	58167.0	0.9	-1067.3	0.7	9.0121828	0.0000006
4	5		B	-	12415.7	0.9		56317.1	1.1	-16497	5	9.0133287	0.0000010
3	6		C		28912	5		39038	5	*		9.031038	0.000006
7	3	10	LI	-N	35340	SYST		43030	SYST	22730	SYST	10.03794	SYST
6	4		BE		12608.1	0.7		64978.9	1.0	555.9	0.8	10.0135352	0.0000008
5	5		B		12052.3	0.4	6.47	64752.3	0.9	-3650.4	1.8	10.0129385	0.0000004
4	6		C		15702.7	1.8		60319.4	2.0	*		10.0168573	0.0000020
8	3	11	LI	-N	43310	SYST		43130	SYST	23130	SYST	11.04649	SYST
7	4		BE		20177	6		65482	6	11509	6	11.021660	0.000007
6	5		B		8667.95	0.29	6.93	76208.3	1.0	-1982.2	1.0	11.00930533	0.00000030
5	6		C	-	10650.2	1.1		73443.6	1.4	-14800	SYST	11.0114333	0.0000011
4	7		N	-	25450	SYST		57860	SYST	*		11.02732	SYST
8	4	12	BE	-N	24950	SYST		68780	SYST	11580	SYST	12.02678	SYST
[illegible]	[illegible]		[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	6		C		0.0	0.0	7.68	92165.5	1.1	-17344	5	12.00000000	0.0
5	7		N		[illegible]	[illegible]		[illegible]	[illegible]			[illegible]	[illegible]



10
5
0
0
100
200
Mass number A
B/A (MeV per nucleon)
56Fe
89Y
110Cd
35Cl
141Pr
20Ne
180Hf
12C
209Bi
19F
75As
100Mo
126Te
14N
160Dy
4He
197Au
11B
238U
9Be
6Li
2H

2.4 Modelo da gota líquida e limites de estabilidade

Na secção anterior ficaram claros os limites da analogia entre o comportamento da matéria nuclear e o de um líquido. Nesta secção, pretende-se desenvolver um modelo nuclear simples, em que apenas se faz uso de propriedades do núcleo análogas às de um líquido. Este *modelo da gota líquida* permite compreender o comportamento das energias de ligação e, por meio delas, as massas nucleares, não conseguindo contudo explicar outros tipos de propriedades.

No que se segue, considera-se o núcleo como uma gota de um líquido incompressível que se mantém coesa sob a acção de forças de alcance curto. A energia de ligação do núcleo, B, obtém-se pela soma de várias parcelas

$$B = B_1 + B_2 + B_3 + B_4 + B_5 \qquad (2.47)$$

correspondentes a outras tantas contribuições que se discutem de seguida, nas alíneas 1) a 5), sendo a energia B expressa como função de Z e de A. Interessa apenas obter, para cada contribuição, a relação funcional com aquelas grandezas e determinar depois, empiricamente, os valores das constantes necessárias.

1) A principal contribuição para a energia de ligação é a "energia de condensação", libertada no momento em que os nucleões se reunem para formar o núcleo. Ela deve ser proporcional ao número de partículas ligadas, de acordo com o valor aproximadamente constante de B/A (Fig.10). Se a_V for a constante de proporcionalidade, tem-se, portanto,

$$B_1 = a_V A \qquad (2.48)$$

Como A é proporcional ao volume do núcleo chama-se a este termo *energia de volume.*

2) Os nucleões que se encontram à superfície do núcleo têm menor número de ligações com os vizinhos do que os que estão no interior, ficando por isso menos ligados e contribuindo menos para uma energia de ligação. Introduz-se, portanto, um termo negativo, B_2, proporcional à superfície $4\pi R^2 = 4\pi r_o^2 A^{2/3}$ e, como importa apenas a dependência funcional em A, vem

$$B_2 = -a_S A^{\frac{2}{3}} \quad \text{(Energia de superfície)} \qquad (2.49)$$

3) A energia de ligação é ainda mais reduzida devido à repulsão entre os protões. A energia de Coulomb duma esfera de raio R e carga q, carregada uniformemente, é $(3/5).(q^2/R)$. Para o núcleo de carga Ze e raio $R = r_o A^{1/3}$, a dependência funcional em Z e A conduz a um termo da forma

$$B_3 = -a_C Z^2 A^{-\frac{1}{3}} \quad \text{(Energia de Colomb)} \qquad (2.50)$$

4) Ao considerar a dependência de B em A e Z, deve-se também considerar que o excesso de neutrões é acompanhado por uma diminuição da energia de ligação em relação à situação simétrica (N = Z). De acordo com (2.46), esta diferença de energia depende do excesso de neutrões, sendo o termo correspondente dado por

$$B_4 = -a_A \frac{T_z^2}{A} = -a_A \frac{(Z - A/2)^2}{A} \quad \text{(Energia de assimetria)} \qquad (2.51)$$

5) Sabe-se, com base na sistemática das energias de separação, que os nucleões do mesmo tipo produzem uma ligação particularmente forte quando surgem aos pares. A *energia de emparelhamento* não pode ser explicada com base na analogia com a gota líquida, sendo necessário, neste contexto, introduzi-la como correcção empírica. Se tanto Z como N são números pares (núcleos par-par) esta energia é particularmente elevada, sendo pelo contrário particularmente baixa para núcleos em que Z e N são ímpares (núcleos ímpar-ímpar). Introduz-se pois da seguinte maneira a contribuição B_5:

$$B_5 = \begin{cases} +\delta & \text{núcleos par-par} \\ 0 & \text{núcleos par-ímpar ou ímpar-par} \\ -\delta & \text{núcleos impar-ímpar} \end{cases} \qquad (2.52)$$

Uma fórmula empírica, válida em boa aproximação, é

$$\delta \approx a_p A^{-\frac{1}{2}} \qquad (2.53)$$

A energia de emparelhamento não pode ser explicada facilmente. Isso torna-se evidente se se pensar que um par de nucleões idênticos não está ligado. De facto nem o "diprotão" nem o "dineutrão" existem como sistemas ligados. Se esses núcleos existissem os seus nucleões apresentariam spins

desemparelhados no estado fundamental, de acordo com o princípio de Pauli. Pelo contrário, o spin do deuterão no estado fundamental é 1, ou seja, o protão e o neutrão têm spins paralelos. Isto significa que a estrutura das forças nucleares é tal que a energia de ligação é maior no caso de spins paralelos. Não se pode portanto compreender a energia de emparelhamento a partir do potencial da ligação entre pares de nucleões. Trata-se efectivamente dum fenómeno que surge somente nos sistemas de muitas partículas e cuja origem será discutida no secção 6.5.

Veja-se agora de que modo as contribuições 1) a 5) se adicionam para dar a energia de ligação. De acordo com (2.10) a massa nuclear m(Z,A) exprime-se por

$$m(Z, A) = Zm_H + (A - Z)m_n - B/c^2 .$$

Introduzindo aqui a expressão de B dada em (2.47) e fazendo uso das relações (2.48) a (2.52), resulta

$$m(Z, A) = Zm_H + (A - Z)m_n - a_V A + a_S A^{\frac{2}{3}} + a_C Z^2 A^{-\frac{1}{3}} + a_A (Z - A/2)^2 A^{-1} \pm \delta \qquad (2.54)$$

onde as constantes a_V até a_p contêm agora um factor $1/c^2$. Esta expressão é conhecida por *fórmula de Weizsaecker* (1935). Para determinar os valores das constantes serão precisas em princípio cinco massas nucleares. Contudo, o ajuste é muito melhor se forem consideradas tantas mais massas quantas for possível, uma vez que a fórmula apenas descreve um comportamento médio. Um conjunto de valores para aquelas constantes é o seguinte [Wap 58]:

$$a_V = 17{,}011 \text{ mu} = 15{,}85 \text{ MeV}/c^2$$

$$a_S = 19{,}691 \text{ mu} = 18{,}34 \text{ MeV}/c^2$$

$$a_C = 0{,}767 \text{ mu} = 0{,}71 \text{ MeV}/c^2$$

$$a_A = 99{,}692 \text{ mu} = 92{,}86 \text{ MeV}/c^2$$

$$a_p = \pm 12{,}3 \text{ mu} = 11{,}46 \text{ MeV}/c^2$$

A contribuição dos termos individuais da expressão (2.54) para a energia de ligação por nucleão está representada na Fig.19. A figura mostra como o decréscimo da energia de superfície e do crescimento da energia de Coulomb, conduz a um máximo de B/A para $A \approx 60$. É claro que a fórmula de Weizsaecker exprime apenas o comportamento médio dos núcleos, não podendo decerto reproduzir quaisquer efeitos da estrutura em camadas. A expressão só é aplicável para $A > 30$ (ver Fig.10), produzindo para $A > 40$ valores de B/A correctos dentro de $\approx 1\%$. É, de facto, notável que um modelo tão simples seja capaz de descrever tão bem a energia de ligação. Para aplicações práticas existem fórmulas de massa que foram refinadas à custa da inclusão de hipóteses suplementares, e que produzem resultados ainda melhores que a expressão (2.54) (ver, por exemplo, [See 61, Mye 66, Gar 69]).

A constante a_A do termo de assimetria pode calcular-se a partir do modelo do gás de Fermi (v. (2.45)), mas o valor assim determinado representa apenas cerca de metade do valor determinado empiricamente a partir das massas nucleares. Existe porém uma outra contribuição para o termo de assimetria, que tem a ver com a já referida dependência que apresentam as forças nucleares relativamente ao spin. A ligação entre um neutrão e um protão que estejam alinhados paralelamente é maior que entre dois neutrões, os quais, devido ao princípio de Pauli, só podem ter orientação anti-paralela. Os núcleos com excesso de neutrões apresentam por isso uma energia de ligação menor. Verifica-se que esta contribuição é proporcional a T_Z/A.

A fórmula de Weizsaecker permite deduzir um certo número de regularidades importantes. Repare-se na variação da massa nuclear ao longo duma série de isóbaros, i.e., tome-se A = const. e faça-

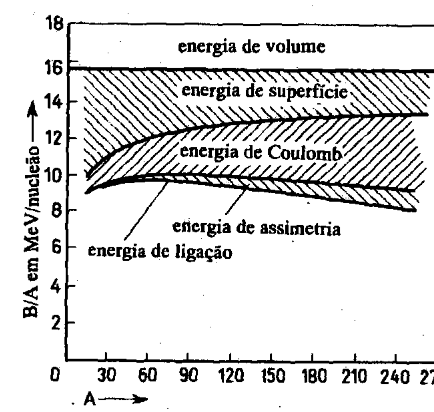


Fig.19
Contribuição dos diferentes termos da fórmula das massas nucleares para a energia de ligação média por nucleão [Eva 55].

-se variar Z em (2.54). Olhando para a expressão vê-se que ela é quadrática em Z. Para A ímpar obtém-se pois uma parábola como a representada na Fig.20a. Nos casos de A par surgem duas parábolas diferentes, devido à energia de emparelhamento $\pm\delta$. O núcleo está numa ou noutra parábola conforme seja do tipo par-par ou ímpar-ímpar (Fig.20b). Como se vê na Fig.20, nuclidos de Z vizinho podem transformar-se uns nos outros por emissão duma partícula β^+ ou β^-. Na Fig.20 lê-se também a regra segundo a qual para A ímpar apenas existe um isóbaro estável, enquanto para A par se têm vários isóbaros estáveis possíveis.

O número de protões, Z_o, para a qual a massa nuclear duma série de isóbaros é mínima ocorre para

$$\left(\frac{\partial m(Z, A)}{\partial Z}\right)_{A = const} = 0$$

Introduzindo aqui (2.54), resulta

$$- m_n + 2Z_0 a_C A^{-1/3} + 2a_A(Z_0 - A/2)A^{-1} = 0$$

e, resolvendo esta equação em ordem a Z_o, tem-se

$$Z_0 = \frac{A}{2}\left[\frac{m_n - m_H + a_A}{a_C A^{2/3} + a_A}\right] = \frac{A}{1{,}98 + 0{,}015\ A^{2/3}} \qquad (2.55)$$

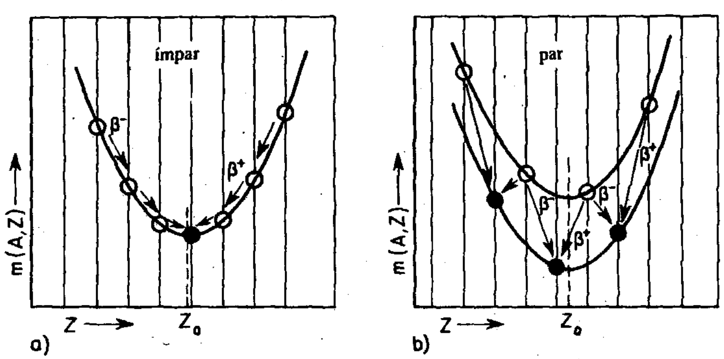


Fig.20 As energias dos núcleos com um mesmo A. Os núcleos estáveis são indicados pelos círculos a cheio.

Representando estes valores num diagrama de N em função de Z, obtém-se a Fig.21. Se, além disso, as massas nucleares forem representadas segundo o eixo perpendicular ao plano NZ, a linha a cheio na Fig.21 corresponde à localização aproximada dos núcleos estáveis, i.e., que não estão sujeitos ao decaimento β. Esses núcleos são os que se encontram no fundo do "vale" das massas nucleares.

Nos processos de transmutação por decaimento β o número de massa não é alterado. Pode igualmente usar-se a fórmula de Weizsaecker para saber se um dado processo de separação de nucleões pode libertar energia. É de esperar que, frequentemente, se ganhe energia na separação duma partícula α, em particular, devido à sua elevada energia de ligação. É realmente o que acontece sempre que a soma das massas da partícula, m_α, e do núcleo resultante, m(Z-2, A-4), seja inferior à massa do núcleo original. A energia cinética libertada será

$$E_\alpha = [m(Z, A) - m(Z - 2, A - 4) - m_\alpha]c^2 \qquad (2.56)$$

A comparação com (2.19) mostra que isto apenas significa uma energia de separação negativa. Em princípio, ganha-se energia pela separação duma partícula α sempre que $E_\alpha > 0$. Com a ajuda da fórmula de Weizsaecker é possível determinar as regiões do plano NZ que correspondem a $E_\alpha > 0, > 2, > 4, > 6$ MeV, etc. Na Fig.22 representam-se as fronteiras dessas regiões para diferentes valores de E_α. Para maior clareza, não se representa o plano NZ, mas sim N/Z em função de A. Representam-se igualmente na figura os limites das regiões de instabilidade para a separação de neutrões e de protões. Como se vê, esses limites afastam-se bastante da linha dos núcleos estáveis, que aliás nunca cruzam. Resulta daí que a emissão de neutrões ou de

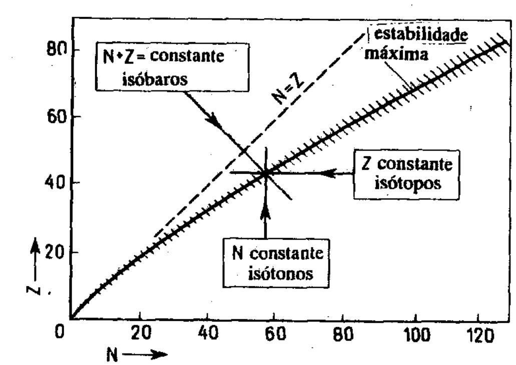


Fig.21
Localização dos núcleos estáveis no plano NZ.

actually observed, it must take this effect into account. (Otherwise it would allow stable isotopes of hydrogen with hundreds of neutrons!) This term is very important for light nuclei, for which $Z \simeq A/2$ is more strictly observed. For heavy nuclei, this term becomes less important, because the rapid increase in the Coulomb repulsion term requires additional neutrons for nuclear stability. A possible form for this term, called the symmetry term because it tends to make the nucleus symmetric in protons and neutrons, is $-a_{\text{sym}}(A-2Z)^2/A$ which has the correct form of favoring nuclei with $Z = A/2$ and reducing in importance for large A.

Finally, we must include another term that accounts for the tendency of like nucleons to couple pairwise to especially stable configurations. When we have an odd number of nucleons (odd Z and even N, or even Z and odd N), this term does not contribute. However, when both Z and N are odd, we gain binding energy by converting one of the odd protons into a neutron (or vice versa) so that it can now form a pair with its formerly odd partner. We find evidence for this *pairing force* simply by looking at the stable nuclei found in nature—there are only four nuclei with odd N and Z (^{2}H, ^{6}Li, ^{10}B, ^{14}N), but 167 with even N and Z. This pairing energy δ is usually expressed as $+a_p A^{-3/4}$ for Z and N even, $-a_p A^{-3/4}$ for Z and N odd, and zero for A odd.

Combining these five terms we get the complete binding energy:

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{\text{sym}}\frac{(A-2Z)^2}{A} + \delta \quad (3.28)$$

and using this expression for B we have the *semiempirical mass formula*:

$$M(Z,A) = Zm(^1\text{H}) + Nm_n - B(Z,A)/c^2 \quad (3.29)$$

The constants must be adjusted to give the best agreement with the experimental curve of Figure 3.16. A particular choice of $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV, $a_{\text{sym}} = 23$ MeV, $a_p = 34$ MeV, gives the result shown in Figure 3.17, which reproduces the observed behavior of B rather well.

The importance of the semiempirical mass formula is not that it allows us to predict any new or exotic phenomena of nuclear physics. Rather, it should be regarded as a first attempt to apply nuclear models to understand the systematic behavior of a nuclear property, in this case the binding energy. It includes several different varieties of nuclear models: the *liquid-drop model*, which treats some of the gross collective features of nuclei in a way similar to the calculation of the properties of a droplet of liquid (indeed, the first three terms of Equation 3.28 would also appear in a calculation of the energy of a charged liquid droplet), and the *shell model*, which deals more with individual nucleons and is responsible for the last two terms of Equation 3.28.

For constant A, Equation 3.29 represents a parabola of M vs. Z. The parabola will be centered about the point where Equation 3.29 reaches a minimum. To compare this result with the behavior of actual nuclei, we must find the minimum, where $\partial M/\partial Z = 0$:

$$Z_{\text{min}} = \frac{[m_n - m(^1\text{H})] + a_c A^{-1/3} + 4a_{\text{sym}}}{2a_c A^{-1/3} + 8a_{\text{sym}} A^{-1}} \quad (3.30)$$

With $a_c = 0.72$ MeV and $a_{\text{sym}} = 23$ MeV, it follows that the first two terms in the numerator are negligible, and so

$$Z_{\text{min}} \simeq \frac{A}{2}\frac{1}{1 + \frac{1}{4}A^{2/3}a_c/a_{\text{sym}}} \quad (3.31)$$

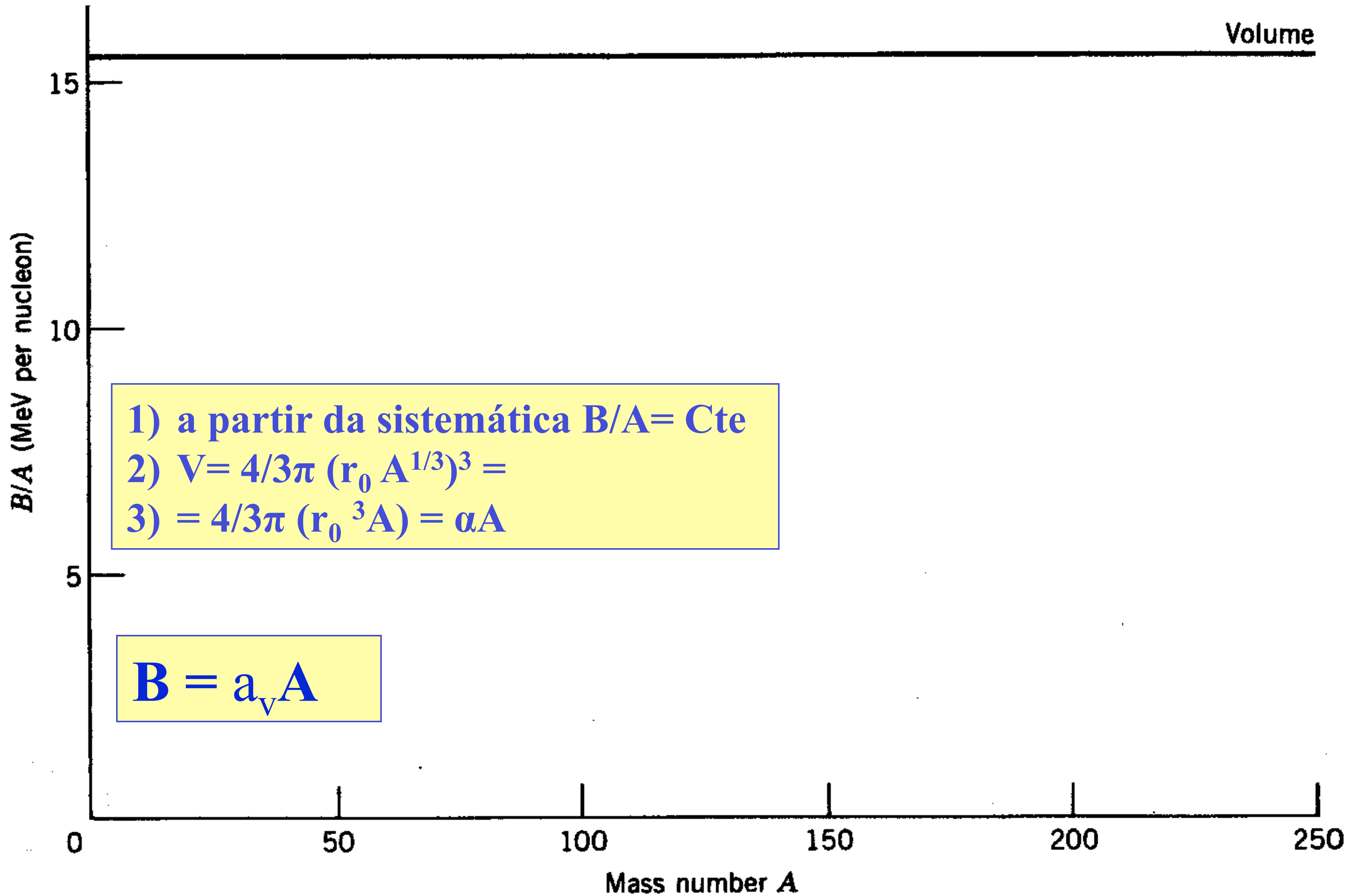
For small A, $Z_{\text{min}} \simeq A/2$ as expected, but for large A, $Z_{\text{min}} < A/2$. For heavy nuclei, Equation 3.31 gives $Z/A \simeq 0.41$, consistent with observed values for heavy stable nuclei.

Figure 3.18 shows a typical odd-A decay chain for $A = 125$, leading to the stable nucleus at $Z = 52$. The unstable nuclei approach stability by converting a neutron into a proton or a proton into a neutron by radioactive β decay. Notice how the decay energy (that is, the mass difference between neighboring isobars) increases as we go further from stability. For even A, the pairing term gives two parabolas, displaced by 2δ. This permits two unusual effects, not seen in odd-A decays: (1) some odd-Z, odd-N nuclei can decay in either direction, converting a neutron to a proton or a proton to a neutron; (2) certain *double β decays* can become energetically possible, in which the decay may change 2 protons to 2 neutrons. Both of these effects are discussed in Chapter 9.

3.4 NUCLEAR ANGULAR MOMENTUM AND PARITY

In Section 2.5 we discussed the coupling of orbital angular momentum ℓ and spin s to give total angular momentum j. To the extent that the nuclear potential is central, ℓ and s (and therefore j) will be constants of the motion. In the quantum mechanical sense, we can therefore label every nucleon with the corresponding quantum numbers ℓ, s, and j. The total angular momentum of a nucleus containing A nucleons would then be the vector sum of the angular momenta of all the nucleons. This total angular momentum is usually called the *nuclear spin* and is represented by the symbol I. The angular momentum I has all of the usual properties of quantum mechanical angular momentum vectors: $I^2 = \hbar^2 I(I+1)$ and $I_z = m\hbar$ ($m = -I, \ldots, +I$). For many applications involving angular momentum, the nucleus behaves as if it were a single entity with an intrinsic angular momentum of I. In ordinary magnetic fields, for example, we can observe the nuclear Zeeman effect, as the state I splits up into its $2I+1$ individual substates $m = -I, -I+1, \ldots, I-1, I$. These substates are equally spaced, as in the atomic normal Zeeman effect. If we could apply an incredibly strong magnetic field, so strong that the coupling between the nucleons were broken, we would see each individual j splitting into its $2j+1$ substates. Atomic physics also has an analogy here: when we apply large magnetic fields we can break the coupling between the electronic ℓ and s and separate the $2\ell+1$ components of ℓ and the $2s+1$ components of s. No fields of sufficient strength to break the coupling of the nucleons can be produced. We therefore observe the behavior of I as if the nucleus were only a single "spinning" particle. For this reason, the spin (total angular momentum) I and the corresponding spin quantum number I are used to describe nuclear states.

To avoid confusion, we will always use I to denote the nuclear spin; we will use j to represent the total angular momentum of a single nucleon. It will often



1) a partir da sistemática B/A= Cte
2) $V = 4/3\pi\,(r_0\,A^{1/3})^3 =$
3) $= 4/3\pi\,(r_0{}^3 A) = \alpha A$

$$B = a_v A$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

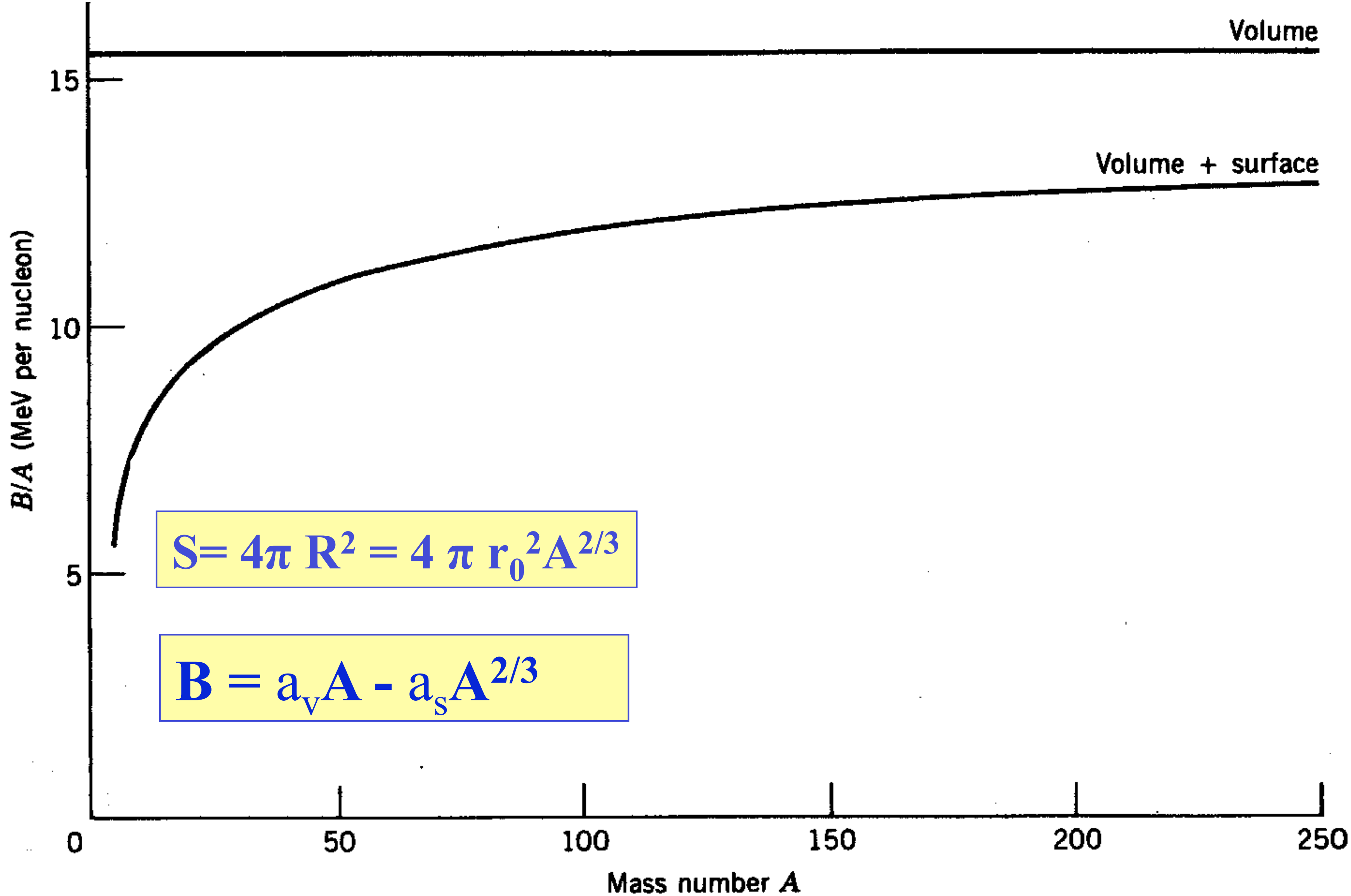
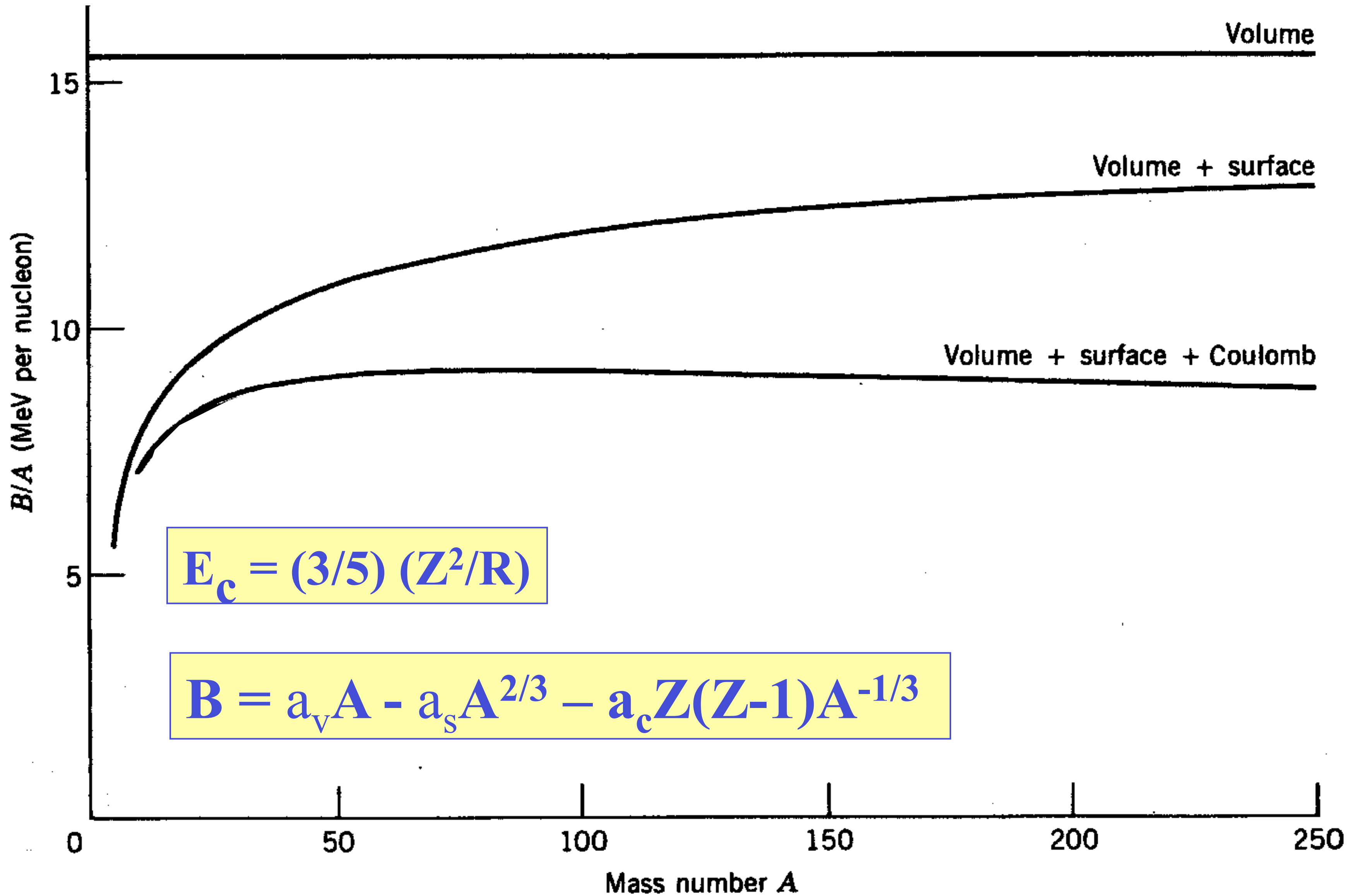


Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.



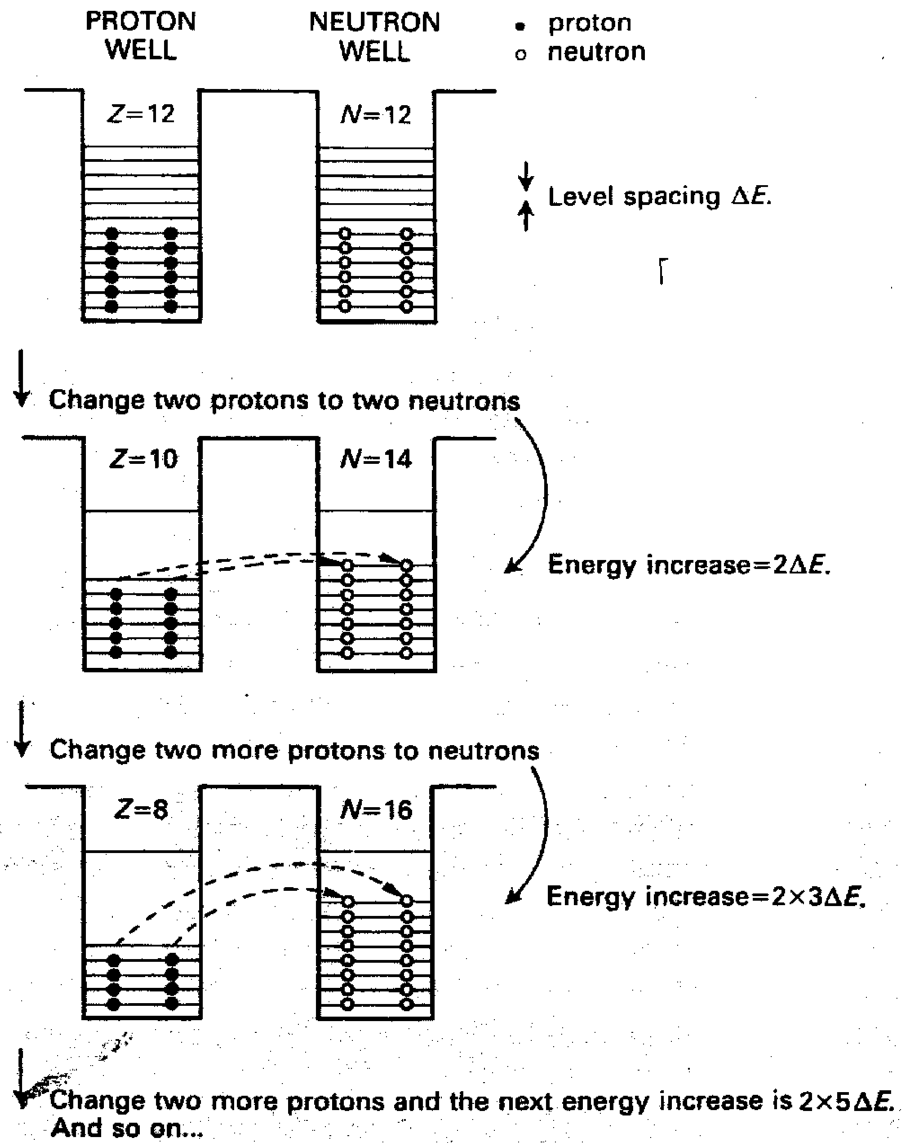
$$E_c = (3/5)\,(Z^2/R)$$

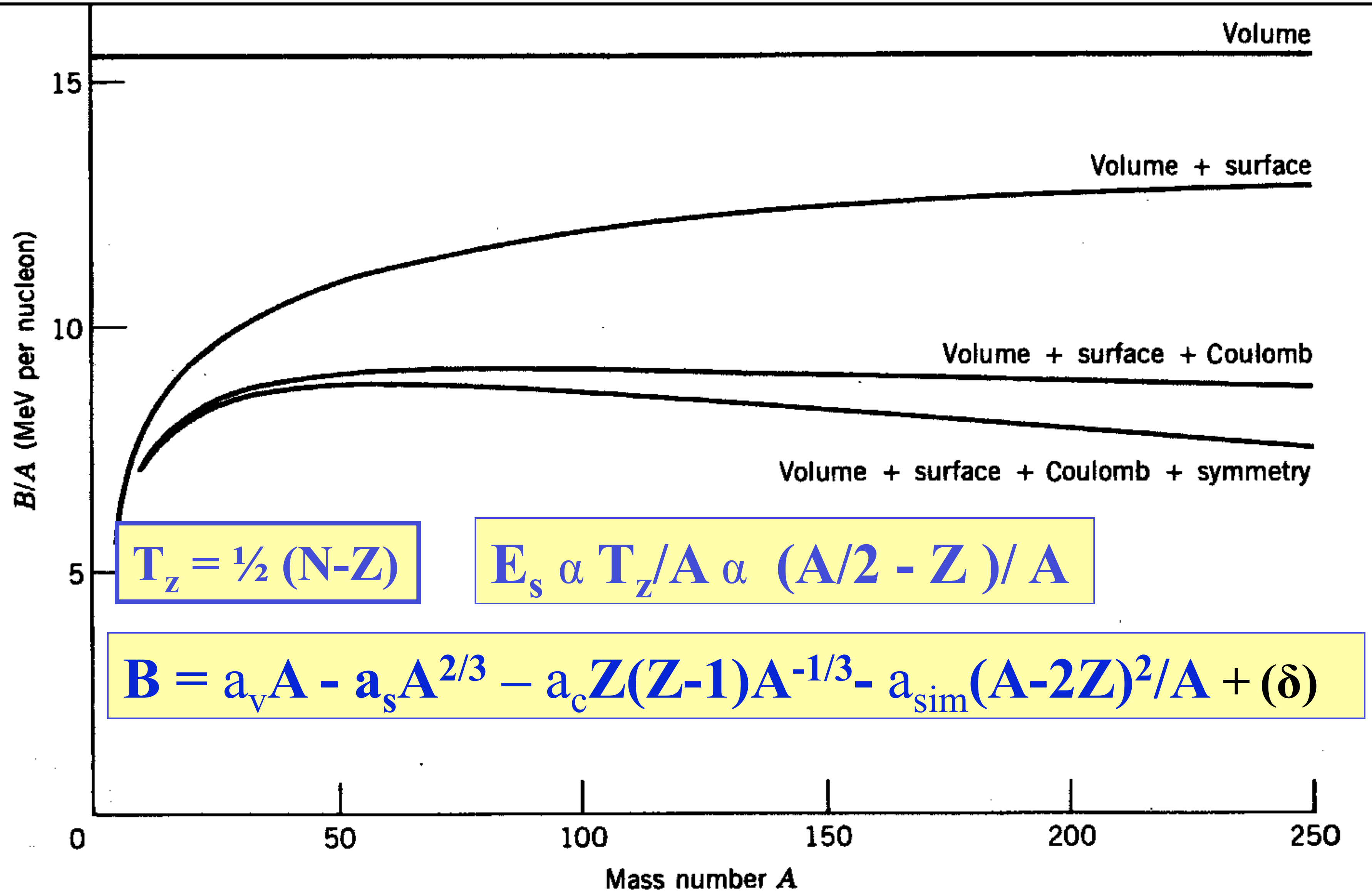
$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1) A^{-1/3}$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

TERMO DE SIMETRIA

Fig. 4.3 The occupation of energy levels of a nucleus by protons (●) and by neutrons (○) according to the Pauli exclusion principle in a nucleus which is changing from $Z=N$ to $N>Z$, while $A=Z+N$ remains constant. The cost in energy of making the change of two protons into two neutrons and placing the latter in unoccupied neutron levels increases at every change. (The cost or gain in energy due to the neutron–proton mass difference is not included.)



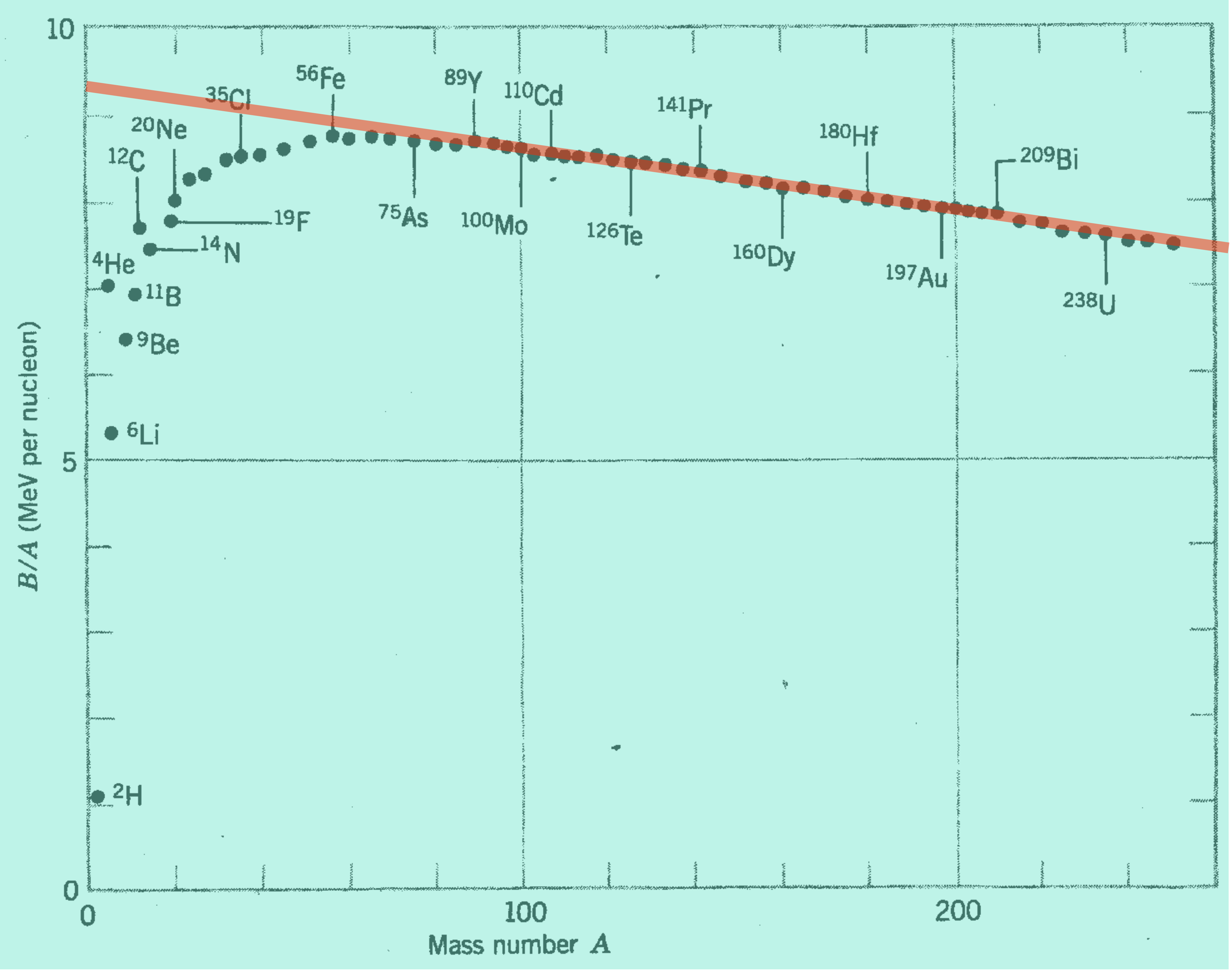


$$T_z = \frac{1}{2}(N-Z)$$

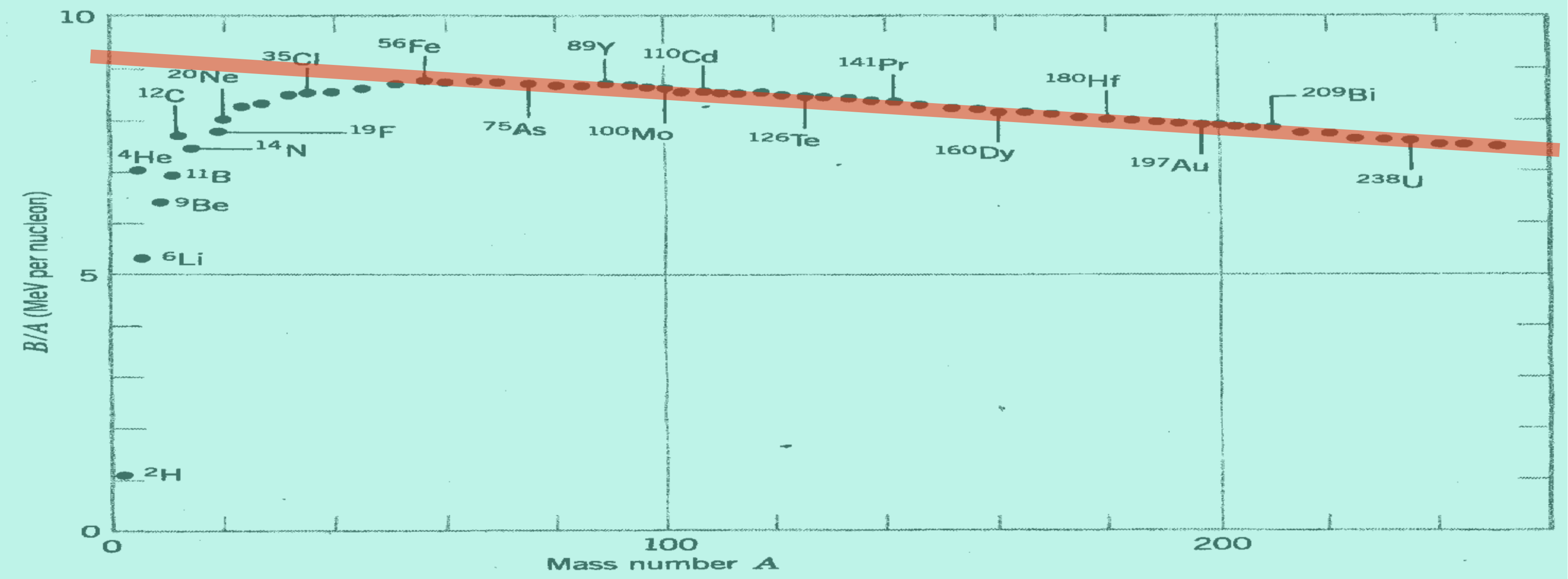
$$E_s \propto T_z/A \propto (A/2 - Z)/A$$

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim}(A-2Z)^2/A + (\delta)$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.



10
5
0
0
100
200
Mass number A
B/A (MeV per nucleon)
56Fe
89Y
110Cd
35Cl
141Pr
20Ne
180Hf
12C
209Bi
19F
75As
100Mo
126Te
14N
160Dy
4He
197Au
11B
238U
9Be
6Li
2H



B/A (MeV per nucleon)
Mass number A
10
5
0
0
100
200
56Fe
89Y
110Cd
35Cl
20Ne
12C
141Pr
180Hf
209Bi
19F
14N
4He
11B
9Be
6Li
2H
75As
100Mo
126Te
160Dy
197Au
238U

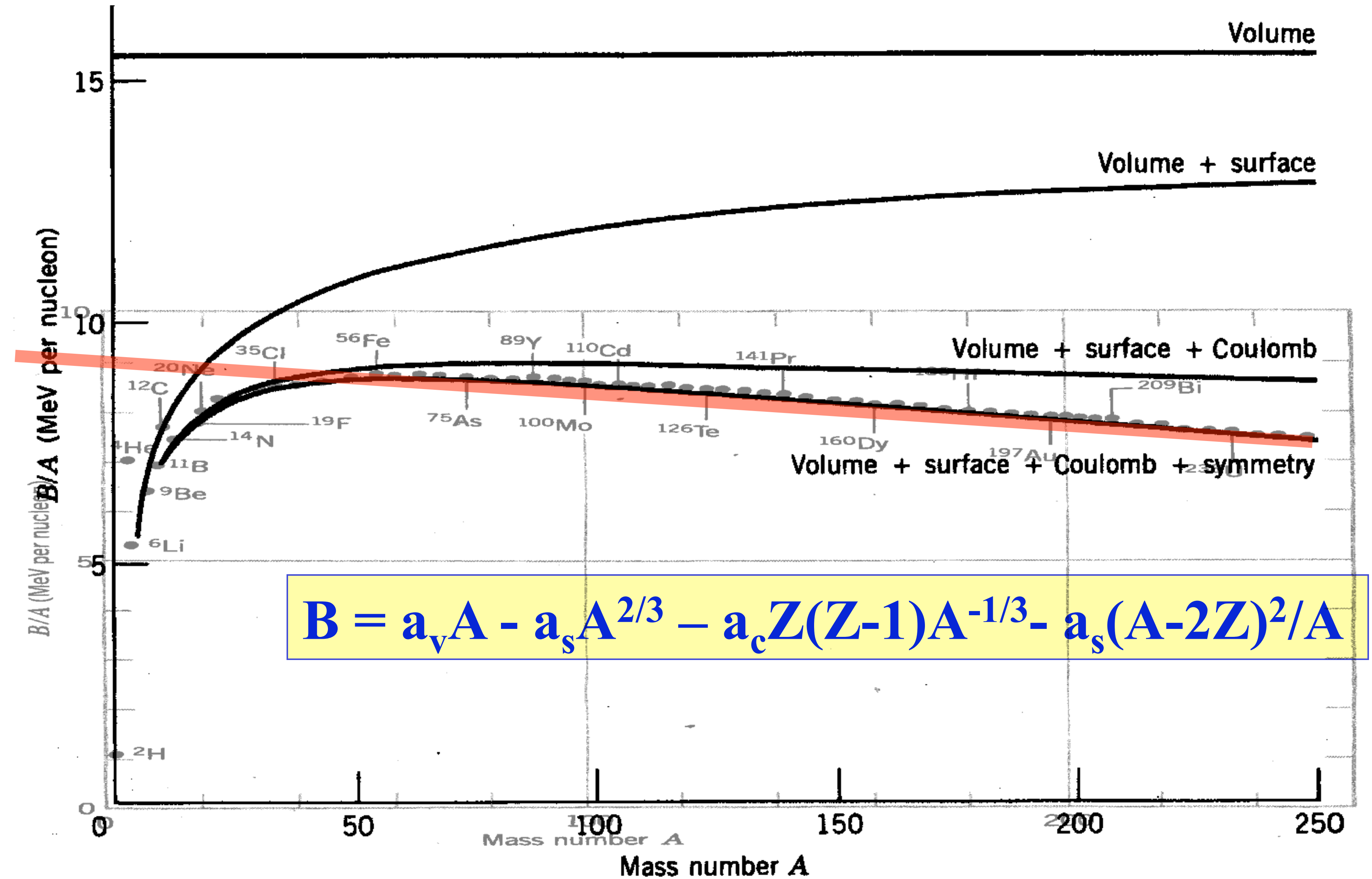


Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

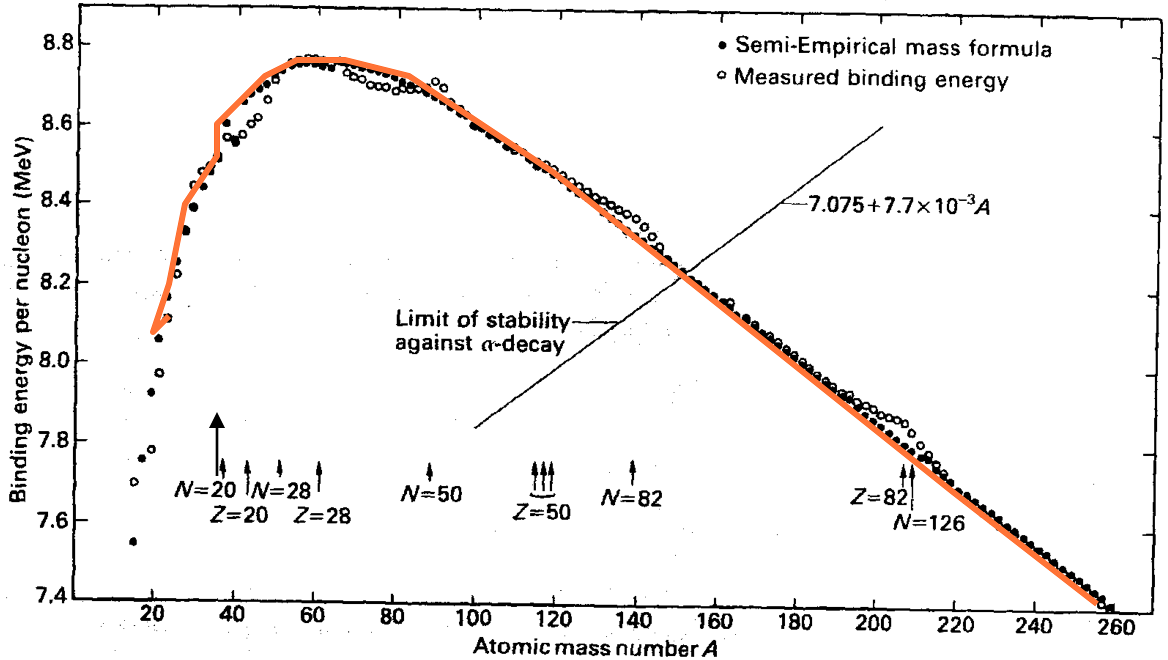
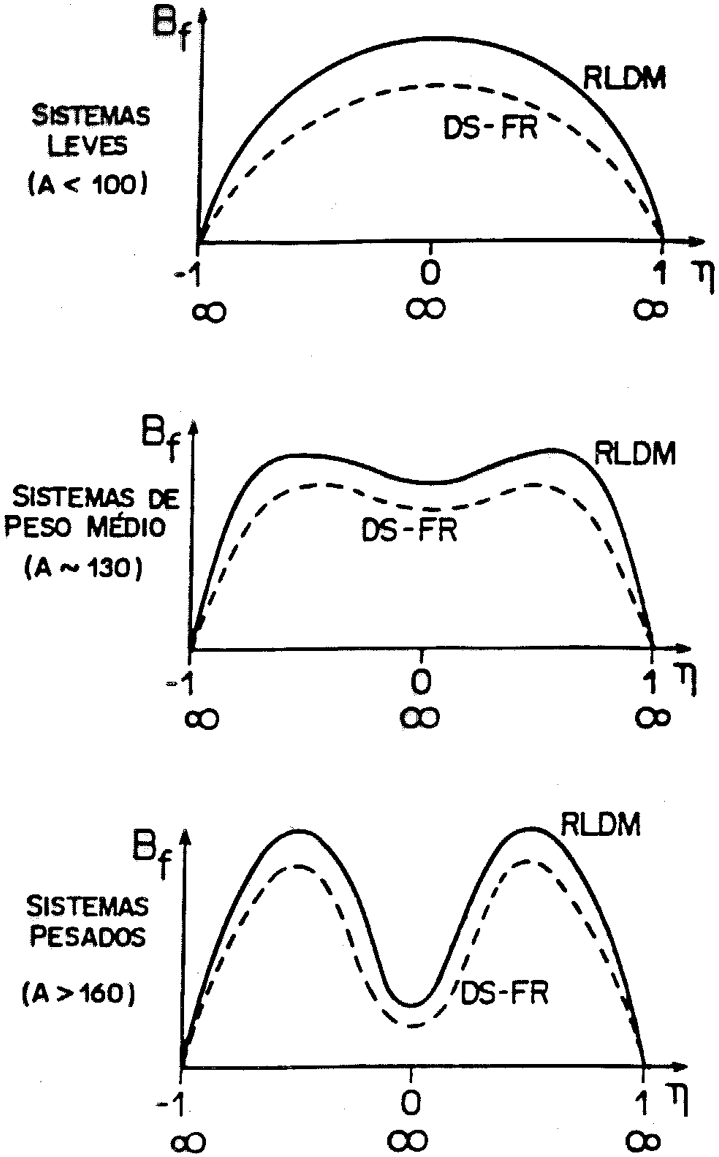


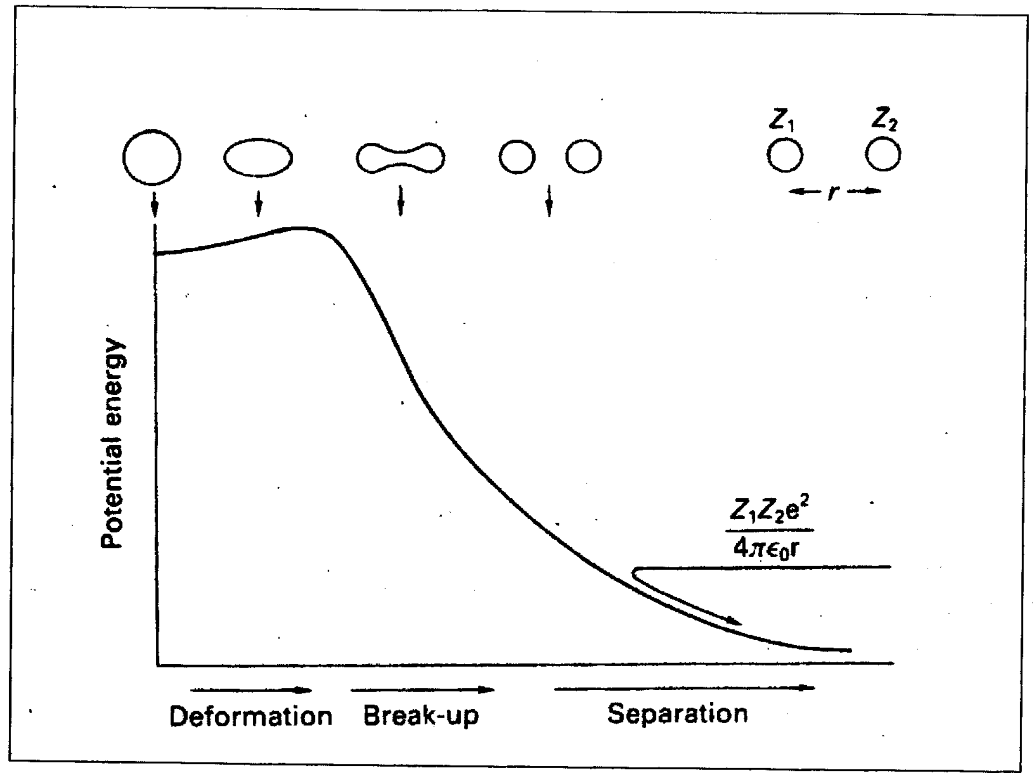
Fig. 4.6 The binding energy as a function of A for the odd-A nuclei from $A = 15$–259. The solid points are the prediction of the semi-empirical mass formula as given in Table 4.1. The open points are the measured values. The points for the formula do not lie on a smooth curve because Z for these nuclei is not a smooth function of A (see Fig. 4.1). Note that the zero of the ordinate is suppressed and its scale is much enlarged. Thus, in spite of the deviations from the formula, it is clear that the formula predicts the binding energy per nucleon for $A > 20$ with a precision which is, for the majority of cases, better than 0.1 MeV. The straight line crossing the curve at $A = 151$ gives the limit of stability of nuclei to α-decay (see Section 5.4).

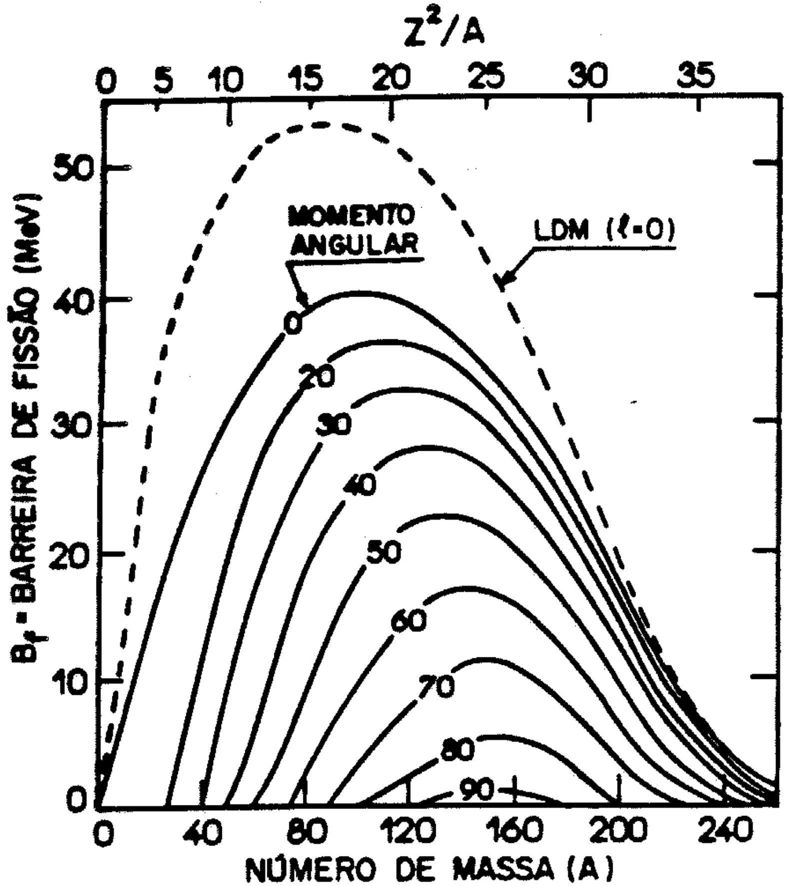
$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim}(A-2Z)^2/A + \delta(Z,A)$$

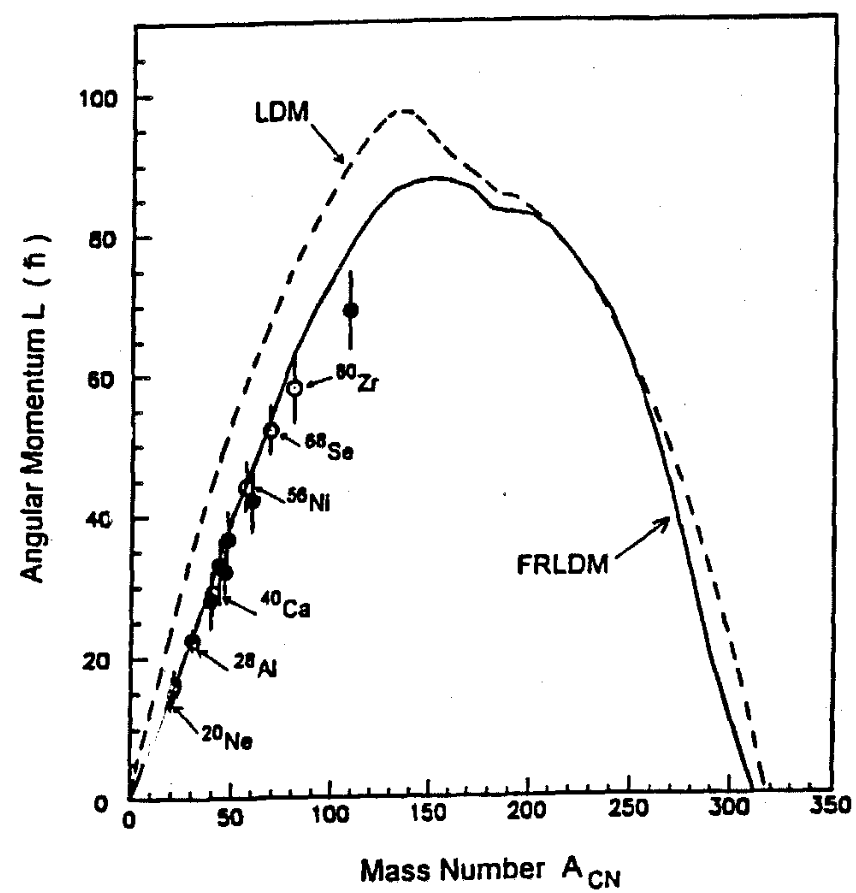
$$\delta(Z,A) = \begin{cases} -34A^{-3/4}\ \text{MeV} & \text{impar-impar} \\ 0 & \text{impar-par ou par-impar} \\ +34A^{-3/4}\ \text{MeV} & \text{par-par} \end{cases}$$



$$\eta = \frac{A_1 - A_2}{A_1 - A_2}$$




Z²/A
0 5 10 15 20 25 30 35
MOMENTO ANGULAR
LDM (ℓ=0)
0
20
30
40
50
60
70
80
90
50
40
30
20
10
0
Bℓ = BARREIRA DE FISSÃO (MeV)
0 40 80 120 160 200 240
NÚMERO DE MASSA (A)


LDM
FRLDM
100
80
60
40
20
0
Angular Momentum L (ħ)
80Zr
68Se
56Ni
40Ca
28Al
20Ne
0 50 100 150 200 250 300 350
Mass Number A CN

momentos eletromagneticos nucleares

Para um eletron orbitando em uma orbita de raio r e area A

$$|\mu| = iA$$

$$|\mu| = \frac{e}{(2\pi r)/v}\pi r^2 = \frac{evr}{2}$$

quanticamente

Como $|\ell| = r.p = r.m.v$
$r = |\ell|/mv$

$$|\mu| = \frac{e}{2m}|\ell|$$ ou $$|\mu| = \frac{e\hbar}{2m}|\ell|$$

$$\frac{e\hbar}{2m} \equiv \textbf{magneton}$$

$\mu_B = 5.7884 \times 10^{-5}$ eV/T

$\mu_N = 3.1525 \times 10^{-8}$ eV/T

reescrevendo $\mu = g_\ell \, \ell \, \mu_N$

$g_\ell = 1$ *para protons*
$g_\ell = 0$ *para neutrons*

o calculo de g_ℓ *considera exclusivamente o momento angu*

No caso do momento angular intrínseco (spin) s

$\mu = g_s s\, \mu_N$ *onde s= ½ para protons, neutrons e elétrons*

Previsões de Dirac para $g_s = 2$

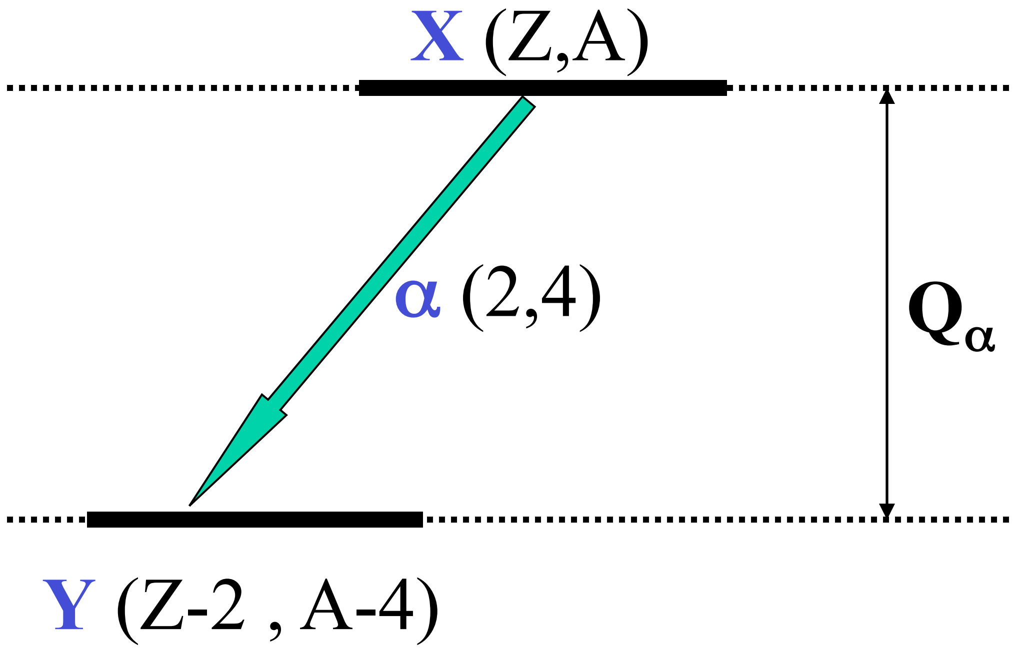
Valores experimentais: $\mathbf{g_s}$ **(elétron)** = 2.0023

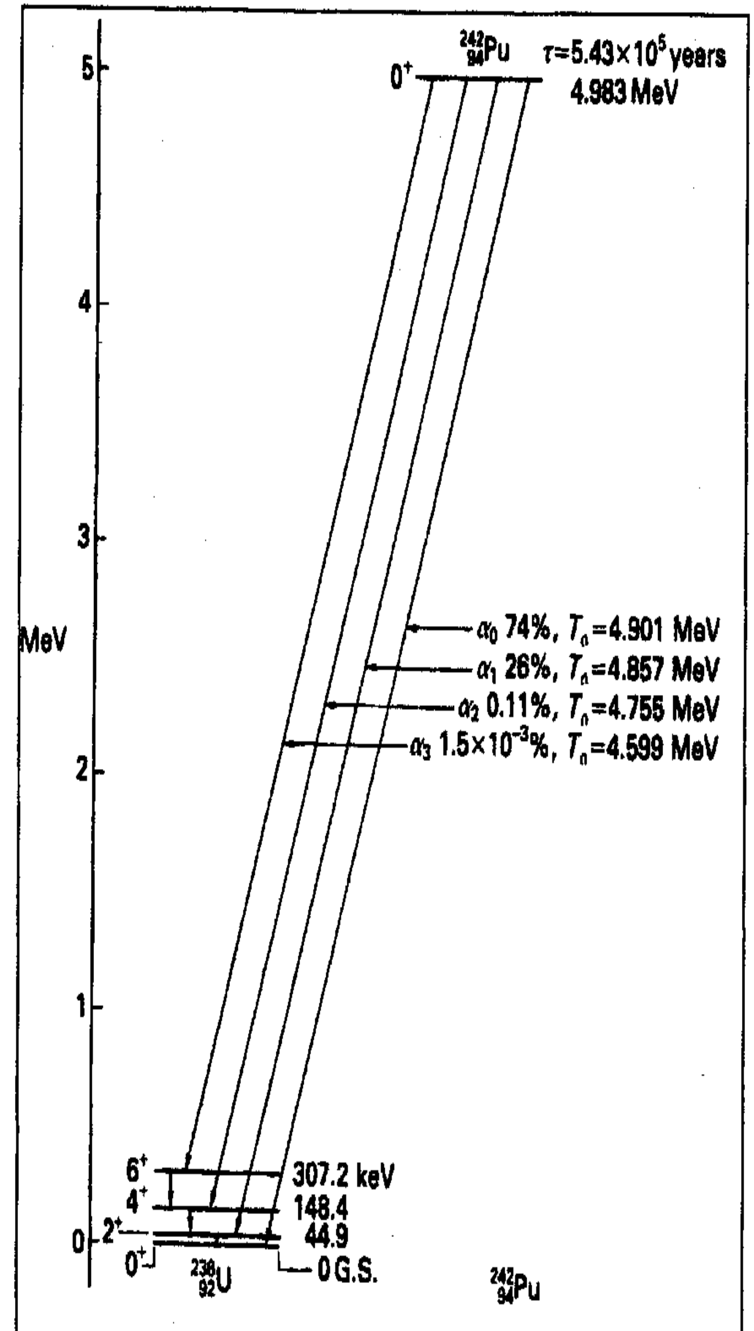
$\mathbf{g_s}$ **(proton)** = 5.5856912

$\mathbf{g_s}$ **(neutron)** = -3.8260837

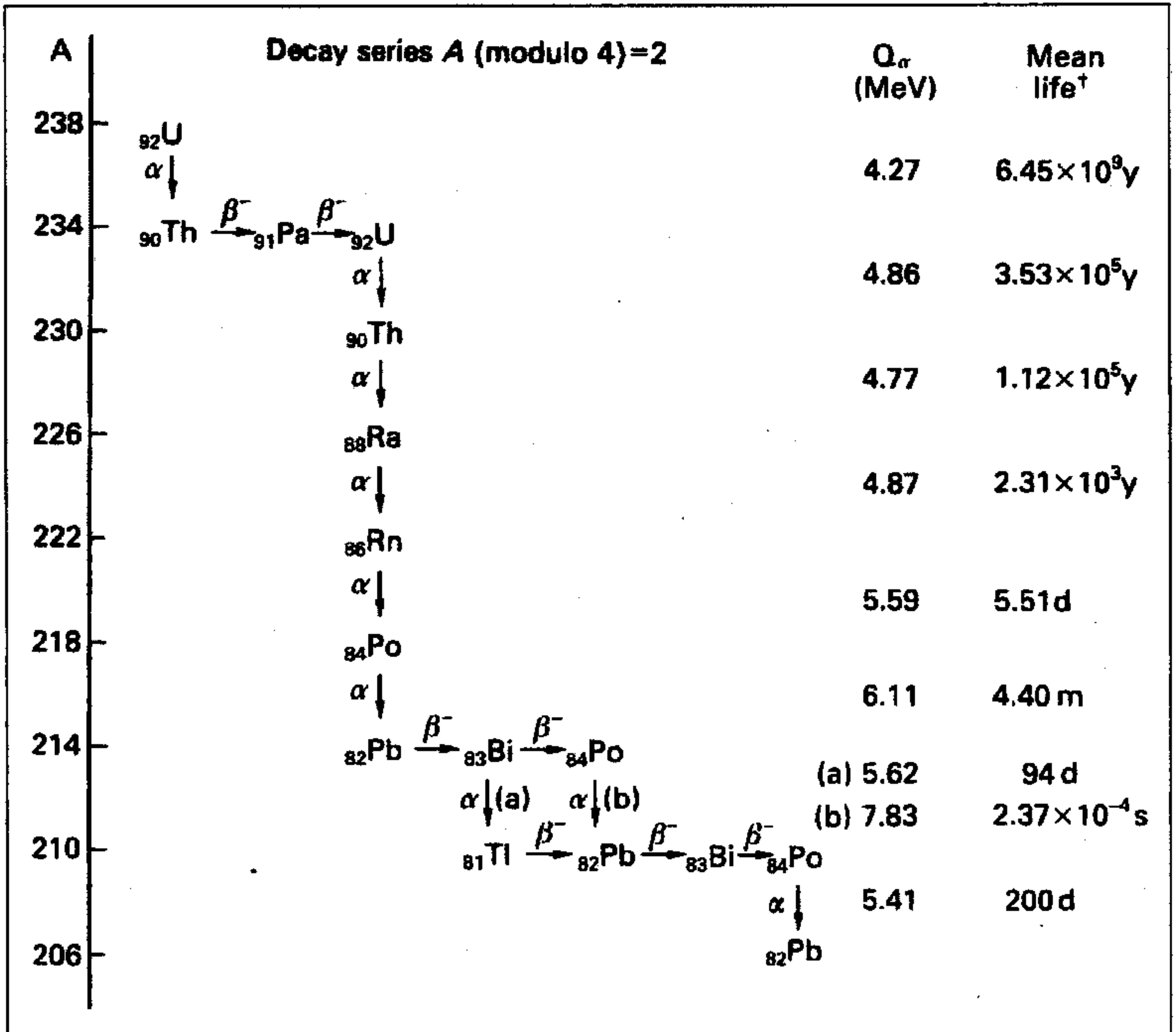
Note que $\mathbf{g_s}$ **(neutron) ≠ 0**

decaimento radiativo

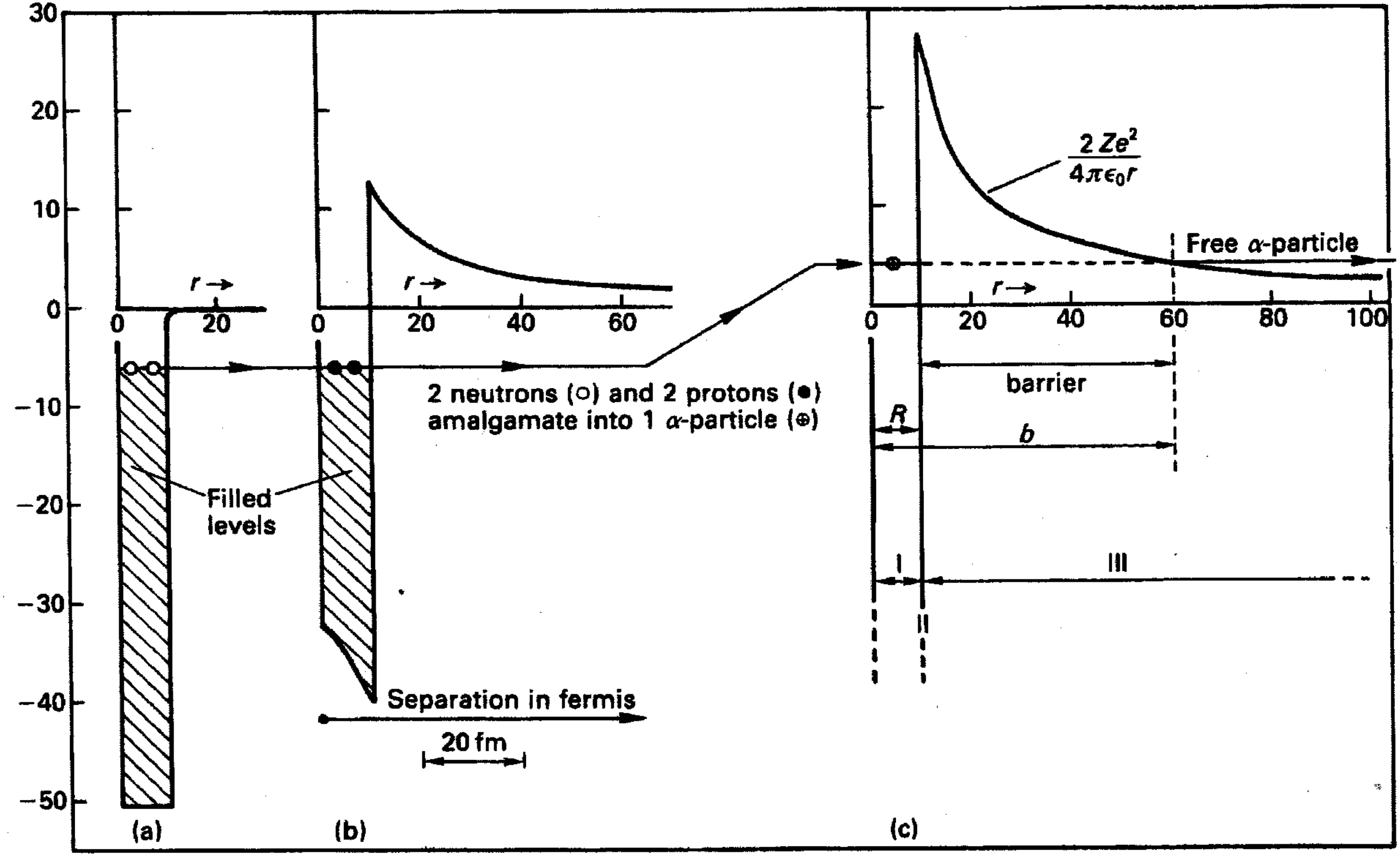




DECAIMENTO ALPHA (α)

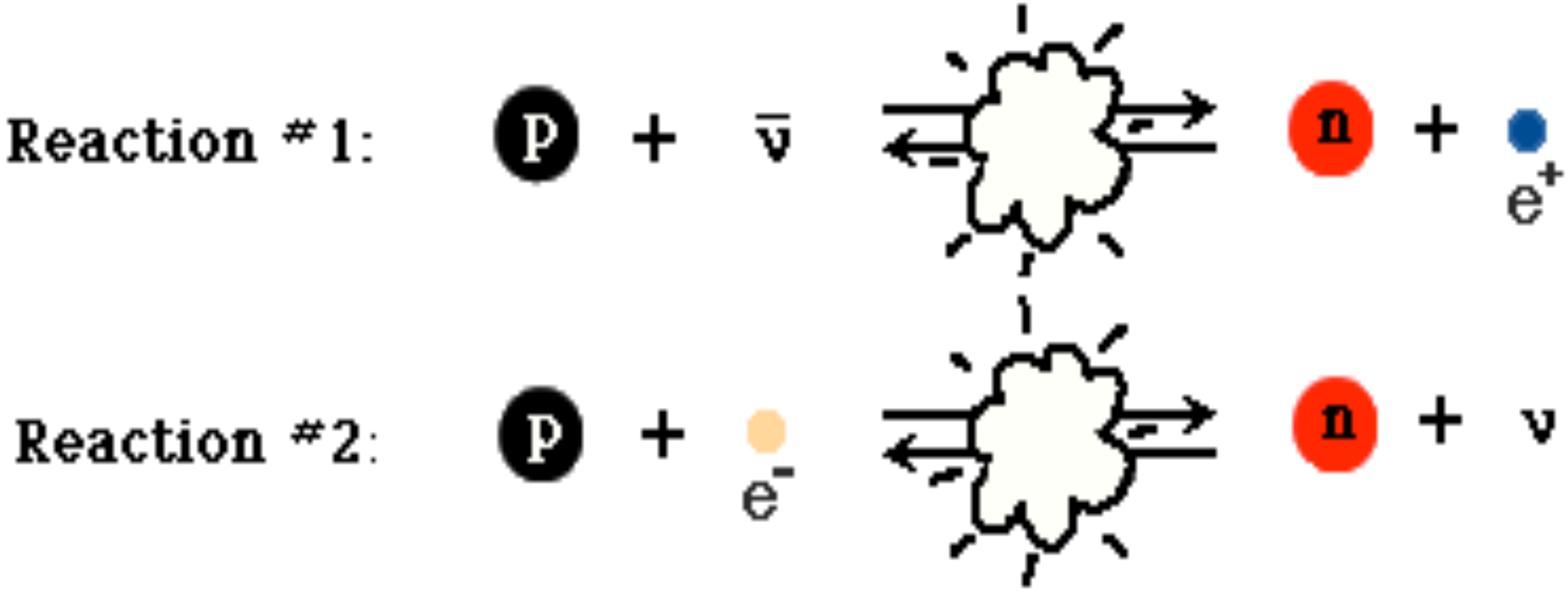


A
Decay series A (modulo 4)=2
Qα (MeV)
Mean life†
238
234
230
226
222
218
214
210
206
92U
α
90Th
β−
91Pa
β−
92U
α
90Th
α
88Ra
α
86Rn
α
84Po
α
82Pb
β−
83Bi
β−
84Po
α (a)
α (b)
81Tl
β−
82Pb
β−
83Bi
β−
84Po
α
82Pb
4.27
6.45×10^9 y
4.86
3.53×10^5 y
4.77
1.12×10^5 y
4.87
2.31×10^3 y
5.59
5.51 d
6.11
4.40 m
(a) 5.62
94 d
(b) 7.83
2.37×10^-4 s
5.41
200 d



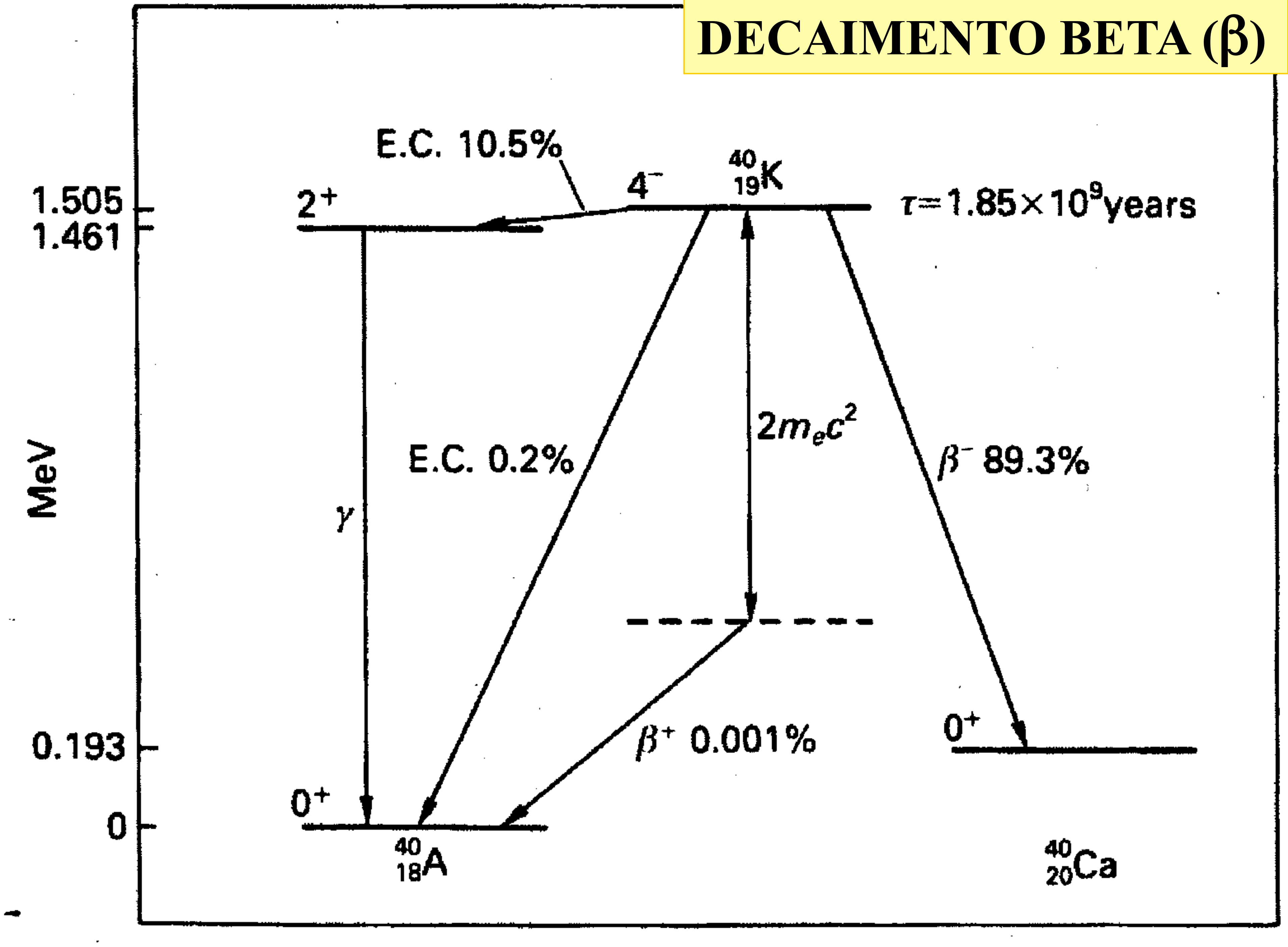
30
20
10
0
−10
−20
−30
−40
−50
r →
0
20
40
60
80
100
$\frac{2Ze^2}{4\pi\epsilon_0 r}$
Free α-particle
barrier
R
b
I
II
III
2 neutrons (○) and 2 protons (●)
amalgamate into 1 α-particle (⊕)
Filled
levels
Separation in fermis
20 fm
(a)
(b)
(c)

proton/neutron conversions

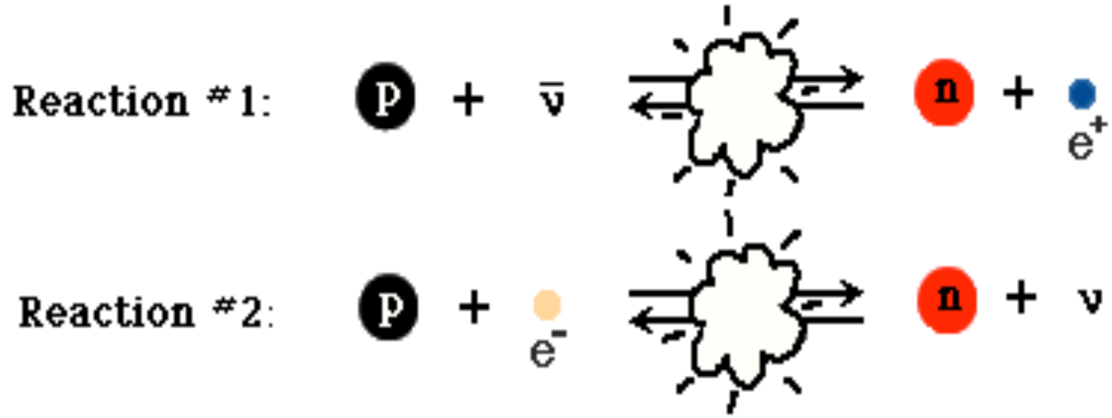


(The double arrows indicate these reactions go both ways.)

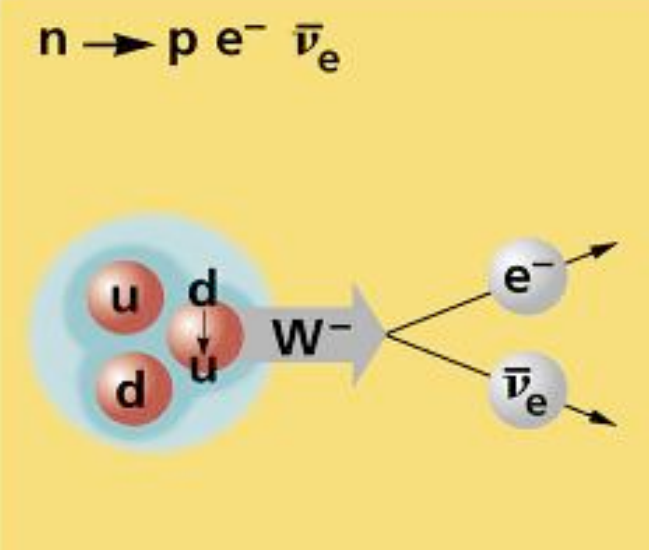
DECAIMENTO BETA (β)



proton/neutron conversions



(The double arrows indicate these reactions go both ways.)



A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.

BOSONS

Unified Electroweak spin = 1

Name	Mass GeV/c²	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

FERMIONS

matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c²	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c²	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2

Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	−1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	−1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	−1

Quarks spin = 1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	−1/3
c charm	1.3	2/3
s strange	0.1	−1/3
t top	175	2/3
b bottom	4.3	−1/3

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$

Baryons are fermionic hadrons.
There are about 120 types of baryons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	anti-proton	$\bar{u}\bar{u}\bar{d}$	−1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	−1	1.672	3/2

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.4	−1
W^+	80.4	+1
Z^0	91.187	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Mesons $q\bar{q}$

Mesons are bosonic hadrons.
There are about 140 types of mesons.

Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	−1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.770	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2 .980	0

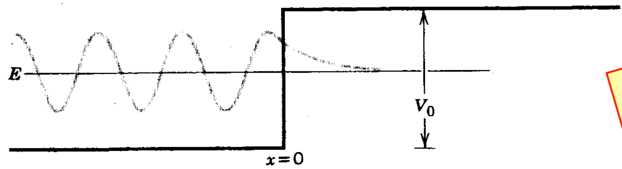


Figure 2.3 The wave function of a particle of energy E encountering a step of height V_0, for the case $E < V_0$. The wave function decreases exponentially in the classically forbidden region, where the classical kinetic energy would be negative. At $x = 0$, ψ and $d\psi/dx$ are continuous.

ref. Krane

the classically forbidden region. All (classical) *particles* are reflected at the boundary; the quantum mechanical *wave packet*, on the other hand, can penetrate a short distance into the forbidden region. The (classical) particle is never directly *observed* in that region; since $E < V_0$, the kinetic energy would be negative in region 2. The solution is illustrated in Figure 2.3

Barrier Potential, $E > V_0$

The potential is

$$\begin{aligned} V(x) &= 0 && x < 0 \\ &= V_0 && 0 \le x \le a \\ &= 0 && x > a \end{aligned} \tag{2.35}$$

In the three regions 1, 2, and 3, the solutions are

$$\begin{aligned} \psi_1 &= A e^{ik_1 x} + B e^{-ik_1 x} \\ \psi_2 &= C e^{ik_2 x} + D e^{-ik_2 x} \\ \psi_3 &= F e^{ik_3 x} + G e^{-ik_3 x} \end{aligned} \tag{2.36}$$

where $k_1 = k_3 = \sqrt{2mE/\hbar^2}$ and $k_2 = \sqrt{2m(E - V_0)/\hbar^2}$.

Using the continuity conditions at $x = 0$ and at $x = a$, and assuming again that particles are incident from $x = -\infty$ (so that G can be set to zero), after considerable algebraic manipulation we can find the transmission coefficient $T = |F|^2/|A|^2$:

$$T = \frac{1}{1 + \dfrac{1}{4}\dfrac{V_0^2}{E(E - V_0)}\sin^2 k_2 a} \tag{2.37}$$

The solution is illustrated in Figure 2.4.

Barrier Potential, $E < V_0$

For this case, the ψ_1 and ψ_3 solutions are as above, but ψ_2 becomes

$$\psi_2 = C e^{k_2 x} + D e^{-k_2 x} \tag{2.38}$$

where now $k_2 = \sqrt{2m(V_0 - E)/\hbar^2}$. Because region 2 extends only from $x = 0$

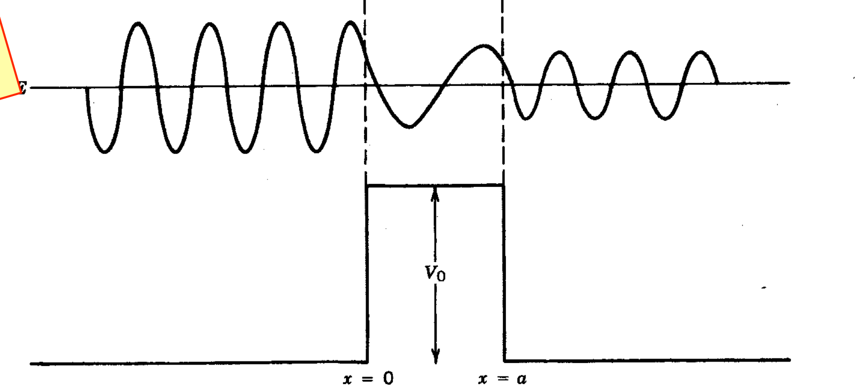


Figure 2.4 The wave function of a particle of energy $E > V_0$ encountering a barrier potential. The particle is incident from the left. The wave undergoes reflections at both boundaries, and the transmitted wave emerges with smaller amplitude.

to $x = a$, the question of an exponential solution going to infinity does not arise, so we cannot set C or D to zero.

Again, applying the boundary conditions at $x = 0$ and $x = a$ permits the solution for the transmission coefficient:

$$T = \frac{1}{1 + \dfrac{1}{4}\dfrac{V_0^2}{E(V_0 - E)}\sinh^2 k_2 a} \tag{2.39}$$

Classically, we would expect $T = 0$—the particle is not permitted to enter the forbidden region where it would have negative kinetic energy. The quantum wave can penetrate the barrier and give a nonzero probability to find the particle beyond the barrier. The solution is illustrated in Figure 2.5.

This phenomenon of *barrier penetration* or quantum mechanical *tunneling* has important applications in nuclear physics, especially in the theory of α decay, which we discuss in Chapter 8.

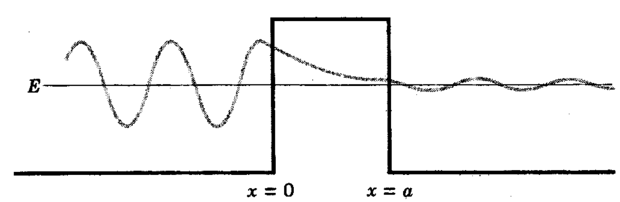
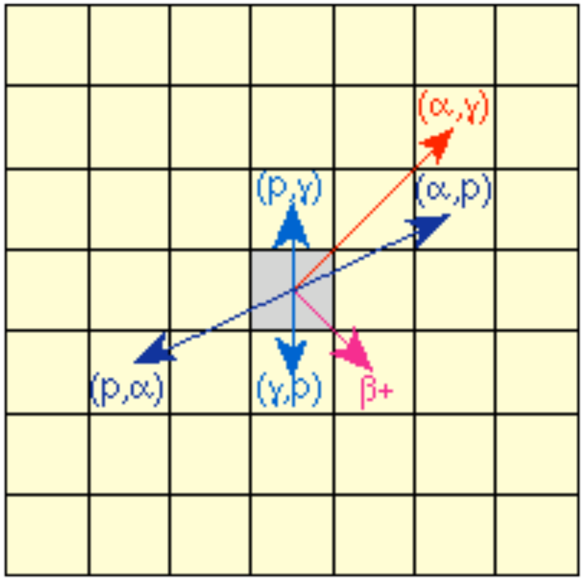
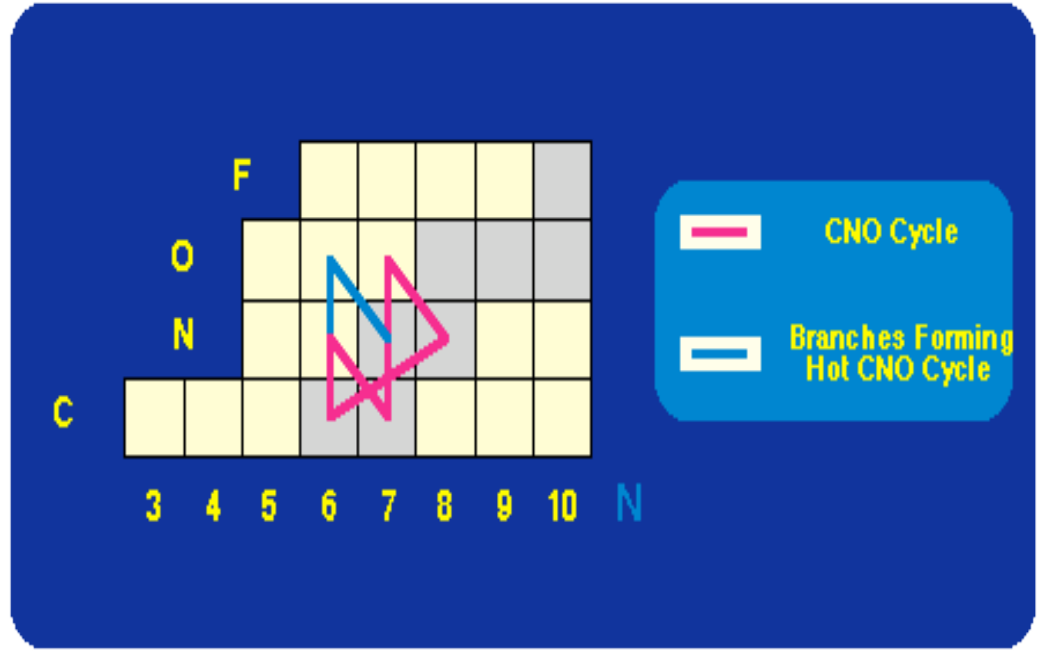
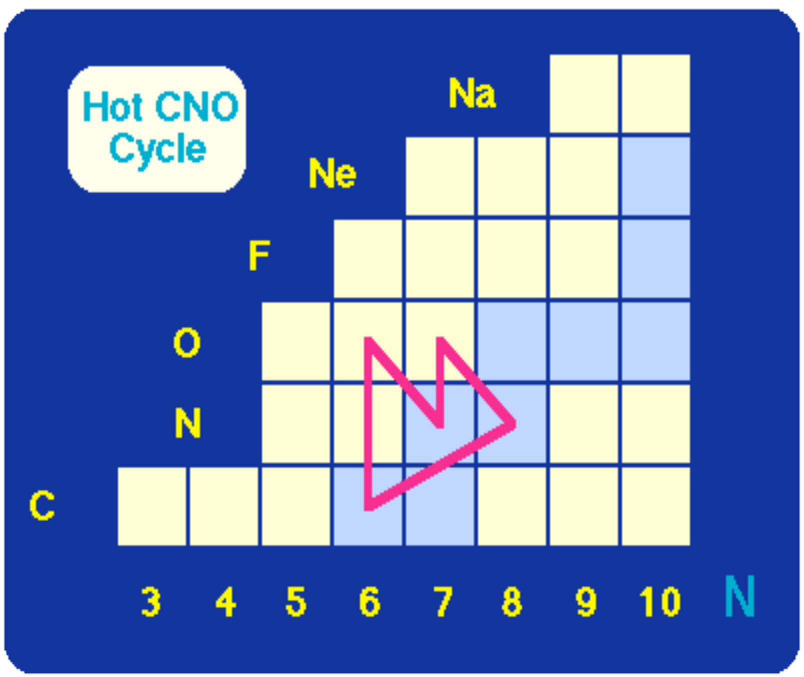


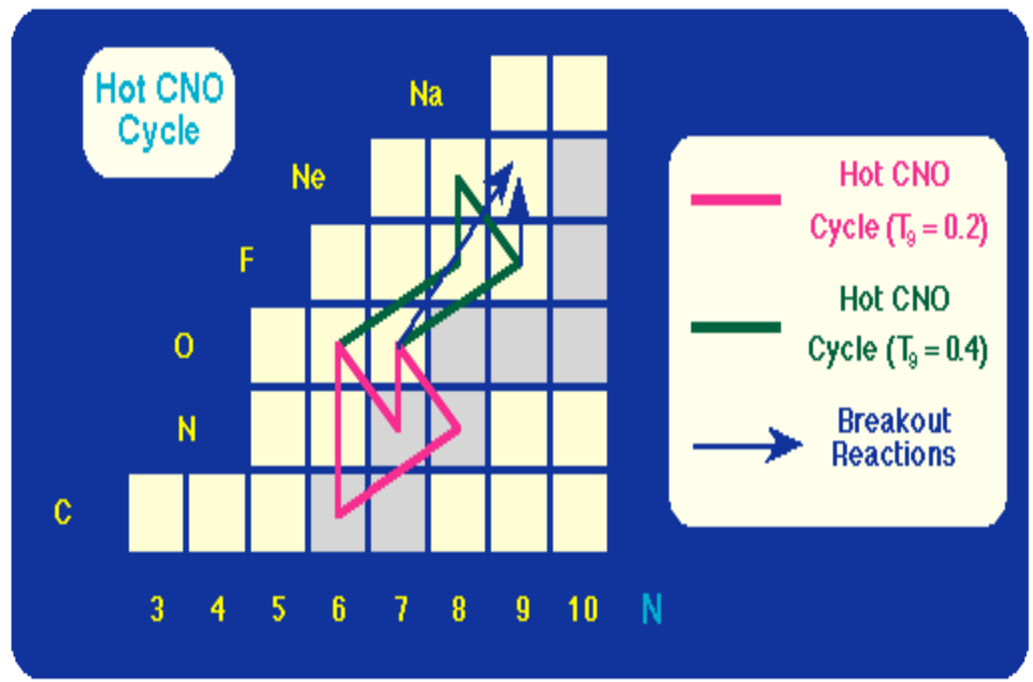
Figure 2.5 The wave function of a particle of energy $E < V_0$ encountering a barrier potential (the particle would be incident from the left in the figure). The wavelength is the same on both sides of the barrier, but the amplitude beyond the barrier is much less than the original amplitude. The particle can never be observed, inside the barrier (where it would have negative kinetic energy) but it can be observed *beyond* the barrier.

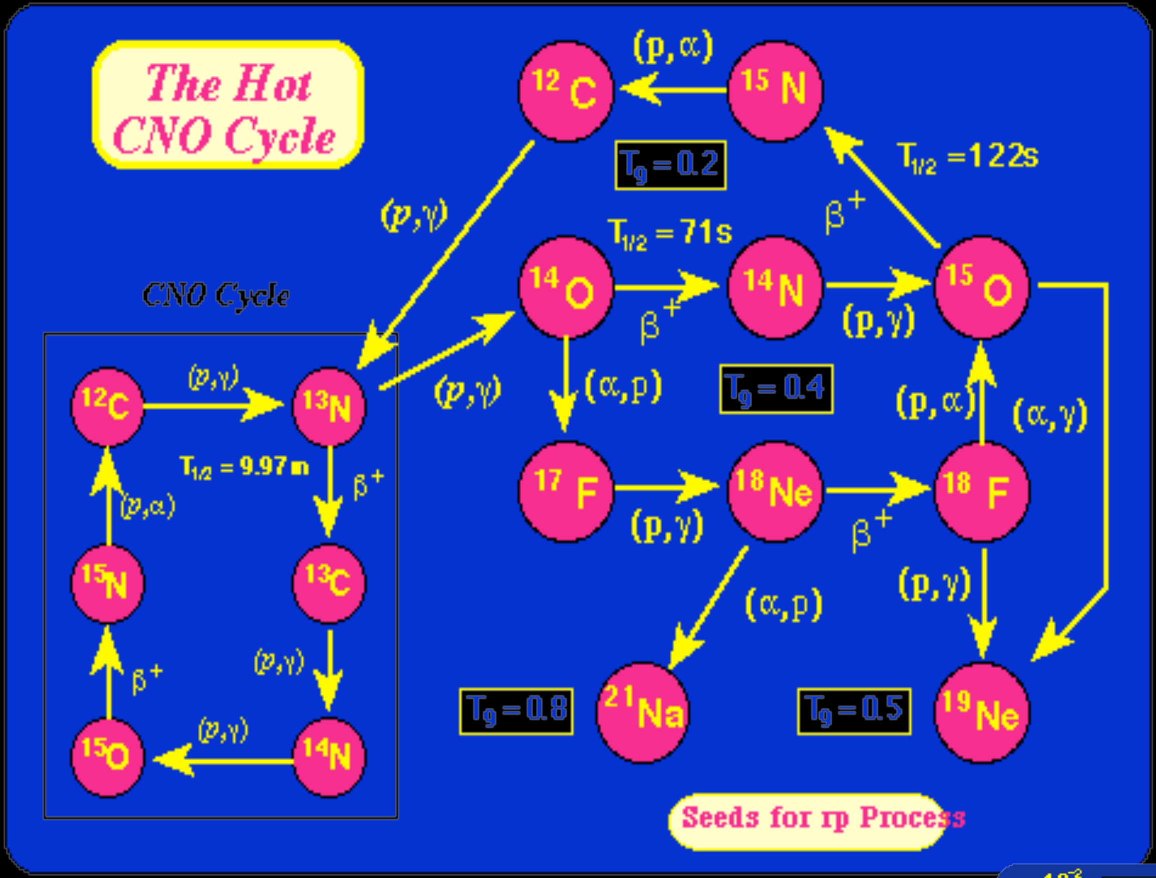


Reaction Vectors

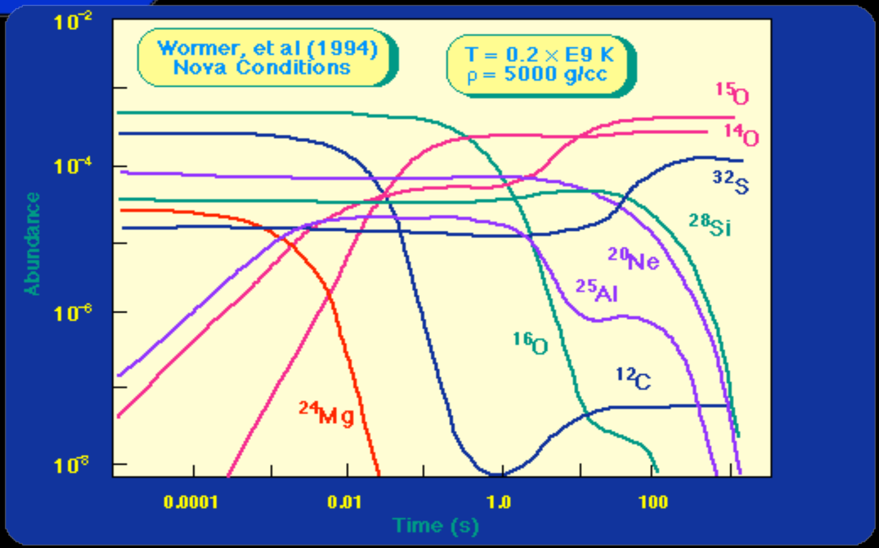




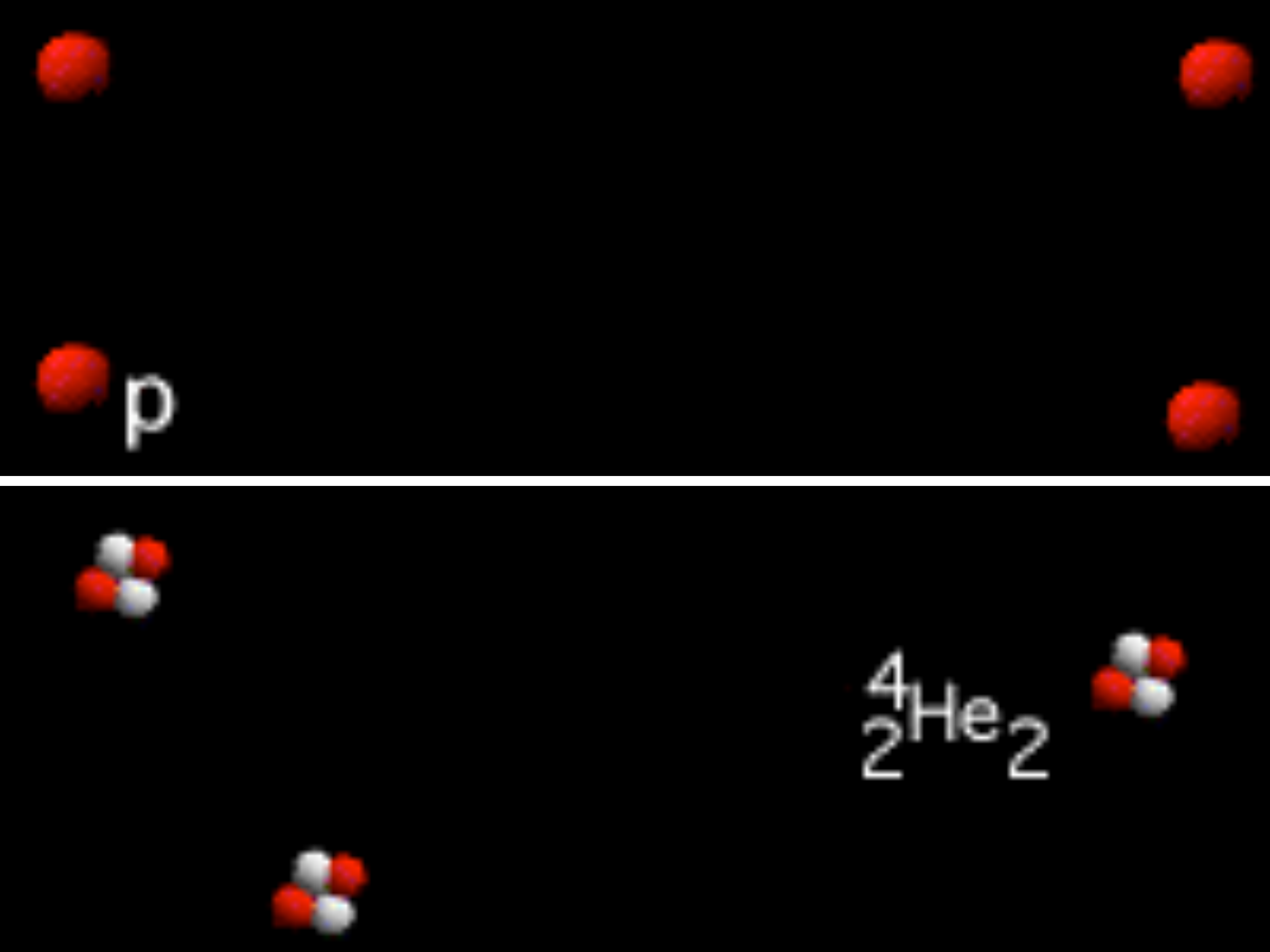


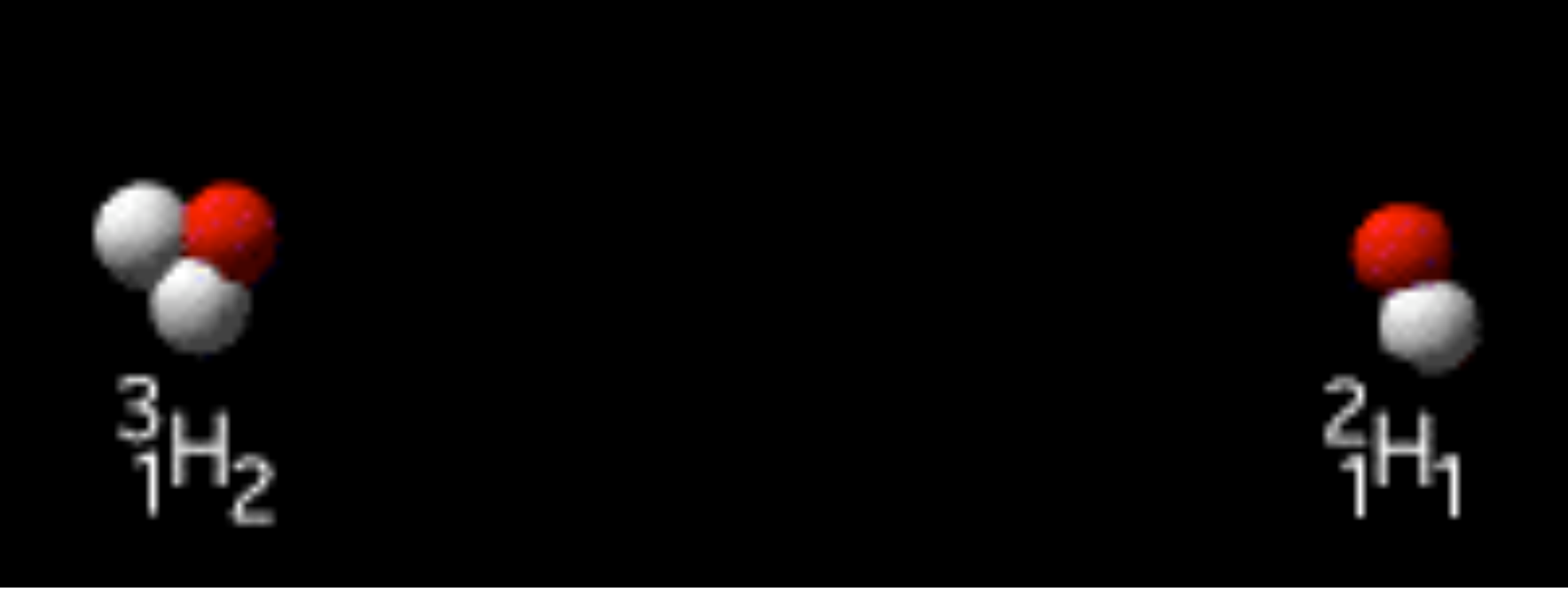


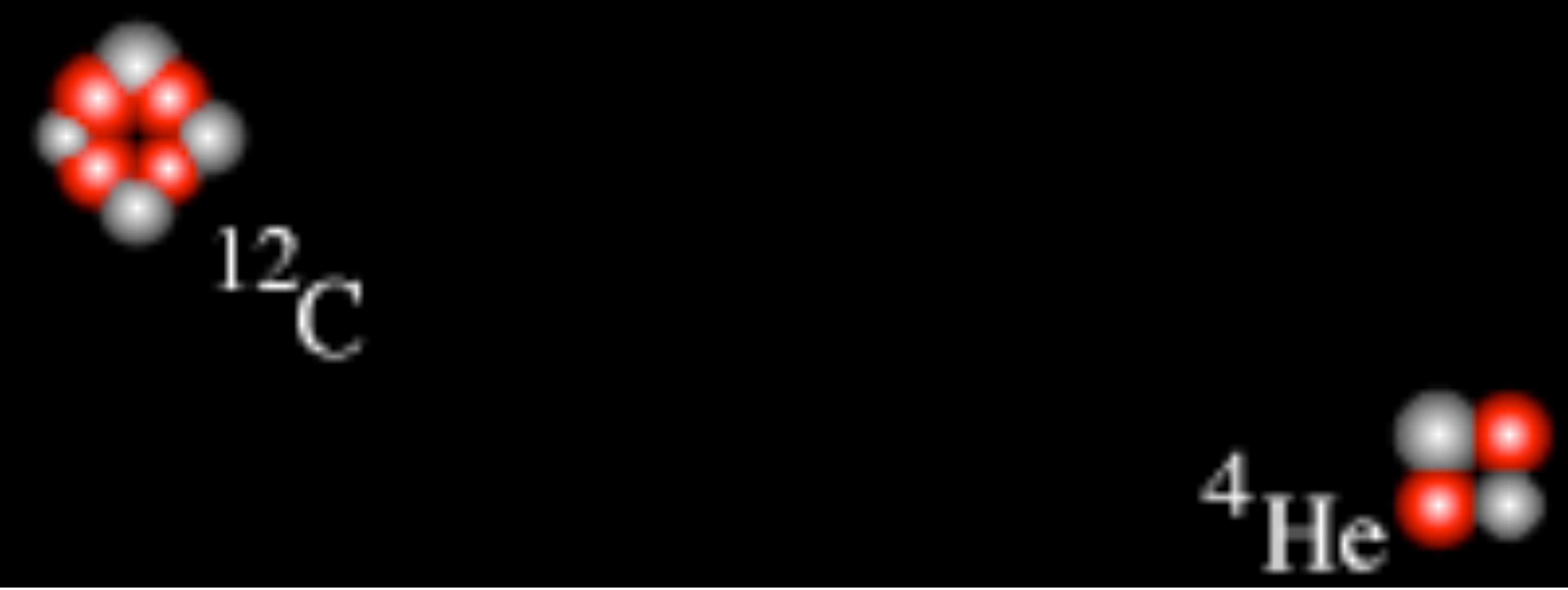
The Hot CNO Cycle
CNO Cycle
^{12}C
^{13}N
^{13}C
^{14}N
^{15}O
^{15}N
(p,γ)
$T_{1/2}$ = 9.97 m
β+
(p,α)
^{12}C
^{15}N
(p,α)
$T_9 = 0.2$
$T_{1/2}$ = 122s
^{14}O
^{14}N
^{15}O
$T_{1/2}$ = 71s
^{17}F
^{18}Ne
^{18}F
(α,p)
$T_9 = 0.4$
(α,γ)
^{21}Na
^{19}Ne
$T_9 = 0.8$
$T_9 = 0.5$
Seeds for rp Process



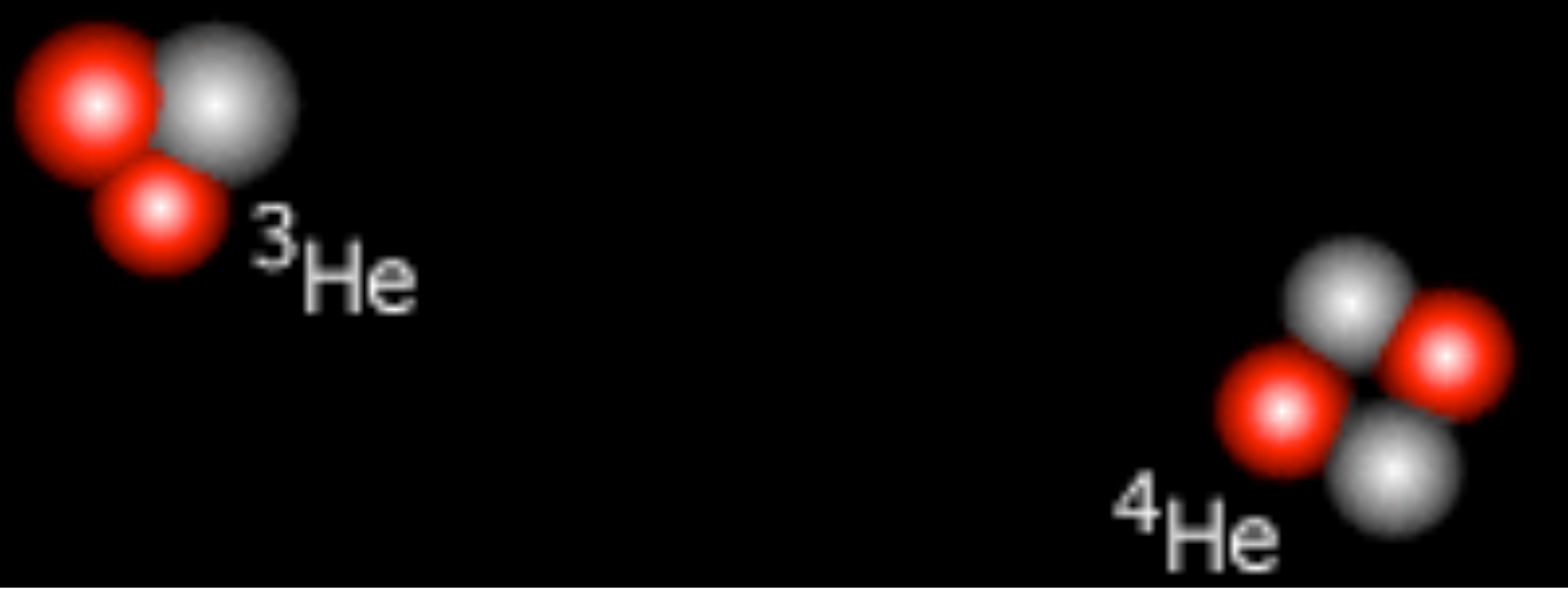
Wormer, et al (1994)
Nova Conditions
T = 0.2 × E9 K
ρ = 5000 g/cc
^{15}O
^{14}O
^{32}S
^{28}Si
^{20}Ne
^{25}Al
^{16}O
^{12}C
^{24}Mg
Abundance
10^{-2}
10^{-4}
10^{-6}
10^{-8}
0.0001
0.01
1.0
100
Time (s)


p
$^{4}_{2}He_{2}$

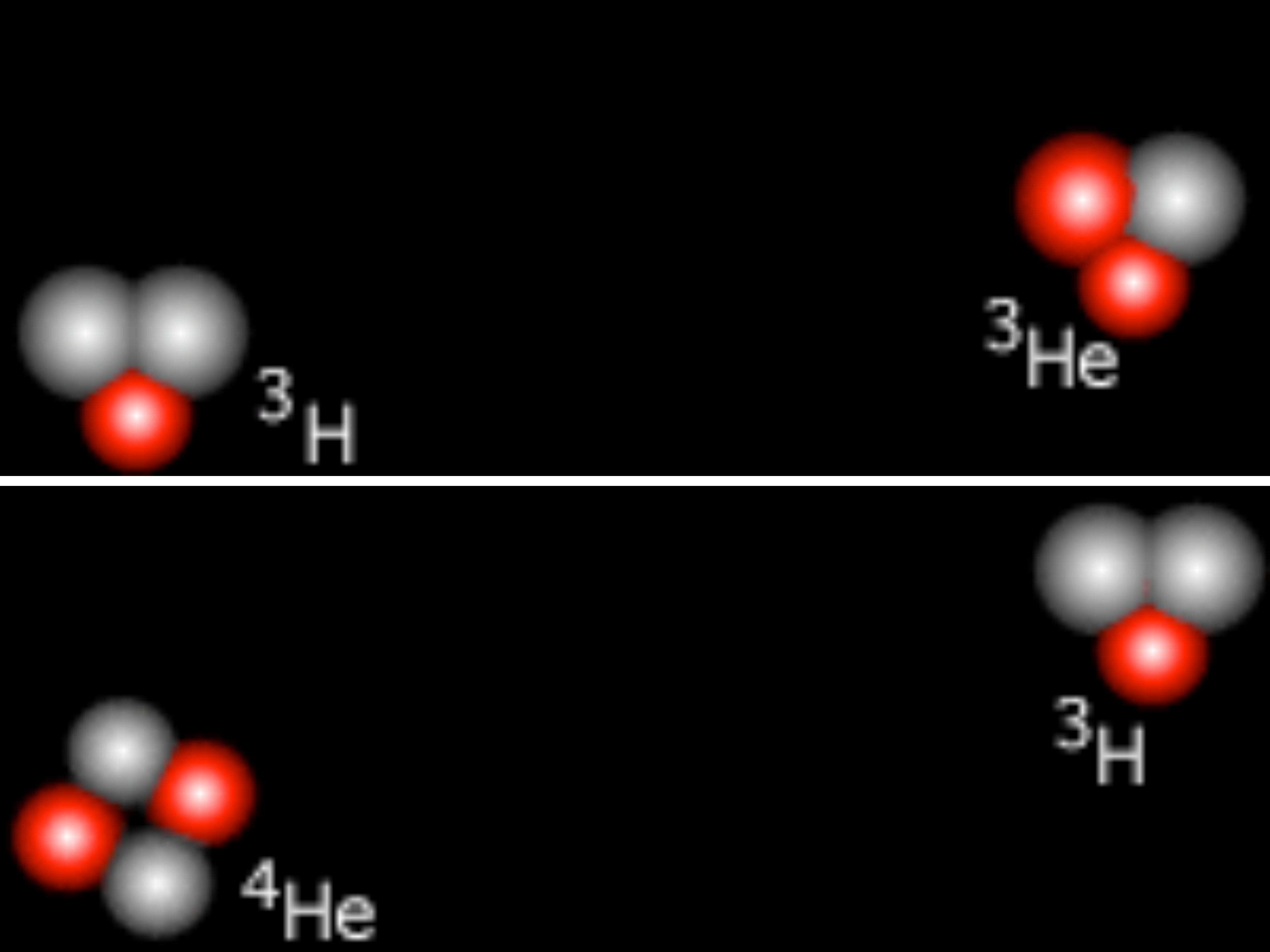

$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$


^{12}C
^{4}He

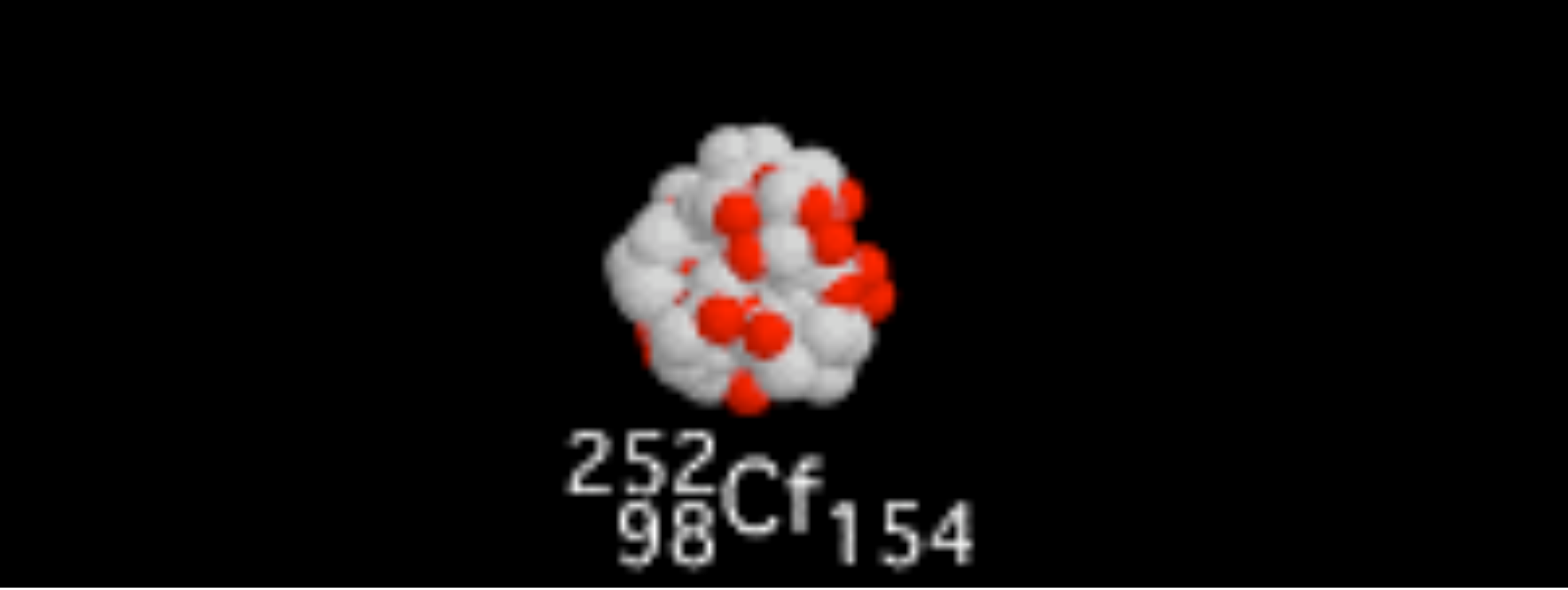

p
^{12}C


^3He
^4He

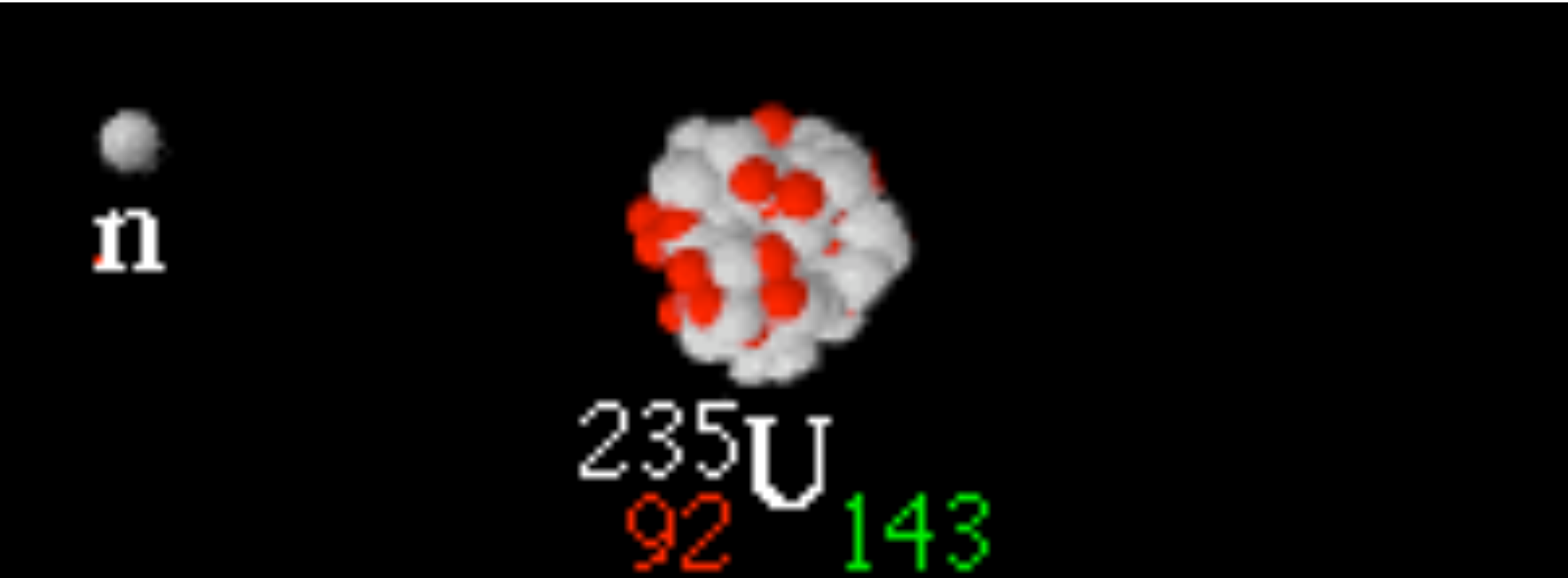

p
^{12}C



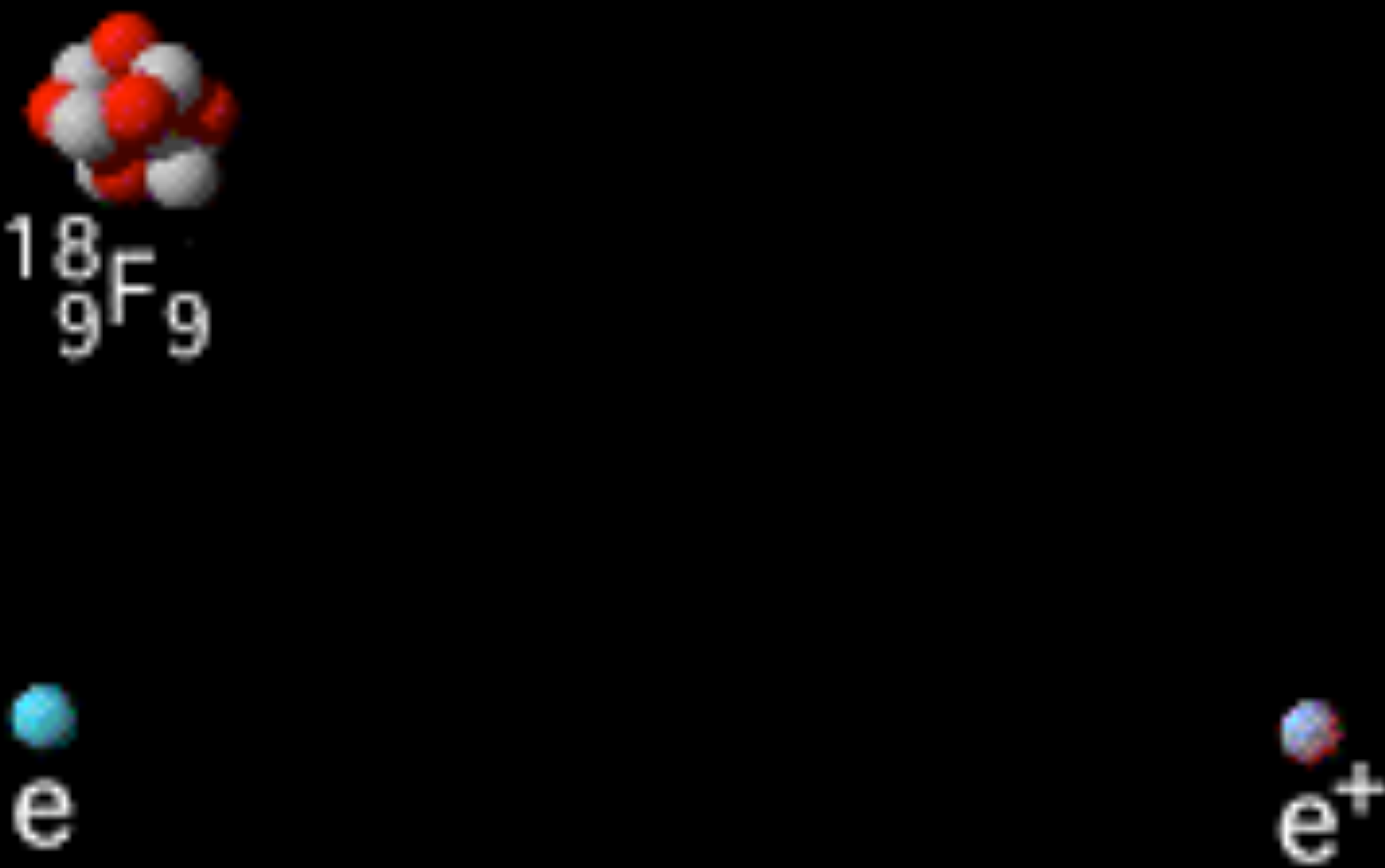
^{3}H
^{3}He
^{4}He
^{3}H



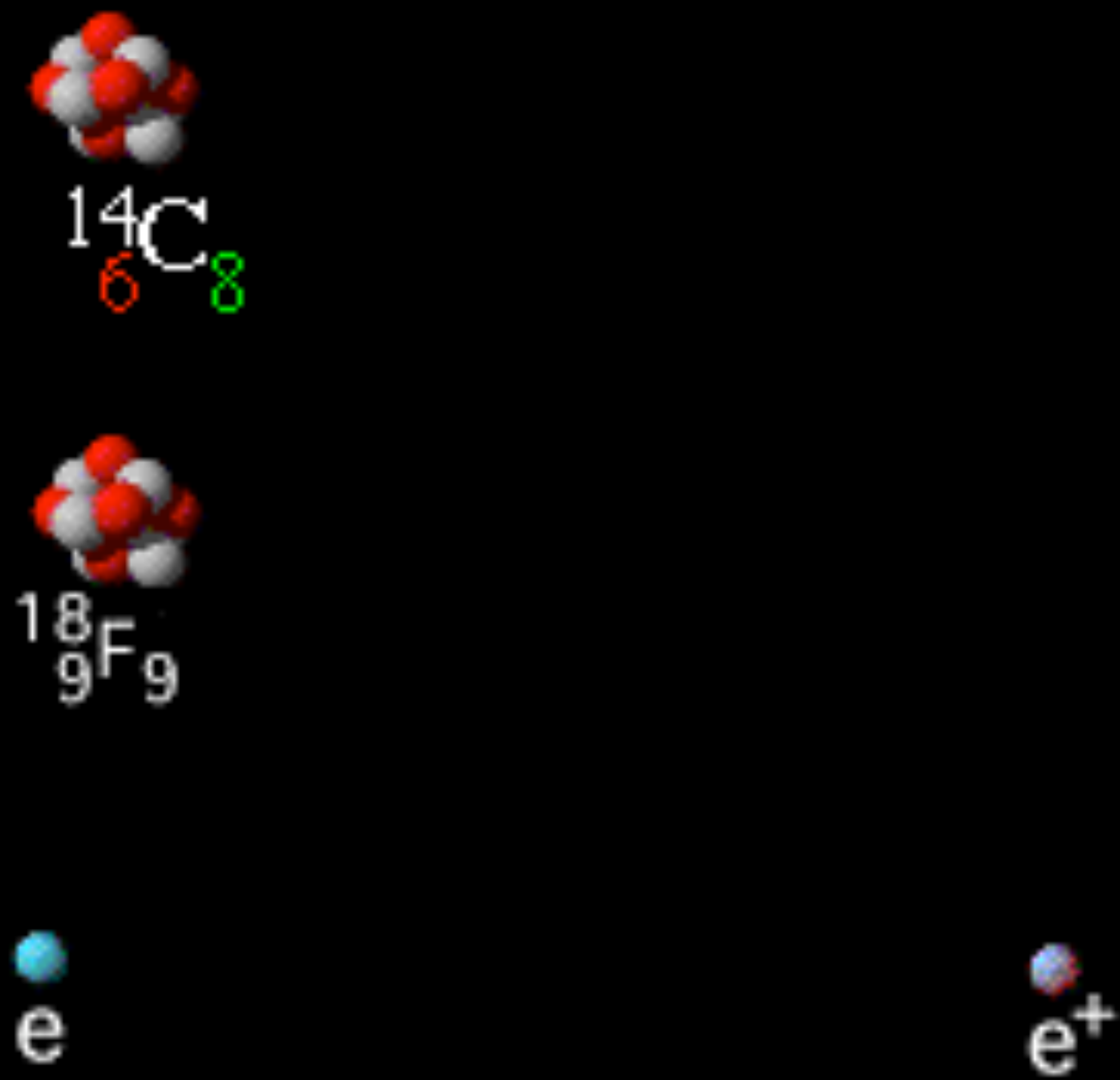
252 98 Cf 154



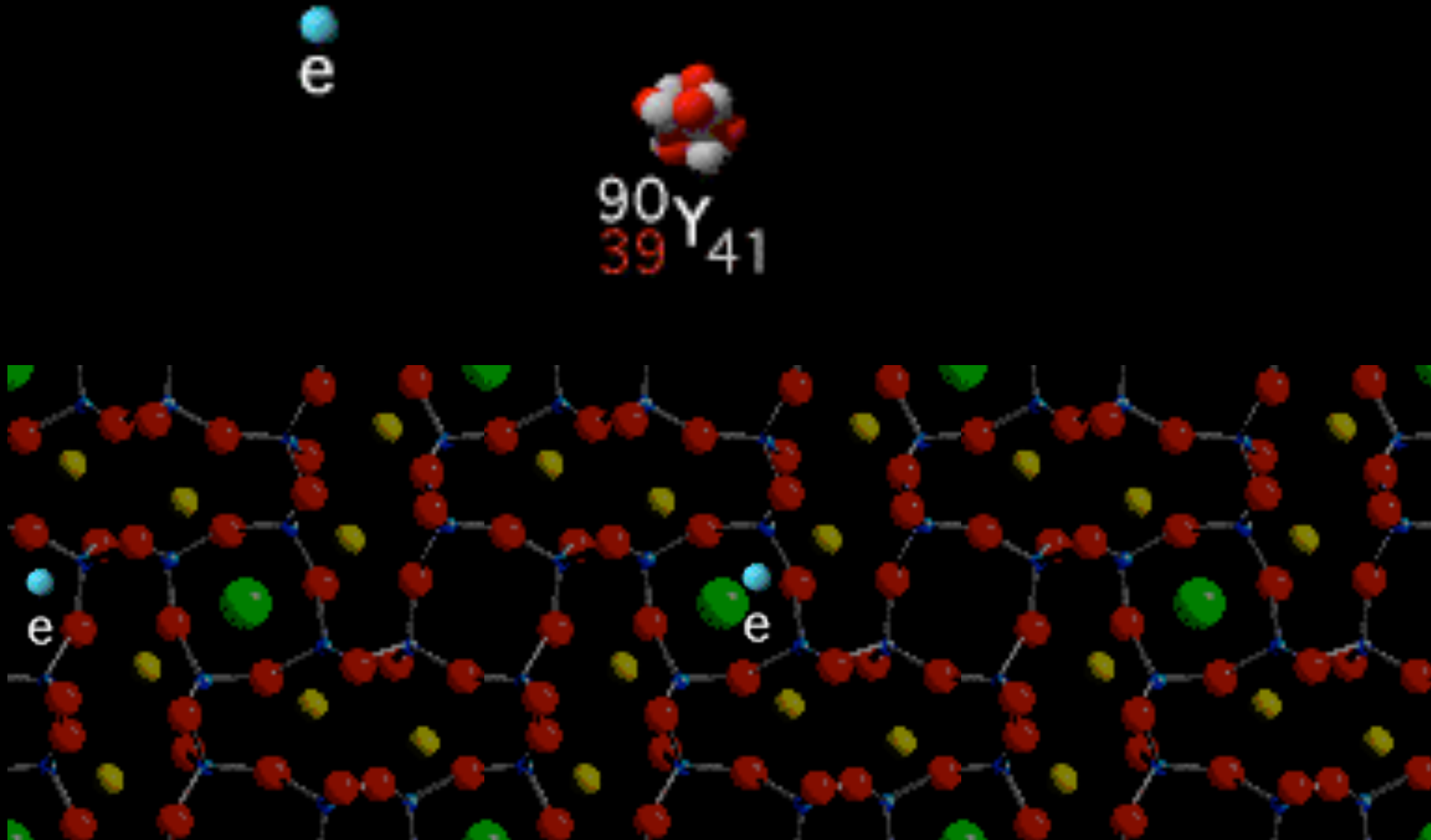
n
235 U 92 143

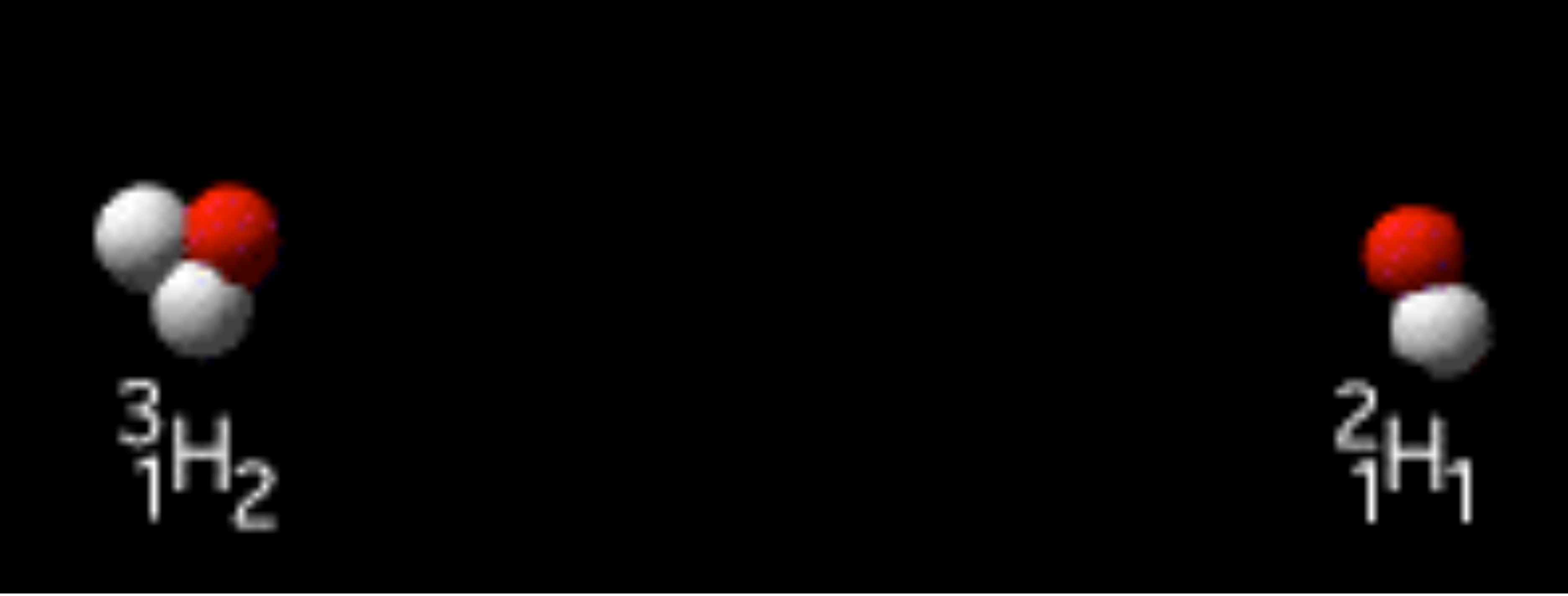


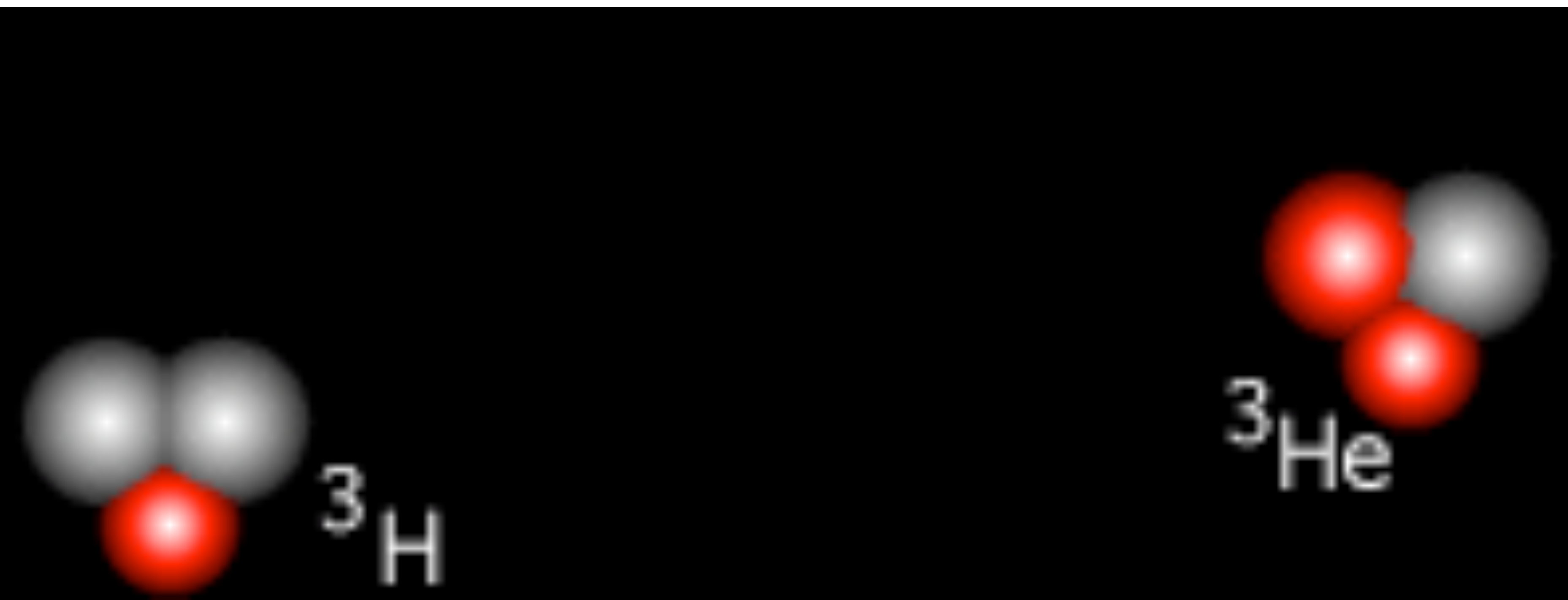
$^{18}_{9}F_{9}$
e
e^{+}

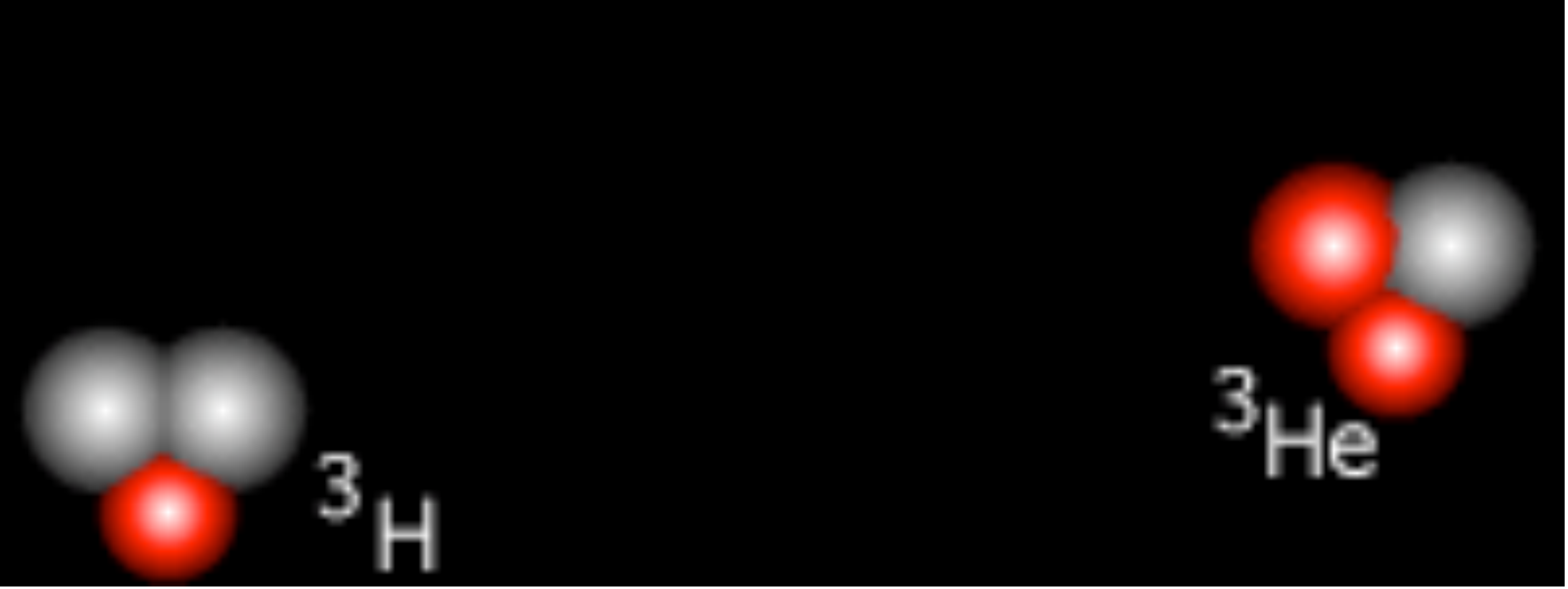


14C
6 8
18F
9 9
e
e+

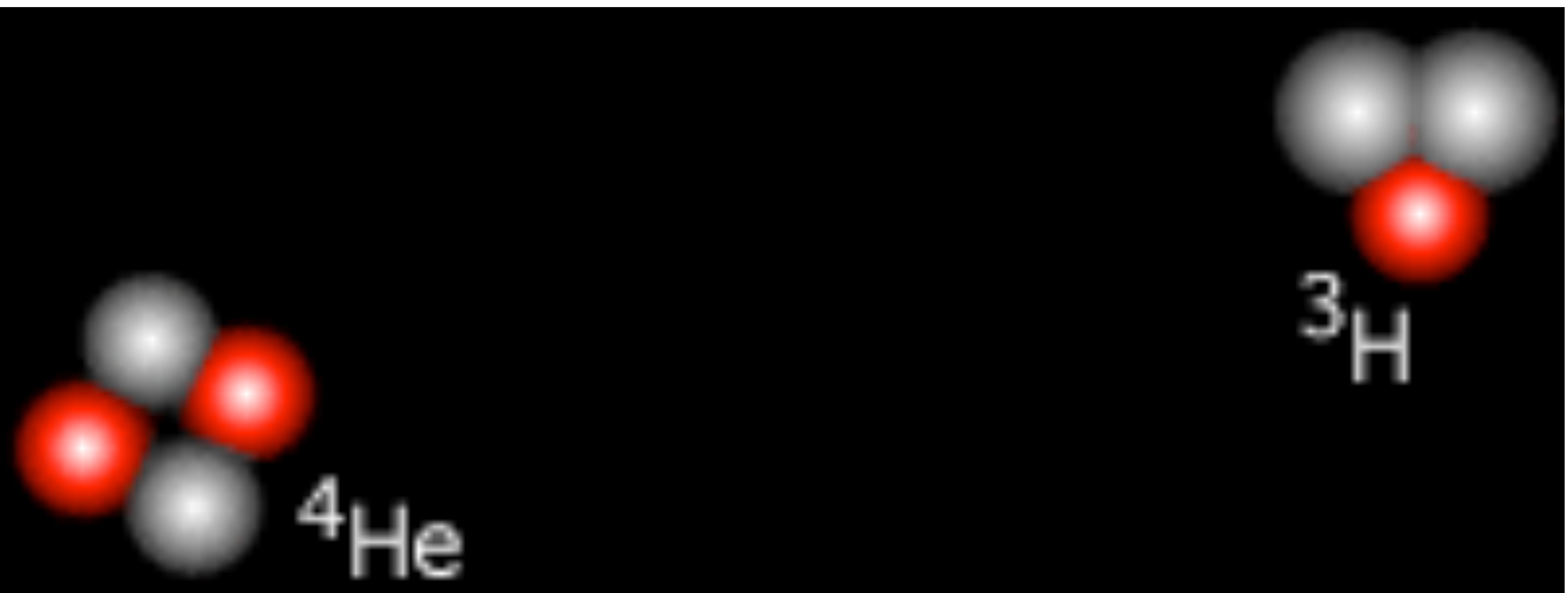

e
$^{90}_{39}Y_{41}$
e
e


$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$

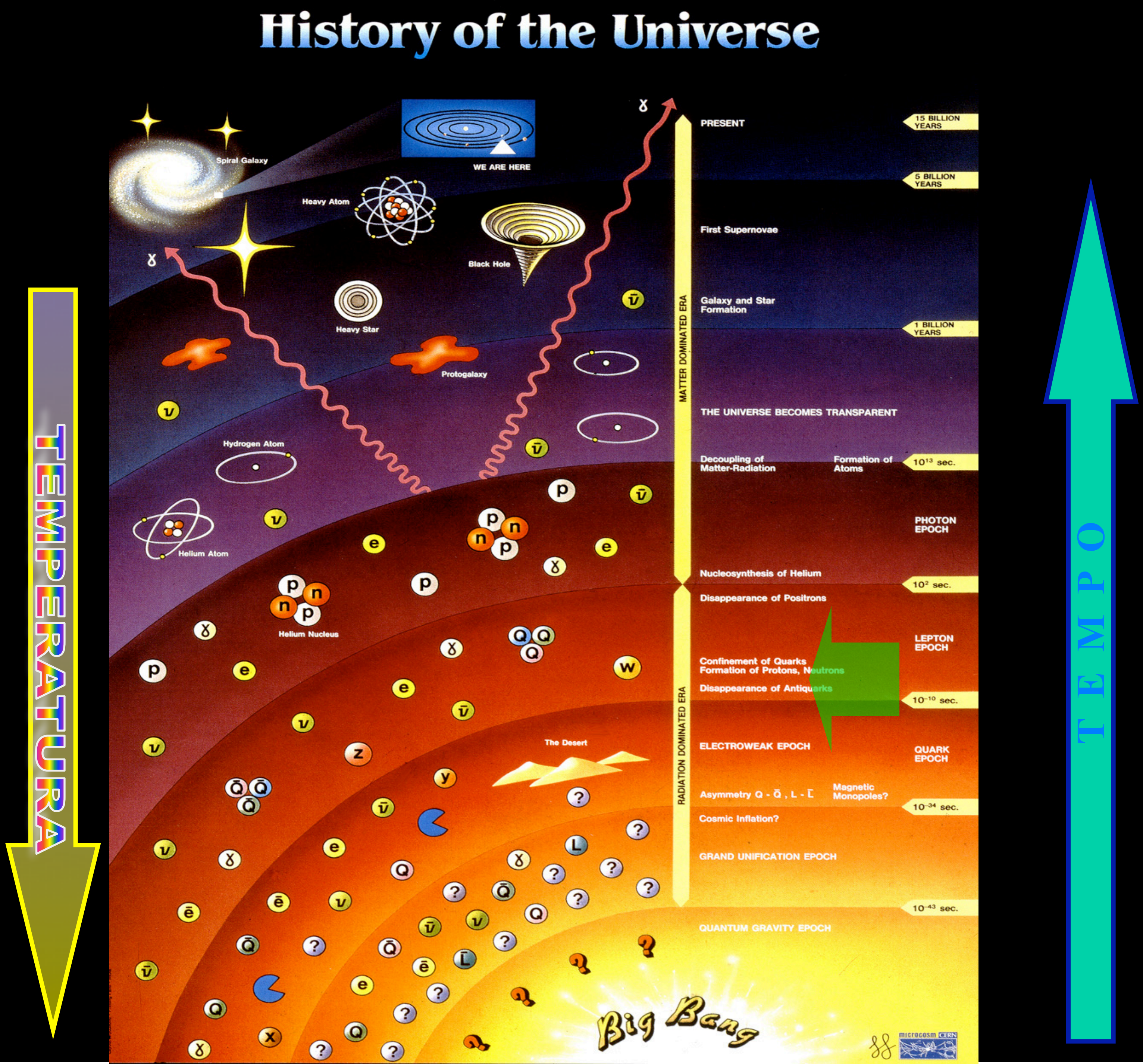

^{3}He
^{3}H



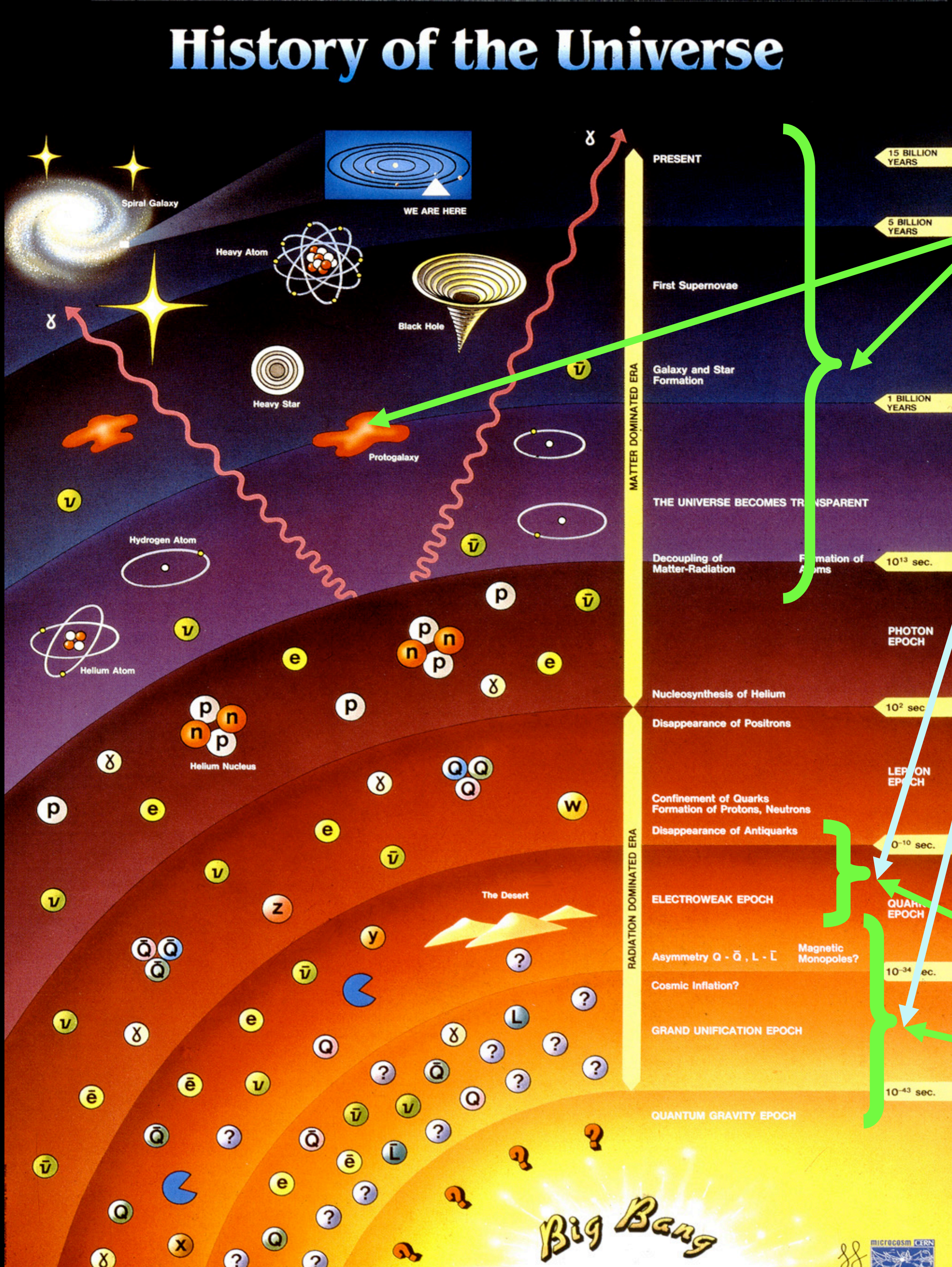
3H
3He



3H
4He



History of the Universe
TEMPERATURA
TEMPO
PRESENT
15 BILLION YEARS
5 BILLION YEARS
First Supernovae
Galaxy and Star Formation
1 BILLION YEARS
THE UNIVERSE BECOMES TRANSPARENT
Decoupling of Matter-Radiation
Formation of Atoms
10^{13} sec.
PHOTON EPOCH
Nucleosynthesis of Helium
10^{2} sec.
Disappearance of Positrons
LEPTON EPOCH
Confinement of Quarks Formation of Protons, Neutrons
Disappearance of Antiquarks
10^{-10} sec.
ELECTROWEAK EPOCH
QUARK EPOCH
Asymmetry Q - $\bar{Q}$, L - $\bar{L}$
Magnetic Monopoles?
10^{-34} sec.
Cosmic Inflation?
GRAND UNIFICATION EPOCH
10^{-43} sec.
QUANTUM GRAVITY EPOCH
MATTER DOMINATED ERA
RADIATION DOMINATED ERA
Spiral Galaxy
WE ARE HERE
Heavy Atom
Black Hole
Heavy Star
Protogalaxy
Hydrogen Atom
Helium Atom
Helium Nucleus
The Desert
Big Bang
microcosm CERN



ASTROFÍSICA NUCLEAR

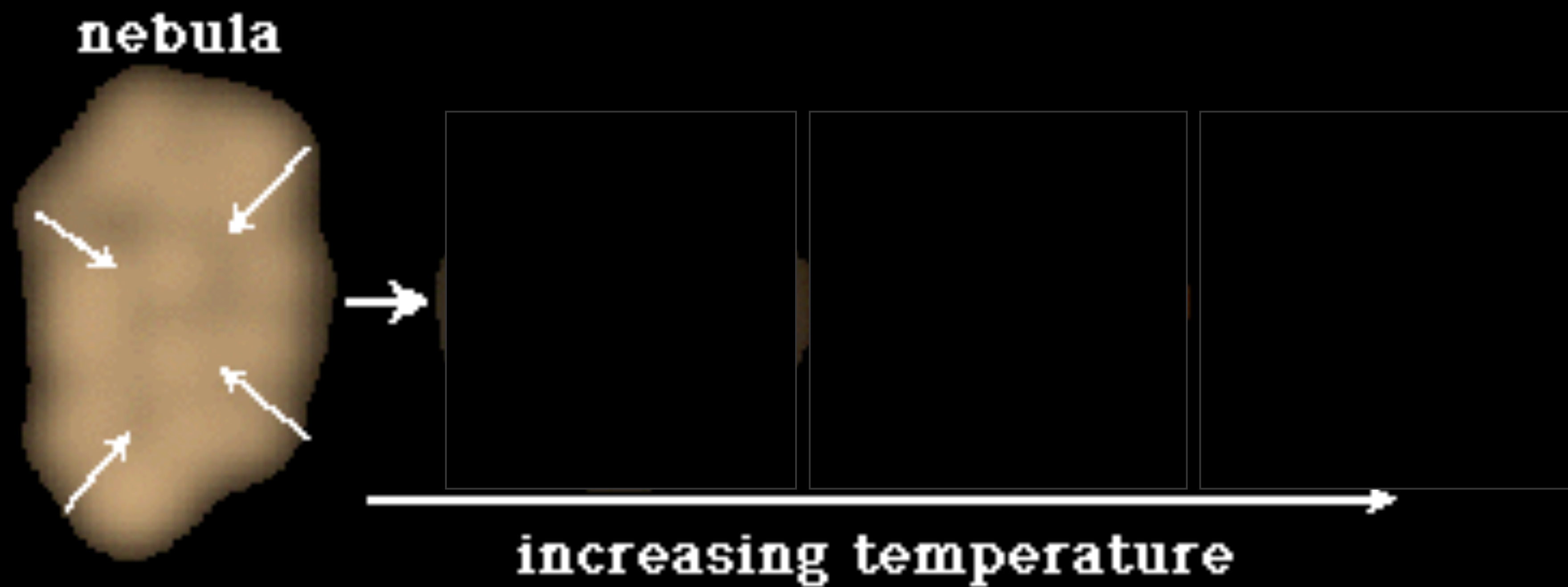
- nucleosíntese
- processos violentos
- limites da matéria nuclear

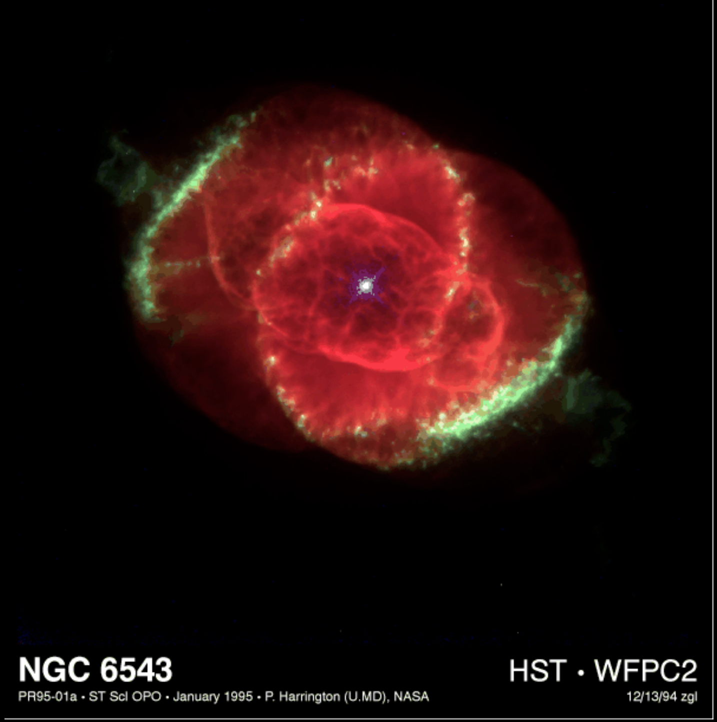
PREVISÕES da QCD:

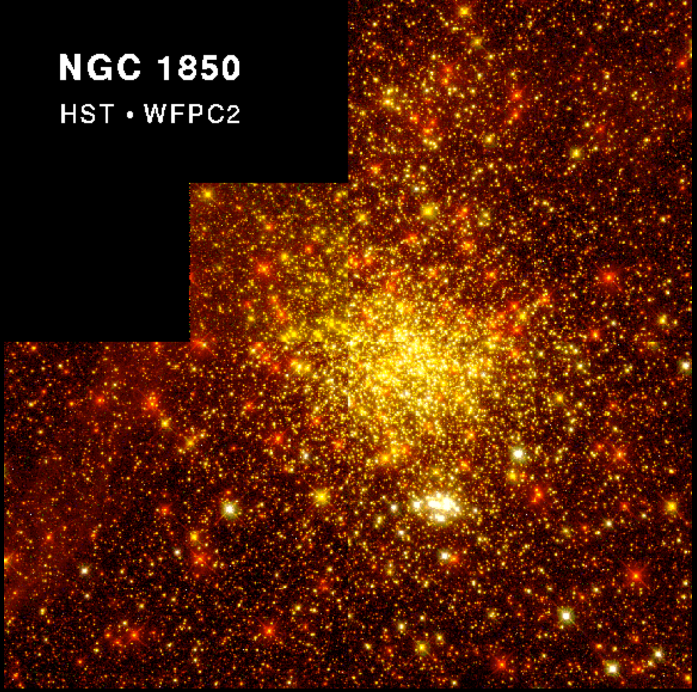
- Desconfinamento (T_c~170 MeV)
- restauração da simetria quiral num QGP

Questões Fundamentais do Modêlo Padrão

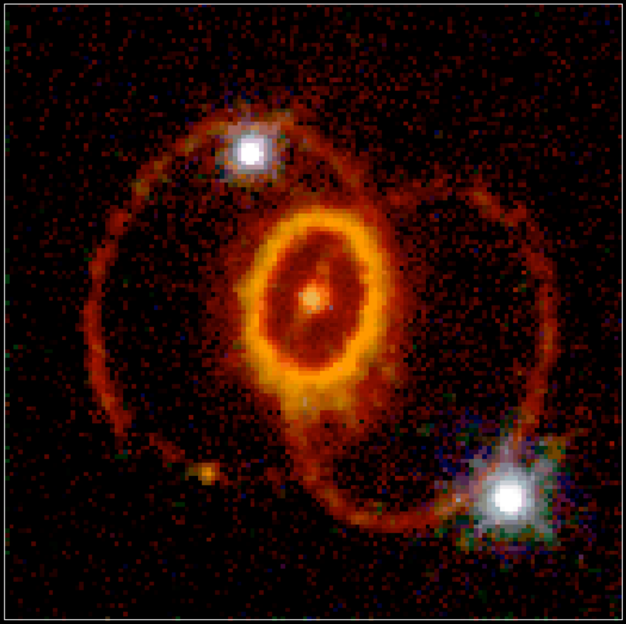
- Transição de fase
- simetrias da
- natureza
- origem das massas n


nebula
increasing temperature

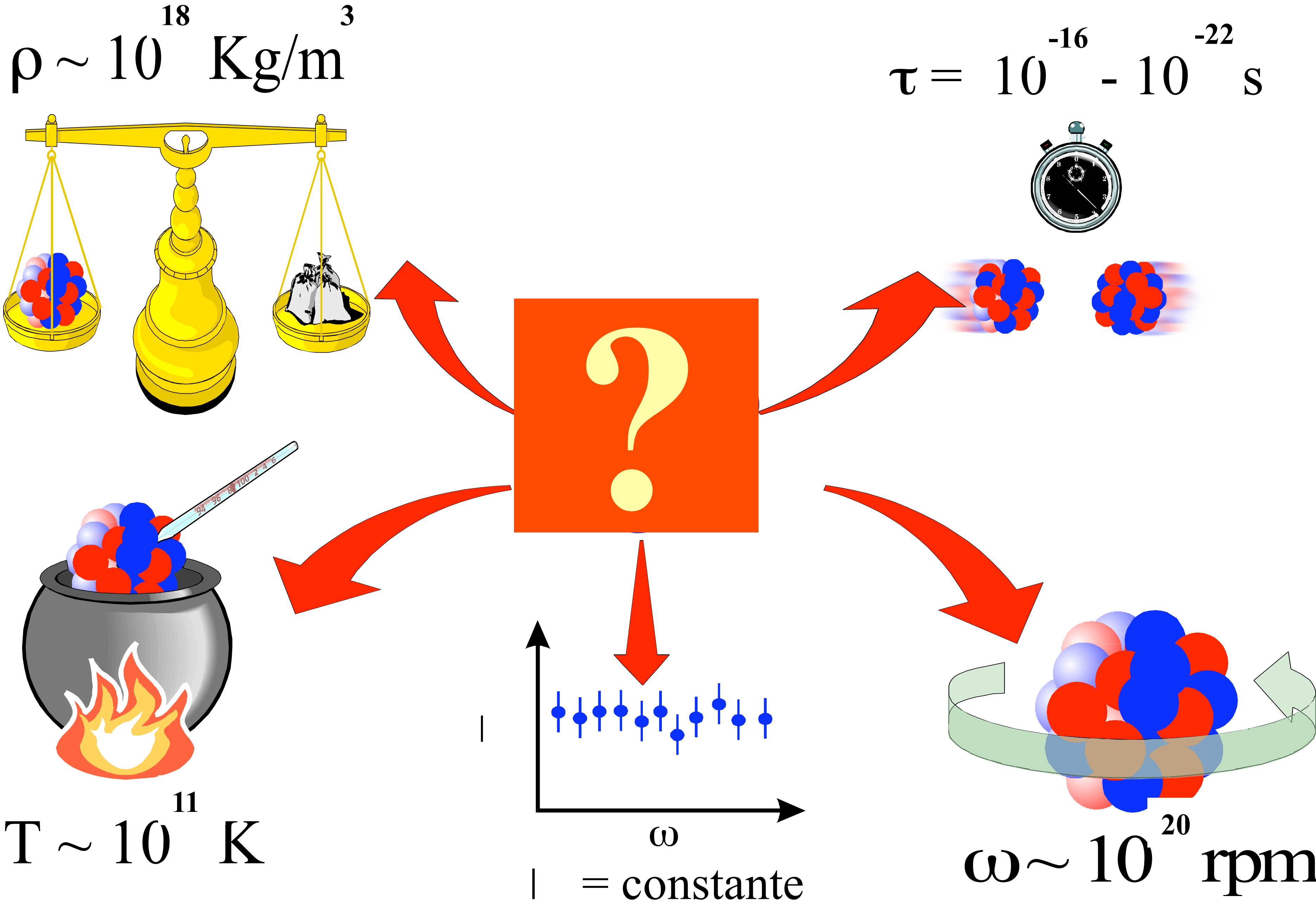




Supernova 1987A Rings



Hubble Space Telescope



ρ ~ 10^18 Kg/m^3
τ = 10^-16 - 10^-22 s
?
T ~ 10^11 K
ω
= constante
ω ~ 10^20 rpm

BLOCO 2

GRANDEZAS MACROSCÓPICAS DO NUCLEO

ESPALHAMENTO RUTHERFORD

- PARÂMETRO DE IM
- ÂNGULO DE DEFLE
- BARREIRA COULOI

RAIO NUCLEAR

- ESPALHAMENTO DE ELETRONS
- DIFUSIVIDADE

MASSA NUCLAR:

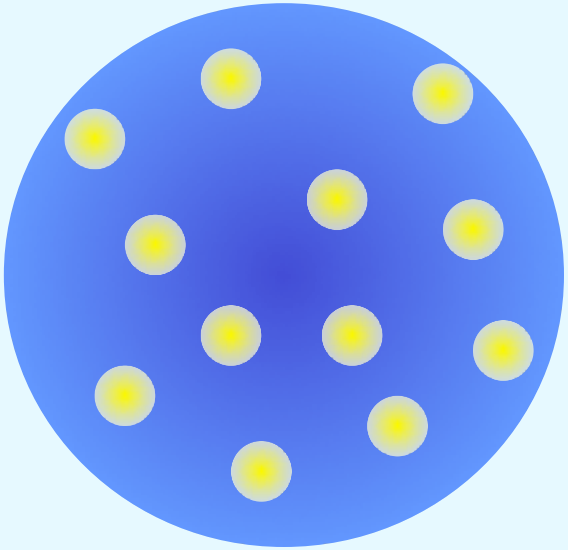
- ENERGIA DE LIGAÇÃO
- EXCESSO DE MASSA
- ENERGIA DE SEPARAÇÃO
- VALOR “Q” de REAÇÃO

CINEMÁTICA DE REAÇÃO - CINEMÁTICA INVERSA

NUCLEO DE THOMSON

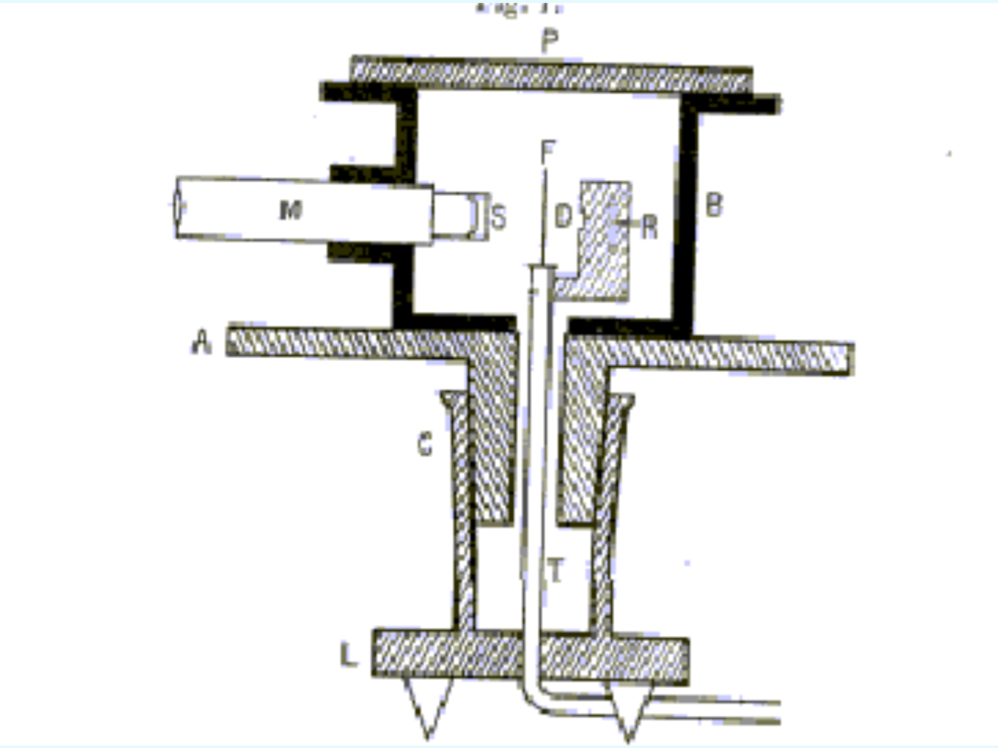
ANIMAÇÃO

http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/rutherford/rutherford2.html



^{24}Mg

NUCLEO DE RUTHERFORD → ***ANIMAÇÃO***



http://galileoandeinstein.physics.virginia.edu/more_stuff/Applet

http://www.nat.vu.nl/~pwgroen/projects/sdm/applets.htm

http://www.nat.vu.nl/~pwgroen/sdm/hyper/anim/anim_DI.html

http://physics.uwstout.edu/physapplets/

http://micro.magnet.fsu.edu/electromag/java/rutherford/

espalhamento Rutherford

The symbols we shall use are defined in Table 1.1. Figure 1.5 shows an orbit. The incident particle, if undeflected, would pass the centre (at *O*) of the target nucleus at a distance *b*, the **impact parameter**. In fact, the orbit is hyperbolic and at *D* the incident particle is at its distance of closest approach, *d*. The orbit is

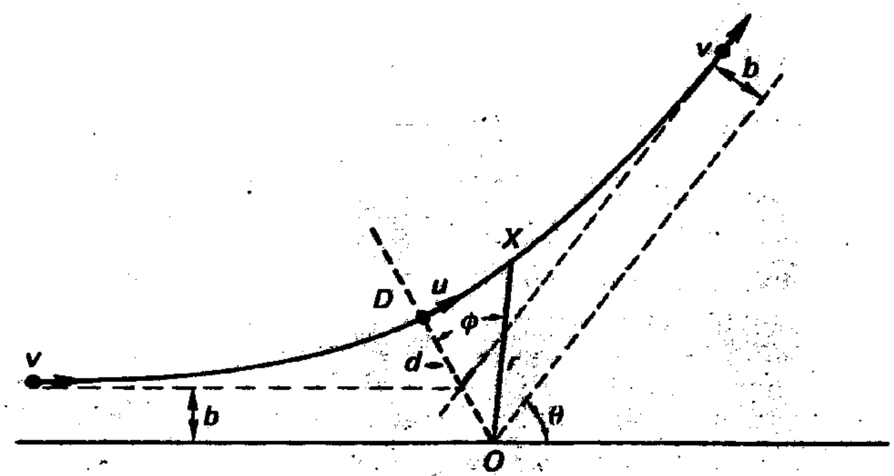


Fig. 1.5 The classical orbit of the incident particle in Rutherford scattering for non-zero impact parameter *b*.

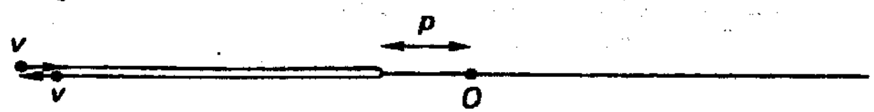


Fig. 1.6 The classical orbit in Rutherford scattering for zero impact parameter. Conservation of energy requires that the incident particle's distance of closest approach *p*, is given by

$$p = Zze^2/4\pi\varepsilon_0 T.$$

clearly symmetric about the line *OD*. If *b* was zero the incident particles would approach to a distance *p* (see Fig. 1.6). At this point the incident kinetic energy is transformed into mechanical potential energy in the Coulomb field, therefore:

$$\tfrac{1}{2}mv^2 p = Zze^2/4\pi\varepsilon_0. \quad (1.1)$$

This is the relation required from Step 1.

Step 2 To derive a first cross-section.

The relation (1.4) tells us that as b decreases θ increases. Therefore to suffer an angle of scatter greater than Θ the impact parameter b must be less than $(p/2)\cot(\Theta/2)$. That means the incident particle must strike a disc of this radius centred at O and perpendicular to v. The area, σ, presented by the nucleus for scattering through an angle greater than Θ is the area of this disc. That is

$$\sigma(\theta > \Theta) = \frac{\pi p^2}{4}\cot^2\frac{\Theta}{2}, \tag{1.5}$$

or in its full glory:

$$\sigma(\theta > \Theta) = \frac{\pi}{4}\left(\frac{Zze^2}{4\pi\varepsilon_0 T}\right)^2\cot^2\frac{\Theta}{2}.$$

The area σ is called a cross-section: if the reader is concerned about the meaning and use of this term we suggest reading Section 2.10, where a fuller description of the concept is given, before proceeding.

Step 3 To obtain the angular differential cross-section.

What we want is $d\sigma/d\Omega$, which is the cross-section per unit solid angle located at an angle θ. The element of solid angle $d\Omega$ between θ and $\theta + d\theta$ is given by

$$d\Omega = 2\pi \sin\theta \, d\theta.$$

Therefore

$$\frac{d\sigma}{d\Omega} = \frac{1}{2\pi\sin\theta}\frac{d\sigma}{d\theta}.$$

The $d\sigma/d\theta$ we need is $(d/d\Theta)\sigma(\theta > \Theta)$ from Equation (1.5) and hence we obtain

$$\frac{d\sigma}{d\Omega} = \left(\frac{Zze^2}{16\pi\varepsilon_0 T}\right)^2 \operatorname{cosec}^4\frac{\theta}{2}. \tag{1.6}$$

Dado um processo: A + a $\rightarrow$ b + B

A (a,b)B

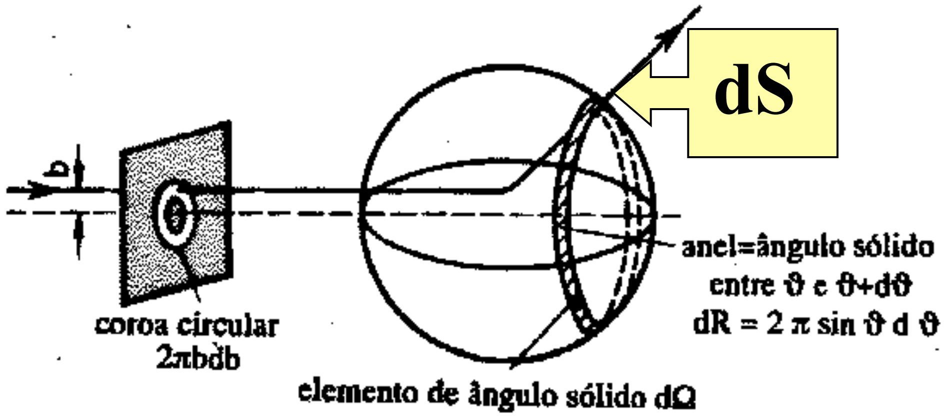


Fig.2 Relações geométricas na dispersão.

$\sigma \rightarrow$ 1 barn = 10^{-28}m^2 = 100 fm^2

1 fm = 10^{-16}m

Definimos:

$$\left(\frac{d\sigma}{d\Omega}\right)_\theta = \frac{\text{n}^\text{o}\ \text{partículas no ângulo}\ \theta\ \text{(no ângulo sólido d}\Omega)}{(\text{n}^\text{o}\ \text{partículas a/unidade de área})(\text{n}^\text{o}\ \text{partículas A no alvo})}$$

Dado um processo: $A + a \rightarrow b + B$

$A\,(a,b)B$

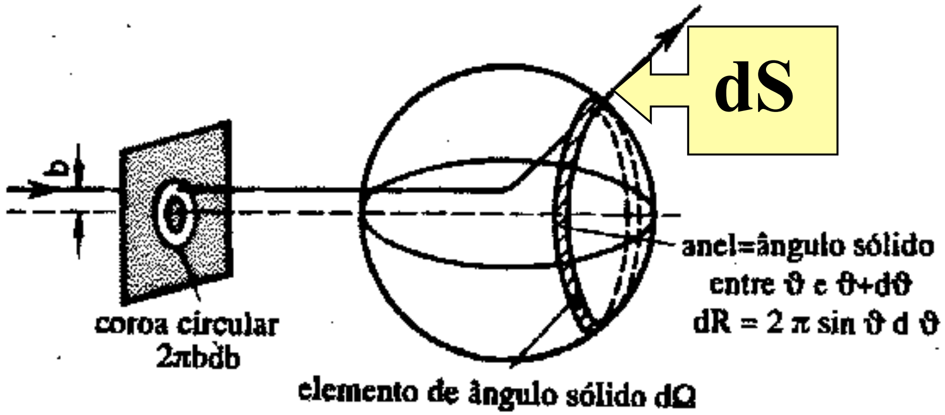


Fig.2 Relações geométricas na dispersão.

$$dS = 2\pi R\ \text{sen}\theta \ .\ R d\theta \ =\ 2\pi R^2\ \text{sen}\theta\ d\theta$$

$$\Delta\Omega = \frac{\Delta S}{R^2} = 2\pi\,\text{sen}\,\theta\, d\theta \Rightarrow d\Omega$$

$$\sigma_{Total} = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\theta}\left(\frac{d\theta}{d\Omega}\right)$$

$$\frac{d\theta}{d\Omega} = \frac{1}{2\pi\,\text{sen}\,\theta} \Rightarrow \frac{d\sigma}{d\Omega} = \frac{1}{2\pi\,\text{sen}\,\theta}\left(\frac{d\sigma}{d\theta}\right)$$

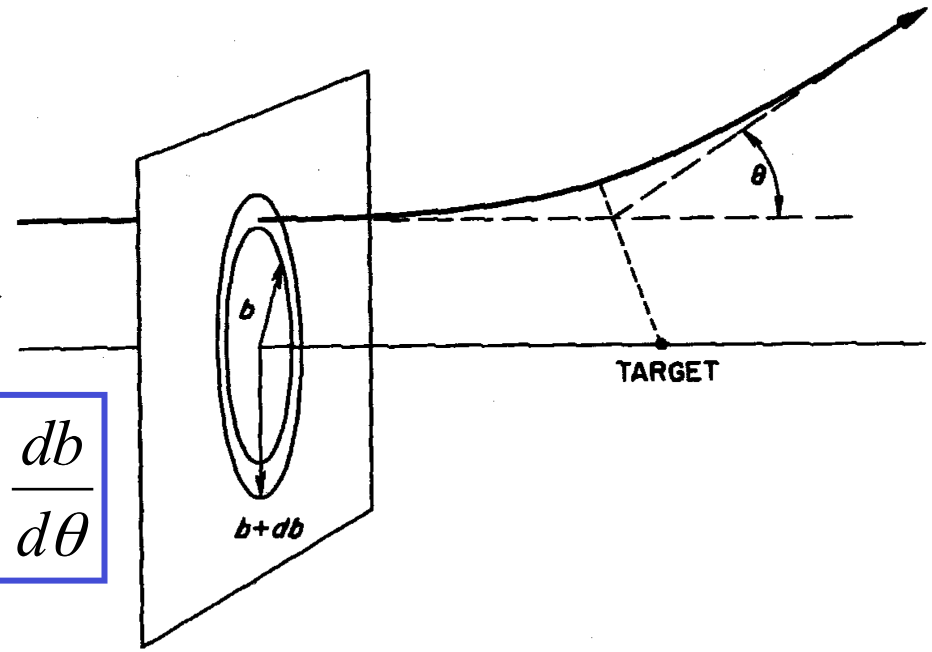
Para um potencial central

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R} \qquad M_A >> m_a$$

$$\frac{d\sigma}{d\theta} = \frac{d\left(\pi b^2\right)}{d\theta} = 2\pi b\frac{db}{d\theta}$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\theta}\left(\frac{d\theta}{d\Omega}\right)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{2\pi \operatorname{sen}\theta}\right)\left(2\pi b\frac{db}{d\theta}\right) = \frac{b}{\operatorname{sen}\theta}\frac{db}{d\theta}$$



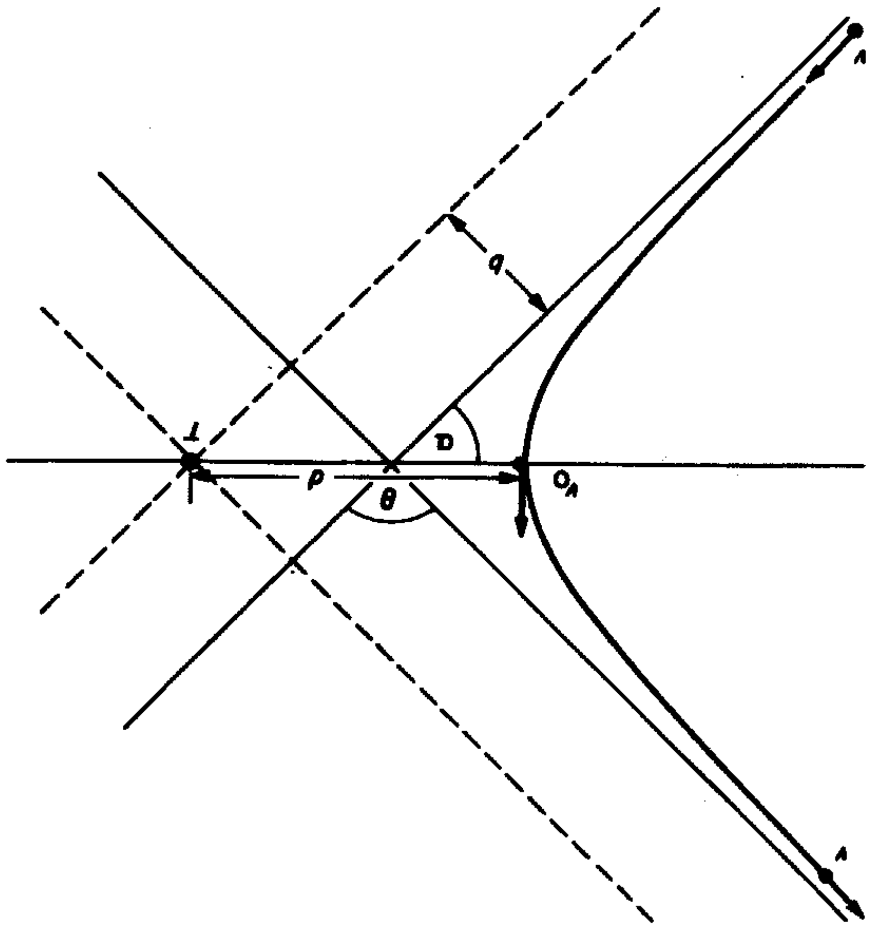
$$b = \frac{1}{4\pi E_0}\frac{Z_A Z_a}{2E}\cot^2\frac{\theta}{2}$$

$$\Downarrow$$

$$\frac{d\sigma}{d\Omega} = \frac{b}{\operatorname{sen}\theta}\frac{db}{d\theta} = \left(\frac{Z_A Z_a e^2}{2E}\right)^2 \frac{1}{\operatorname{sen}^4\frac{\theta}{2}}$$

Para um potencial central

$$\mathbf{V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R}} \qquad \mathbf{M_A >> m_a}$$

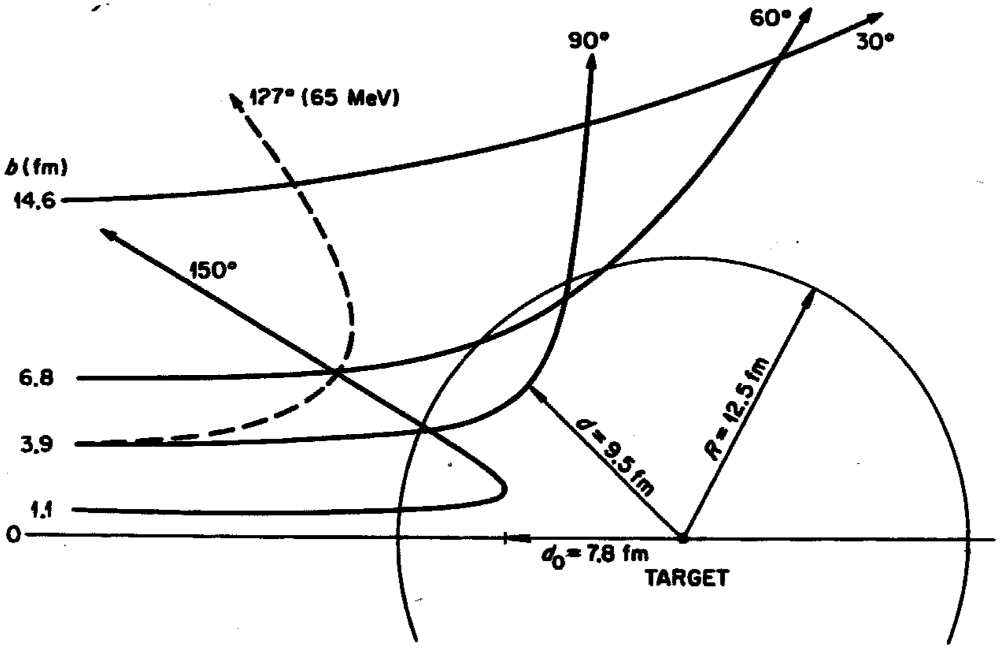


(Distancia de máxima aproximação d)

$$\frac{1}{2} m\,v^2 = \frac{1}{2} m\,v_0^2 + \frac{Z_A Z_a e^2}{d}$$

Para uma colisão frontal (b=0) d = d_0

$$\left(\frac{v_0}{v}\right)^2 = 1 - \frac{d_0}{d} \text{ onde } d_0 = 2\frac{Z_A Z_a e^2}{mv^2} = \frac{Z_A Z_a e^2}{E}$$

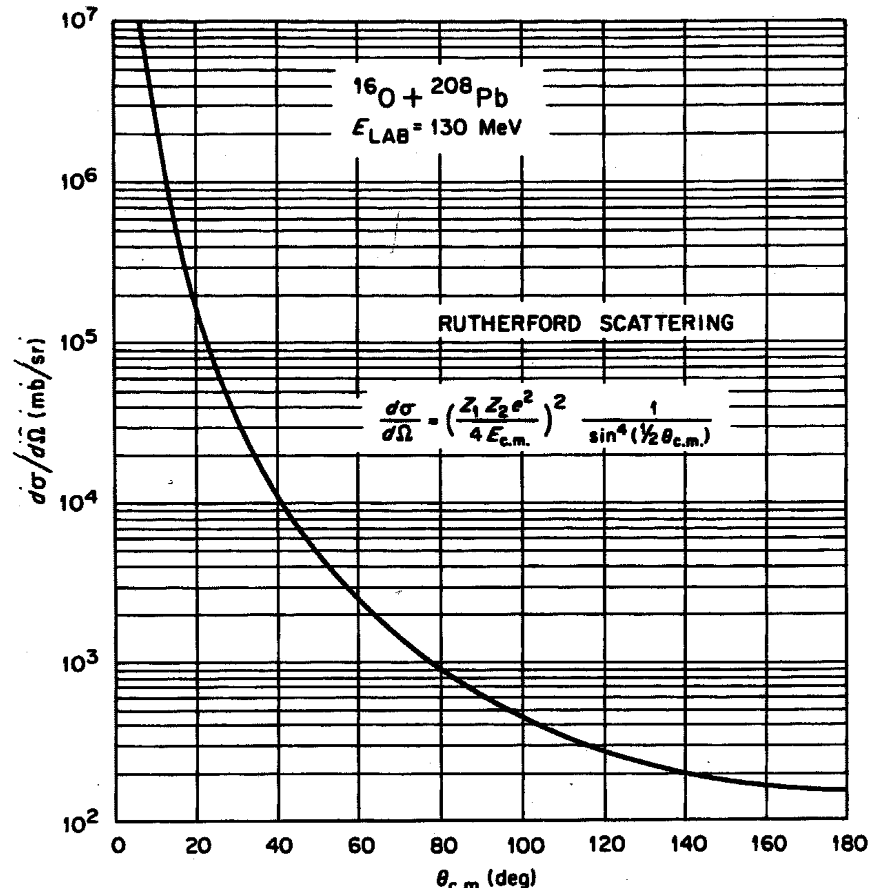


$$\frac{d\sigma}{d\Omega} = \left[\frac{Z_A Z_a e^2}{2E}\right]^2 \frac{1}{\mathrm{sen}^4 \frac{\theta}{2}}$$

espalhamento:
$^{208}Pb(^{16}O,^{16}O)^{208}Pb$

na energia **E** $_{16O}$= 130MeV

a curva pontilhada
corresponde a $E_{(16O)}$ = 65MeV



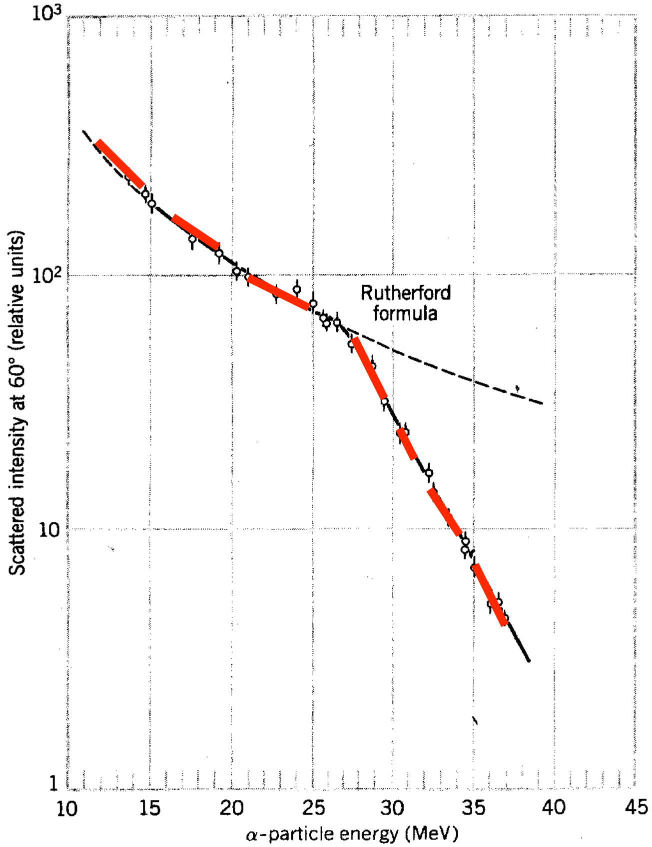


Figure 3.11 The breakdown of the Rutherford scattering formula. When the incident α particle gets close enough to the target Pb nucleus so that they can interact through the nuclear force (in addition to the Coulomb force that acts when they are far apart) the Rutherford formula no longer holds. The point at which this breakdown occurs gives a measure of the size of the nucleus. Adapted from a review of α particle scattering by R. M. Eisberg and C. E. Porter, *Rev. Mod. Phys.* **33**, 190 (1961).

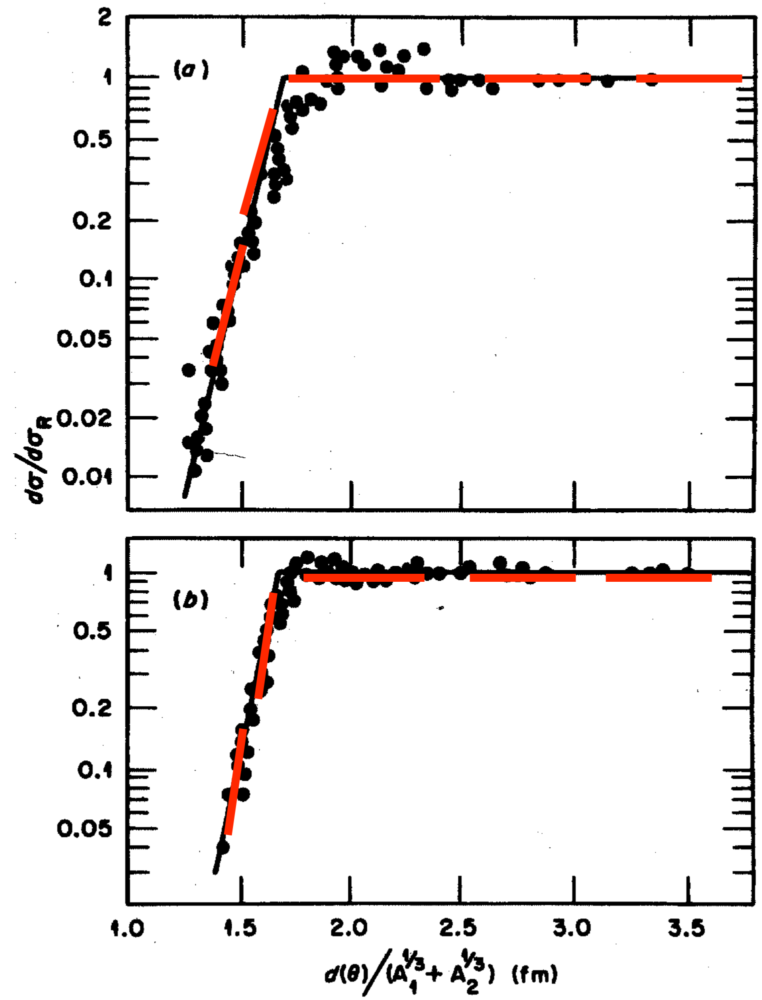


Figure 2.12 The differential cross-sections for O and C ions, in ratio to the Rutherford ones, scattering from various targets plotted against the distance of closest approach d, instead of the scattering angle θ, by using equation 2.20. The distance d has been divided by $(A_1^{1/3} + A_2^{1/3})$, where A_i is the mass number of nucleus i. The measured cross-sections then fall on a universal curve, showing that nuclear radii are approximately proportional to $A^{1/3}$. (a) $^{16}O + {}^{40,48}Ca$ at 49 MeV, $^{16}O + {}^{40,48}Ca$, ^{50}Ti, ^{52}Cr, ^{54}Fe, ^{62}Ni at 60 MeV and $^{18}O + {}^{60}Ni$ at 60 MeV; (b) $^{12}C + {}^{96}Zr$ at 38 MeV, $^{16}O + {}^{96}Zr$ at 47, 49 MeV, $^{16}O + {}^{88}Sr$, ^{93}Zr at 60 MeV and $^{18}O + {}^{90}Zr$ at 60, 66 MeV. (After Christensen *et al.*, 1973)

BARREIRA COULOMBIANA

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R}$$

Para a colisão razante:

$$R_c = r_0 A_A^{1/3} + r_0 A_a^{1/3} = r_0(A_A^{1/3} + A_a^{1/3})$$

$$V_{Coul} = \frac{Z_A Z_a e^2}{r_0(A_A^{1/3} + A_a^{1/3})}$$

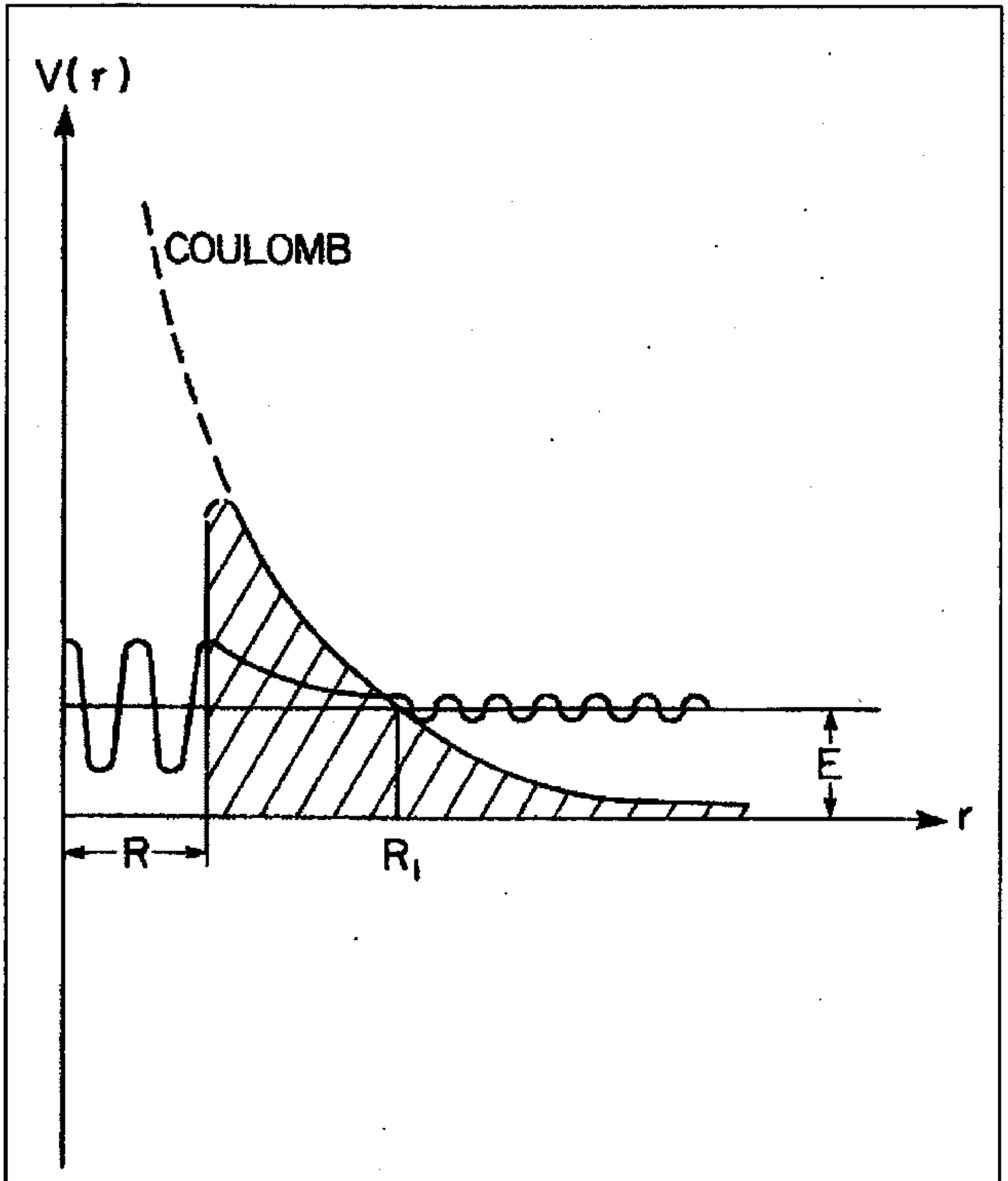
$E_{cm} > V_{coul} \implies T_l(E_{cm}) = 1$

$E_{cm} < V_{coul} \implies T_l(E_{cm}) = 0$

$e^2 = 1.44$ MeV.fm

$r_0 \sim 1.25$ fm

V_{coul} (MeV)


V(r)
COULOMB
E
r
R
R_1

BARREIRA COULOMBIANA

$$V_C = \frac{Z_A Z_a e^2}{R} = \frac{C}{R}$$

Para a colisão razante:

$$R = R_A + R_a$$

$$V_{Coul} = \frac{Z_A Z_a e^2}{R}$$

$E_{cm} > V_{coul} \implies T_l(E_{cm}) = 1$

$E_{cm} < V_{coul} \implies T_l(E_{cm}) = 0$

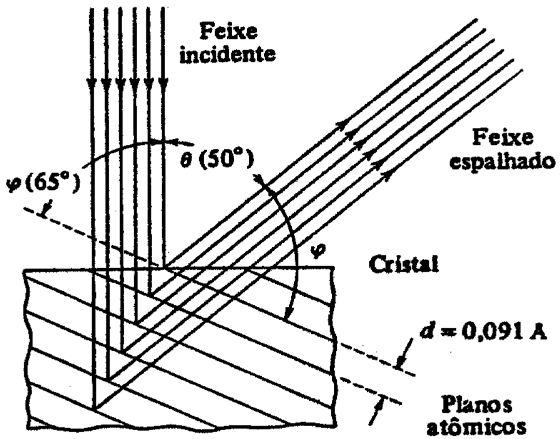
$e^2 = 1.44$ MeV.fm

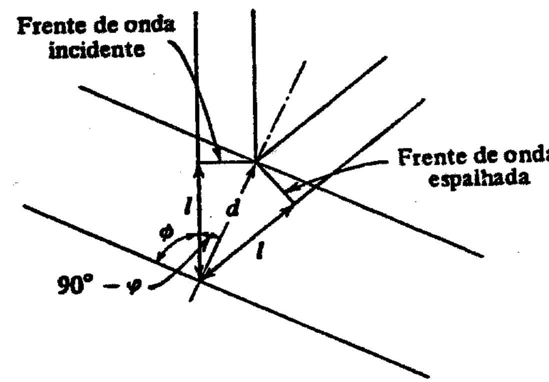
$r_0 \sim 1.25$ fm

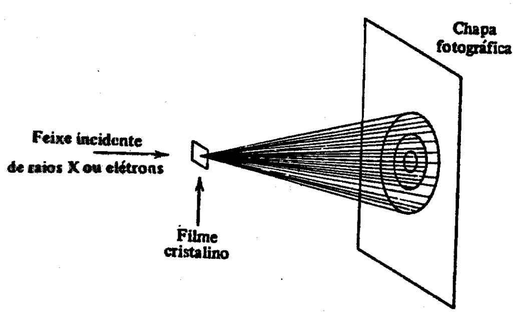
V_{coul} (MeV)

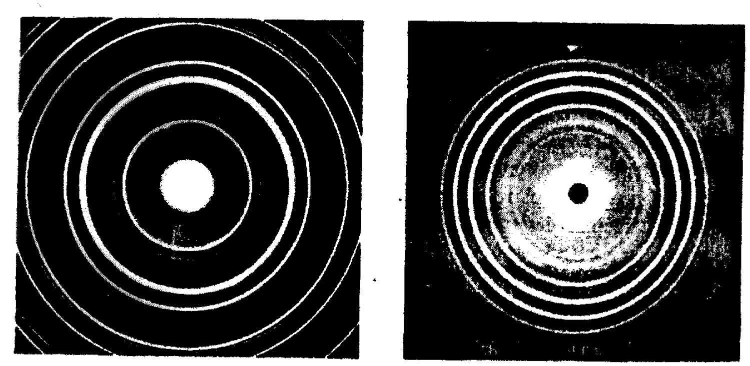
raio nuclear

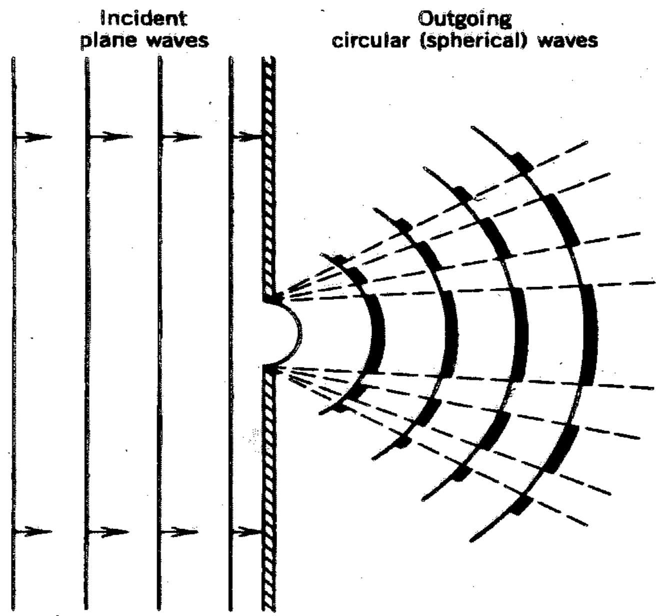
O Postulado de de Broglie Propriedades Ondulatórias das Partículas











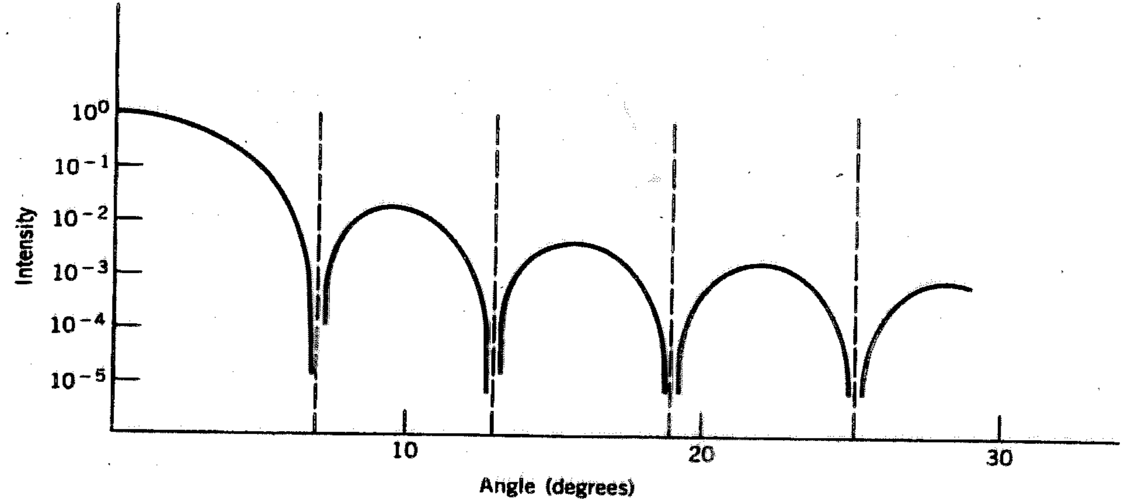
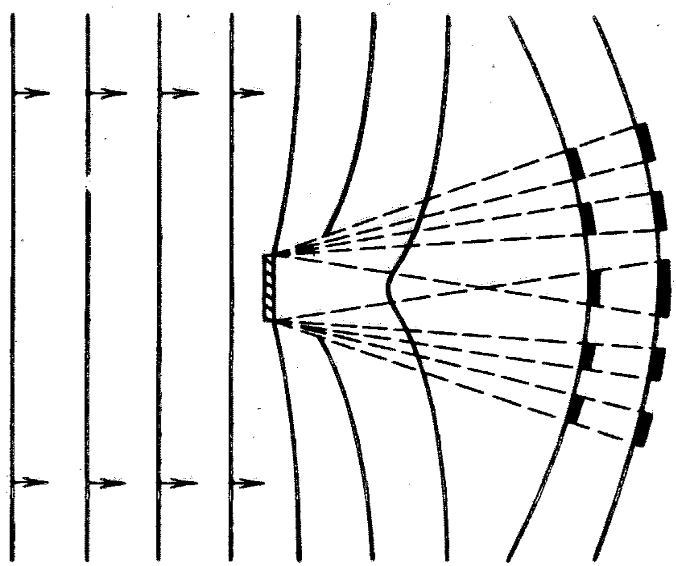


Figure 11.13 Diffraction pattern of light incident on a circular aperture; a circular disk gives a similar pattern. The minima have intensity of zero. The curve is drawn for a wavelength equal to ten times the diameter of the aperture or disk.



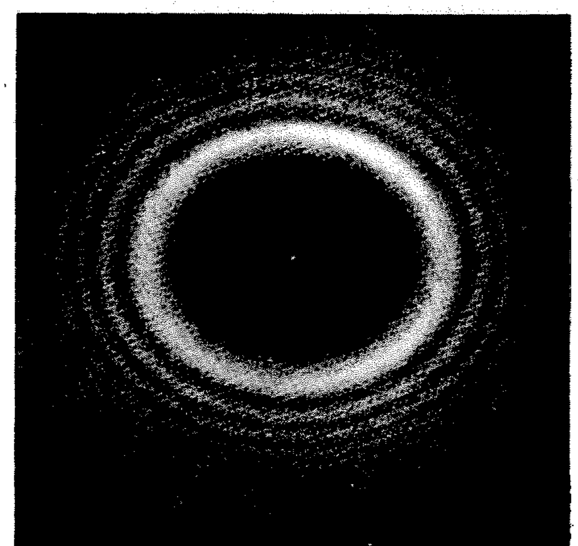


Figure 4.3 Representation of scattering by (top) a small opening and (bottom) a small obstacle. The shading of the wavefronts shows regions of large and small intensity. On the right are shown photographs of diffraction by a circular opening and an opaque circular disk. Source of photographs: M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomena* (Berlin: Springer-Verlag, 1962).

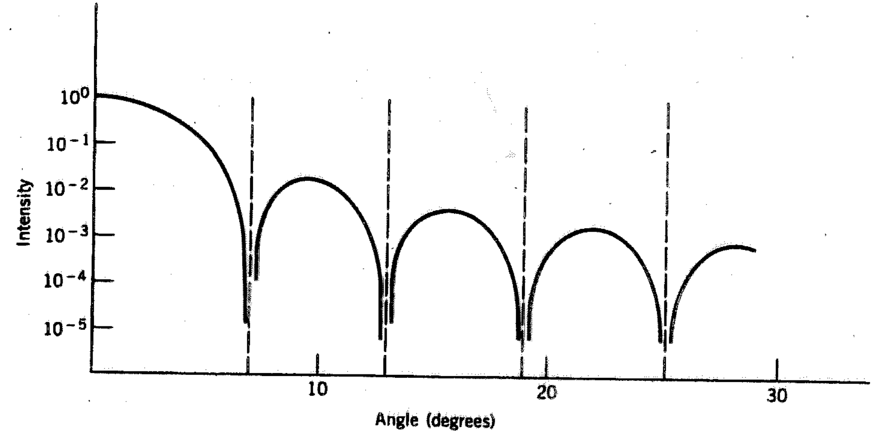


Figure 11.13 Diffraction pattern of light incident on a circular aperture; a circular disk gives a similar pattern. The minima have intensity of zero. The curve is drawn for a wavelength equal to ten times the diameter of the aperture or disk.

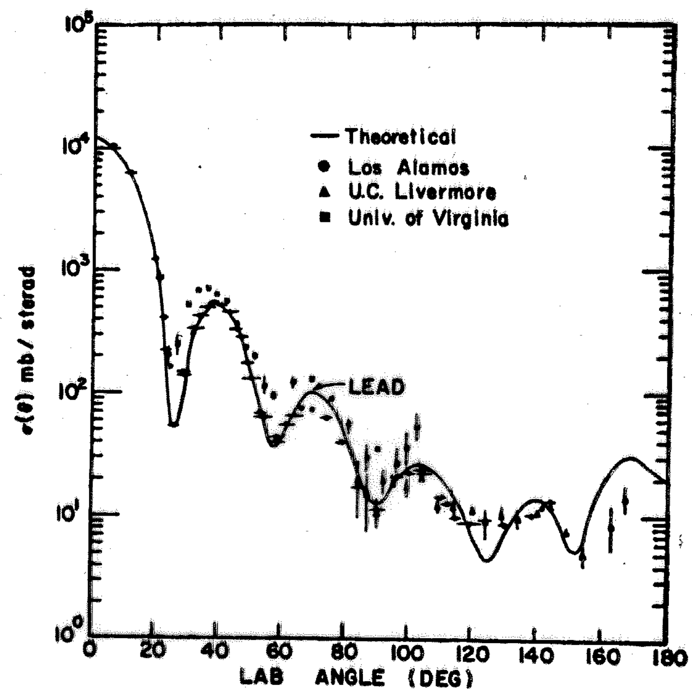


Figure 11.14 Elastic scattering of 14-MeV neutrons from Pb. From S. Fernbach, *Rev. Mod. Phys.* **30**, 414 (1958).

3.3 The nuclear electric charge distribution

ref: williams

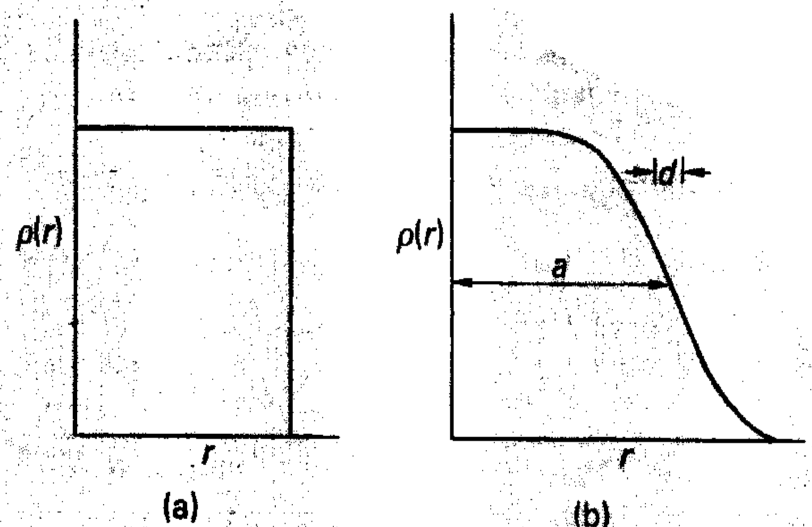
We need a model. The methods used to measure charge distribution find the time average so we can define a time-independent charge density. For the present we shall assume spherically symmetric nuclei so that we can define a radial charge density, $\rho(r)$. Two models are given in Fig. 3.1. Model I with its sharp-edged charge distribution is very unlikely but can be tested. Model II softens the hard edges by assuming a charge distribution with a mathematical form normally associated with the Fermi–Dirac statistics but which, applied to nuclei, is called the **Saxon–Woods** form. So the challenge to experiments is to see if either model makes predictions which fit the data and if so, to determine the parameters ρ_0, a and d, and if not, to find a better model and its parameters.

Fig. 3.1 Two models of the radial electric charge distribution of nuclei.

(a) Model I: $\rho(r) = \rho_0,\ r < a,$
$\rho(r) = 0,\ r > a.$

(b) Model II: $$\rho(r) = \frac{\rho_0}{1+\exp\left(\frac{r-a}{d}\right)}.$$

The $r = a \pm d/2$ points have densities 62.2% and 37.8% of the central density respectively. The 90% to 10% thickness is $4.39d$. This shape is called the Saxon–Woods form. The charge density, ρ_0, is fixed by normalizing to the total nuclear charge $Z|e|$.



3.4 The nuclear electric form-factor

How do we deal with the effect of an extended nuclear charge on the Mott scattering of electrons? The answer is to do as in classical optics where we derive the Fraunhofer diffraction pattern of an aperture in a screen by taking the Fourier transform of that aperture. For electron scattering the aperture is replaced by a spherical distribution of charge. We take the nucleus to have charge Ze where e is the charge on the proton. If that charge was point-like at $r=0$ we can imagine that it gives rise to a scattered wave amplitude $Zef(\theta)$ at large distances at an angle θ, defined so that

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Mott}} = Z^2e^2|f(\theta)|^2. \tag{3.1}$$

If that charge is spread out then an element of charge $d(Ze)$ at a point $\mathbf{r}$ will give rise to a contribution to the amplitude of $e^{i\delta}f(\theta)d(Ze)$ where δ is the extra 'optical' phase introduced by wave scattering by the element of charge at the point r compared to zero phase for scattering at $r=0$.

Consider now Fig. 3.2. The incident and scattered electron have momentum $\boldsymbol{p}$ and $\boldsymbol{p}'$ with $p=|\boldsymbol{p}|=|\boldsymbol{p}'|$. The momentum transfer $\boldsymbol{q}(|q|=2p\sin(\theta/2)$, see Fig. 1.7) is along OZ, O being the nuclear centre. The 'optical ray' P_1OP_1' is taken to have zero relative path length. The ray P_2SP_2' has equal angles of incidence and reflection at the plane AXA' which is perpendicular to OZ. Therefore the path length d is the same for all rays parallel to P_1OP_1' and reflected at any point in AXA'. This path length is given by $d=2OX\sin(\theta/2)$. The phase, δ, is $2\pi d/\lambda$, where λ is the de Broglie wavelength. Now the reduced wavelength $\lambda/2\pi=\hbar/p$ so

$$\delta = pd/\hbar = \frac{2p\sin(\theta/2)}{\hbar}OX = \frac{q}{\hbar}OX.$$

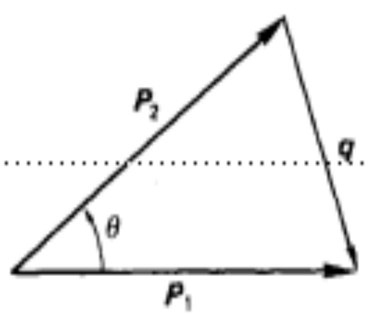


Fig. 1.7 The momentum transfer $\boldsymbol{q}$ ir elastic scattering at a fixed target. Th vectors $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ represent the incid(and scattered particle momenta respectively ($|\boldsymbol{P}_1|=|\boldsymbol{P}_2|=P$). If the angle of scatter is θ, the geometry gi\

$q = 2P\sin(\theta/2)$.

Return to the point S: let the charge density be $\rho(r)$ when $r = OS$ (we assume spherical symmetry). If we have polar coordinate r, α, β, where the polar axis is along Z and $S\hat{O}Z = \alpha$, then a volume element $\mathrm{d}V$ at S is $r^2 \sin\alpha \, \mathrm{d}r \, \mathrm{d}\alpha \, \mathrm{d}\beta$ and the charge is $\rho(r)\mathrm{d}V$. Then this charge element gives rise to an amplitude

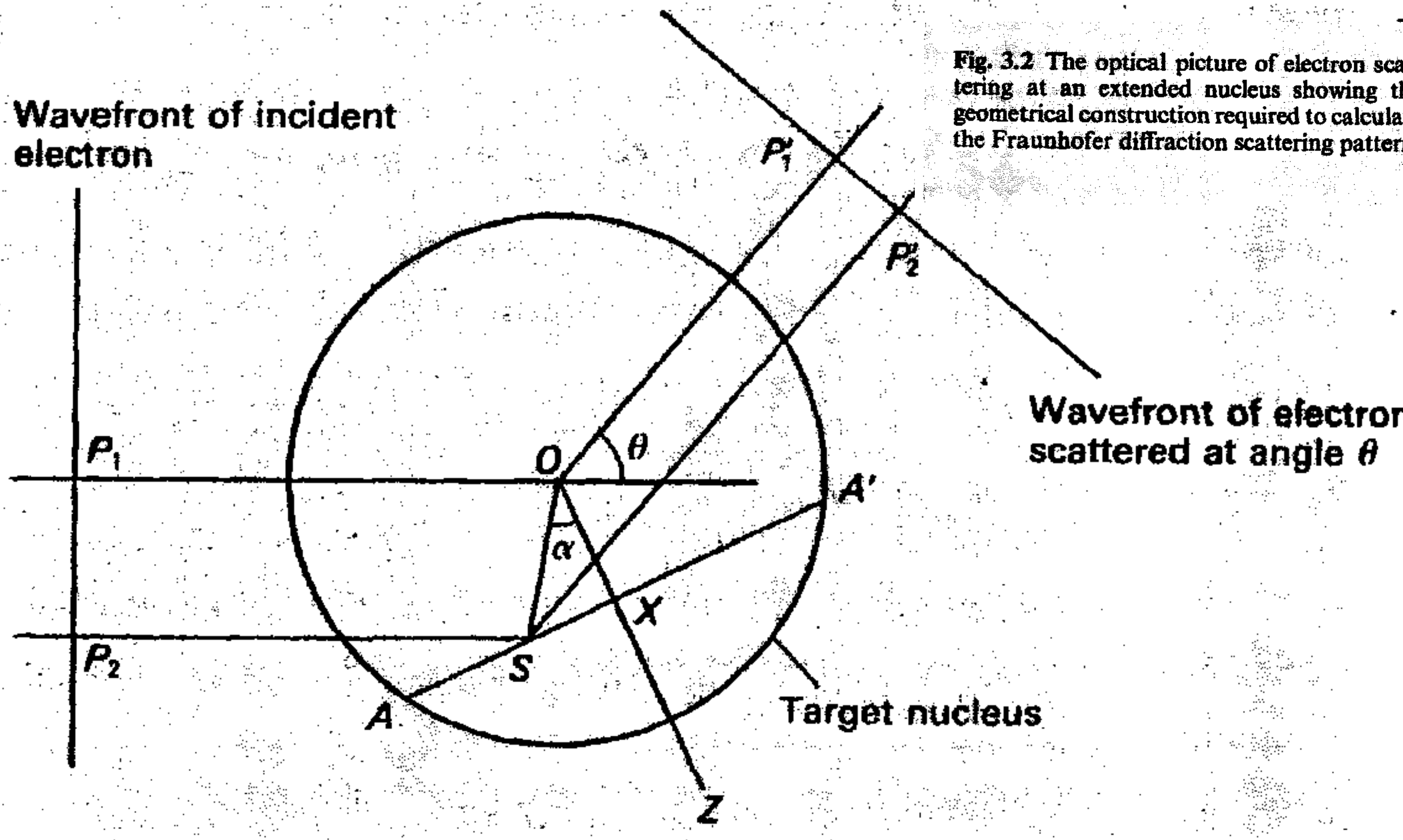


Fig. 3.2 The optical picture of electron scattering at an extended nucleus showing the geometrical construction required to calculate the Fraunhofer diffraction scattering pattern.

$$\text{charge} \times f(\theta) \times e^{i\delta}$$

$$= \rho(r) r^2 \sin\alpha \, dr \, d\alpha \, d\beta \, f(\theta) e^{i\boldsymbol{q}.\boldsymbol{r}/\hbar}.$$

The exponent contains $\boldsymbol{q}.\boldsymbol{r}$ because $q(OX) = qr\cos\alpha = \boldsymbol{q}.\boldsymbol{r}$. Then the total scattered amplitude

$$A(\theta) = f(\theta) \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \rho(r) r^2 \sin\alpha \, dr \, d\alpha \, d\beta \, e^{i\boldsymbol{q}.\boldsymbol{r}/\hbar}, \tag{3.2}$$

the integration being over the whole nucleus. The integration over the azimuthal angle β around OZ is trivial because the point S just traverses the plane AXA' for which $\boldsymbol{q}.\boldsymbol{r}$ is constant.

$$A(\theta) = f(\theta) \int_0^{\pi} \int_0^{\infty} 2\pi\rho(r) r^2 \sin\alpha \, dr \, d\alpha \, e^{i\boldsymbol{q}.\boldsymbol{r}/\hbar}.$$

Now we have for the total charge

$$Ze = \int_0^{\pi} \int_0^{\infty} 2\pi\rho(r) r^2 \sin\alpha \, dr \, d\alpha,$$

so we see that we can write

$$A(\theta) = Zef(\theta)\frac{\iint \rho(r) r^2 \sin\alpha \, dr \, d\alpha \, e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar}}{\iint \rho(r) r^2 \sin\alpha \, dr \, d\alpha} = Zef(\theta) F(\theta).$$

Thus the Mott (or Rutherford) scattering amplitude $Zef(\theta)$ is changed by a factor $F(\theta)$ and the scattering cross-section becomes

$$\frac{d\sigma}{d\Omega} = |F(\theta)|^2 \frac{d\sigma}{d\Omega}\bigg|_{\text{Mott}}.$$

$F(\theta)$ is called a **form factor**. (This is a name inherited from the description of atomic X-ray scattering.) Somewhat formally it could be written

$$F(\theta) = \frac{\int \rho(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar} dV}{\int \rho(\boldsymbol{r}) dV} = \frac{1}{Ze}\int \rho(\boldsymbol{r}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}/\hbar} dV.$$

Clearly as $\theta \to 0$, $q \to 0$ and $F(0) = 1$. Since the form factor is more properly a function of q than of θ it has become usual to write $F(q^2)$ rather than $F(\theta)$ and we note $F(q^2)$ is the Fourier transform of the charge distribution (remember the optics of Fraunhofer diffraction). These ideas and formulae are summarized in Table 3.1.

The effect of an extended nuclear charge is to reduce the differential cross-section for elastic electron scattering from that for a point-like nucleus by a factor which is the square of the form factor:

$$\frac{d\sigma}{d\Omega} \rightarrow |F(q^2)|^2 \frac{d\sigma}{d\Omega},$$

where the form factor

$$F(q^2) = \frac{1}{Ze} \int\int\int \rho(r)\, e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\, dV,$$

and $\mathbf{q}$ is the momentum transfer, $q = |\mathbf{q}|$ and $\rho(\mathbf{r})$ is the charge density. The volume integral is to be taken over the entire nucleus. If the nucleus is spherically symmetric, then

$$F(q^2) = \frac{4\pi\hbar}{Zeq} \int \rho(r) r \sin\left(\frac{qr}{\hbar}\right) dr.$$

point-like nucleus; as q increases the oscillatory nature of the exponential in eq. 3.2 for an extended nucleus reduces $|F(q^2)|$ from 1 and the scattering is reduced. This is not unexpected; an extended electric charge has greater difficulty in taking up the momentum transfer than does the point-like arrangement of the same total charge.

2. Since we know the rough size of nuclei, $R < 10^{-14}$ m ($\simeq 10$ fm), we can now estimate the q needed to see a significant reduction in scattering intensity due to size. We would want $\boldsymbol{q}.\boldsymbol{R}/\hbar$ to be of order 1, hence $q \simeq \hbar/R$. Remember $\hbar c = 197$ MeV fm, so we see that $q > 20$ MeV/c. To reach this at 30° scattering requires incident electrons of 40 MeV. In fact this is hardly adequate since nuclear radii are somewhat less than 10 fm and we want to see detail with a resolution of better than 1 fm. Therefore we should be aiming for $q \simeq \hbar/(1 \text{ fm}) \simeq 200$ MeV/c. The first detailed measurements were made with electrons of 150 MeV but later work has increased the energies used to 500 MeV.

Now what does $F(q^2)$ look like? As an exercise you are asked to show (Problem 3.1) that the form factor for model I is

$$F(q^2) = \frac{3\{\sin x - x\cos x\}}{x^3}, \qquad x = qa/\hbar.$$

This looks rather unmanageable but in fact is the spherical Bessel function $j_1(x)$. The square of this function is the factor by which the point-like Mott differential cross-section is reduced. This factor is plotted in Fig. 3.3 for the case of $a = 4.1$ fm ($^{58}_{28}$Ni nucleus) and the abscissa is marked in units of q MeV/c and of θ degrees for 450 MeV incident electrons. We notice immediately the diffraction zeros near 27°, 48°, 69°, and 95°. This is typical of an object with sharp edges. In Fig. 3.4a we give the measured differential cross-section: it shows diffraction minima at q values expected from model I. On the same figure is a fitted curve using a model close to our model II. The softer edges of the distribution fill in the diffraction minima and give results closer to the data than the model I prediction.

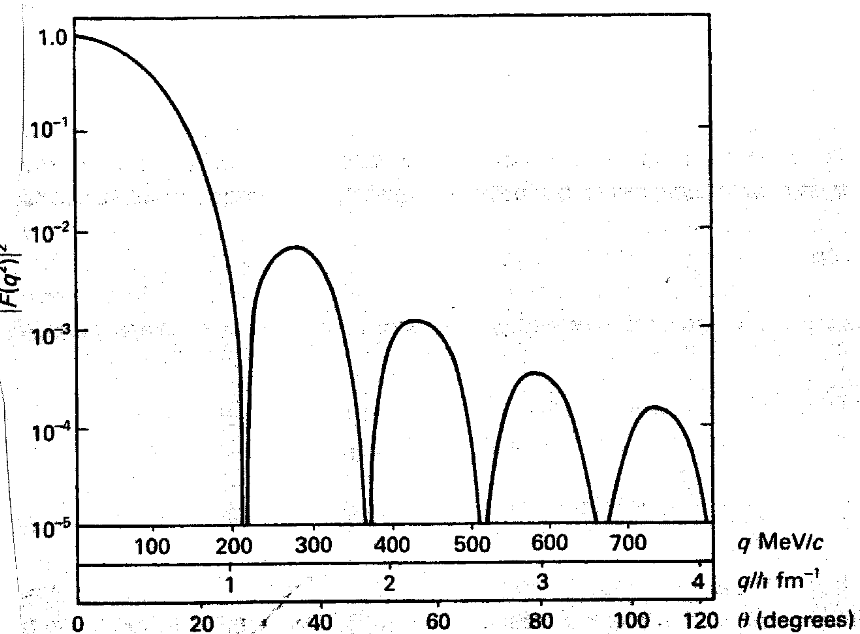
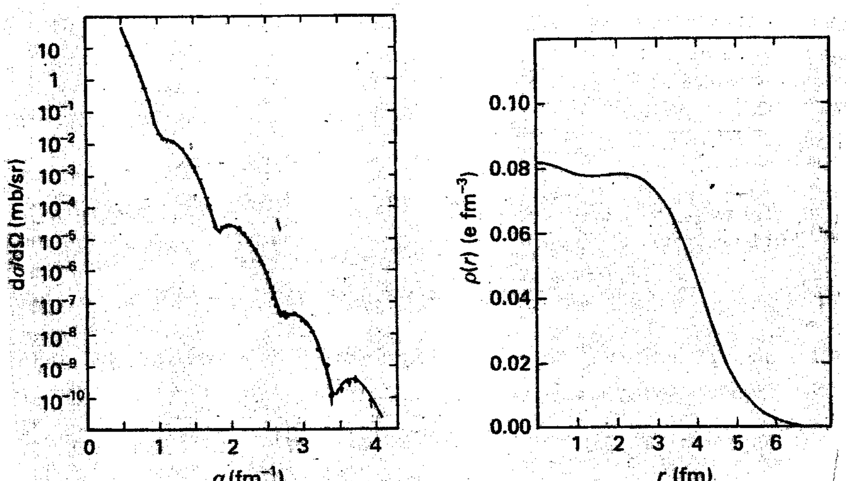
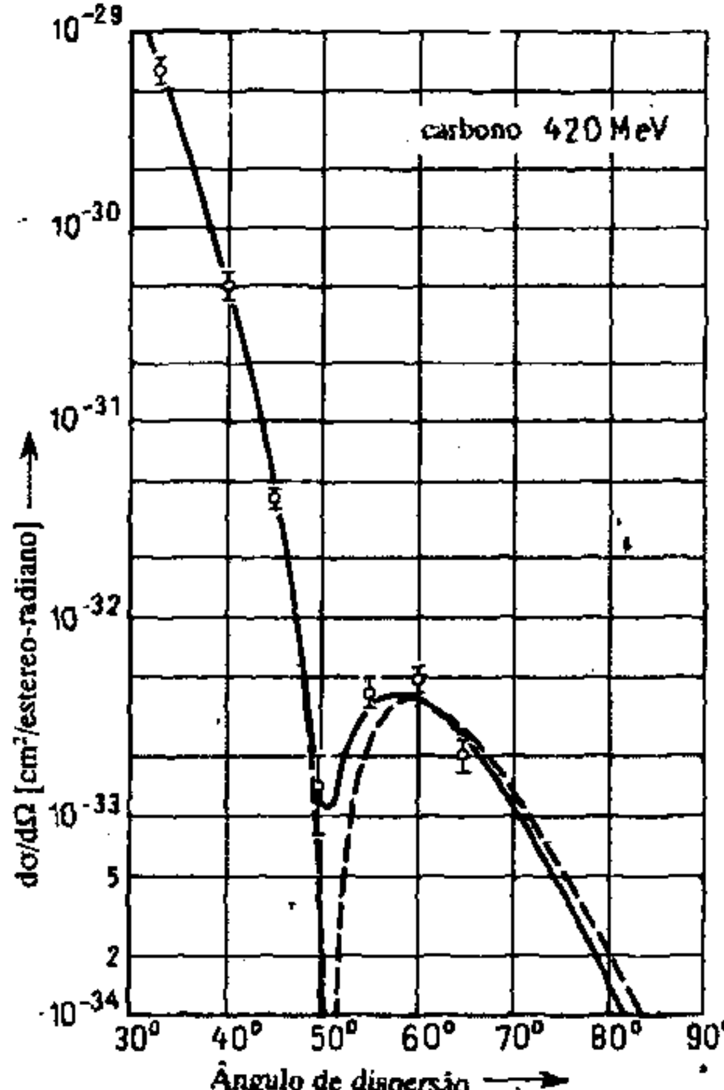


Fig. 3.3 The square of the form factor $|F(q^2)|^2$ as a function of q for a model I nucleus having $a = 4.1$ fm. The abscissa is also marked in inverse fermis ($q/\hbar$) and in degrees for an angle of scatter at a fixed nucleus for incident electrons of 450 MeV. Note that the ordinate is logarithmic.





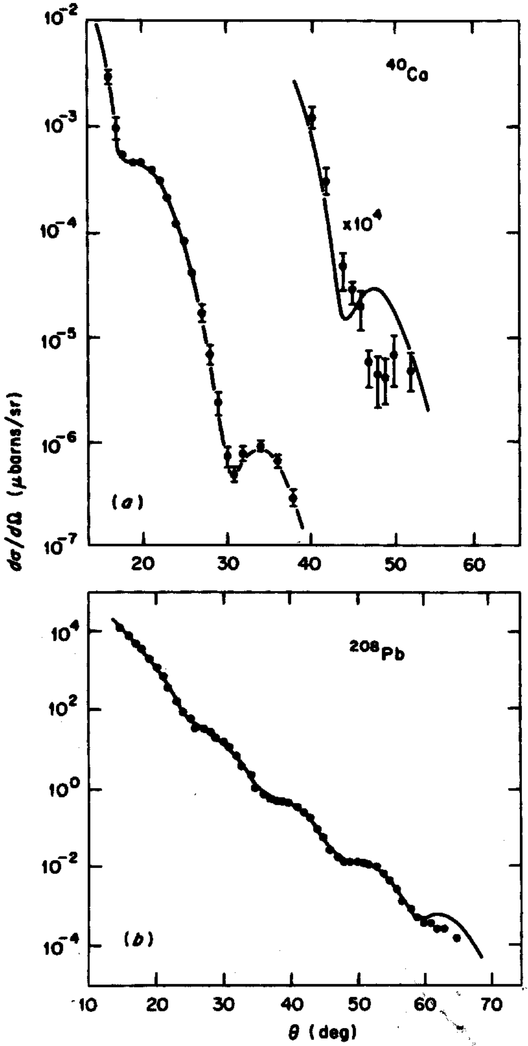


Figure 2.15 The differential cross-sections for electrons scattering elastically from (upper) ^{40}Ca at 750 MeV (after Bellicard *et al.*, 1967) and (lower) ^{208}Pb at 502 MeV. The curves are theoretical fits to the data (after Heisenberg *et al.*, 1969)

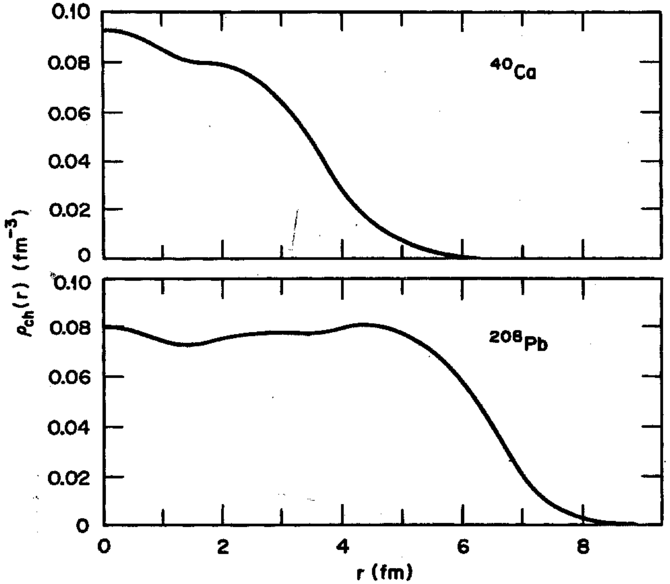


Figure 2.16 The charge distributions of ^{40}Ca and ^{208}Pb nuclei deduced by theoretical fits to the measurements such as those shown in *Figure 2.15*. The shapes at small radii are obtained by fitting the data for the larger angles (that is, for the larger momentum transfers.) (After Friar and Negele, 1973; Sinha *et al.*, 1973)

The charge distribution of the simplest nucleus of all, the proton, has been measured by electron scattering and found to have a root mean square radius of about 0.8 fm. This size can be related to the cloud of virtual mesons which surround the 'bare' proton. For the same reason, the neutron itself is found to have a charge distribution of finite extent; although its total charge is zero, it has a short-ranged distribution of positive charge and a longer-ranged distribution of negative charge, with a net root mean square radius of 0.36 fm.

In addition to the charge distribution we may also determine the distribution

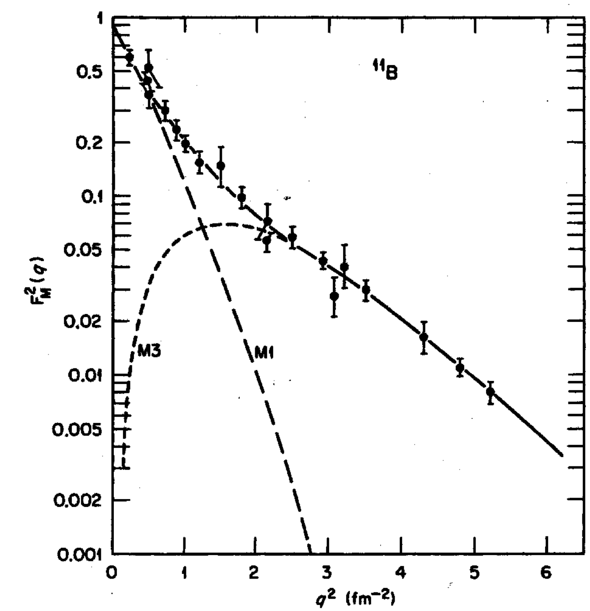
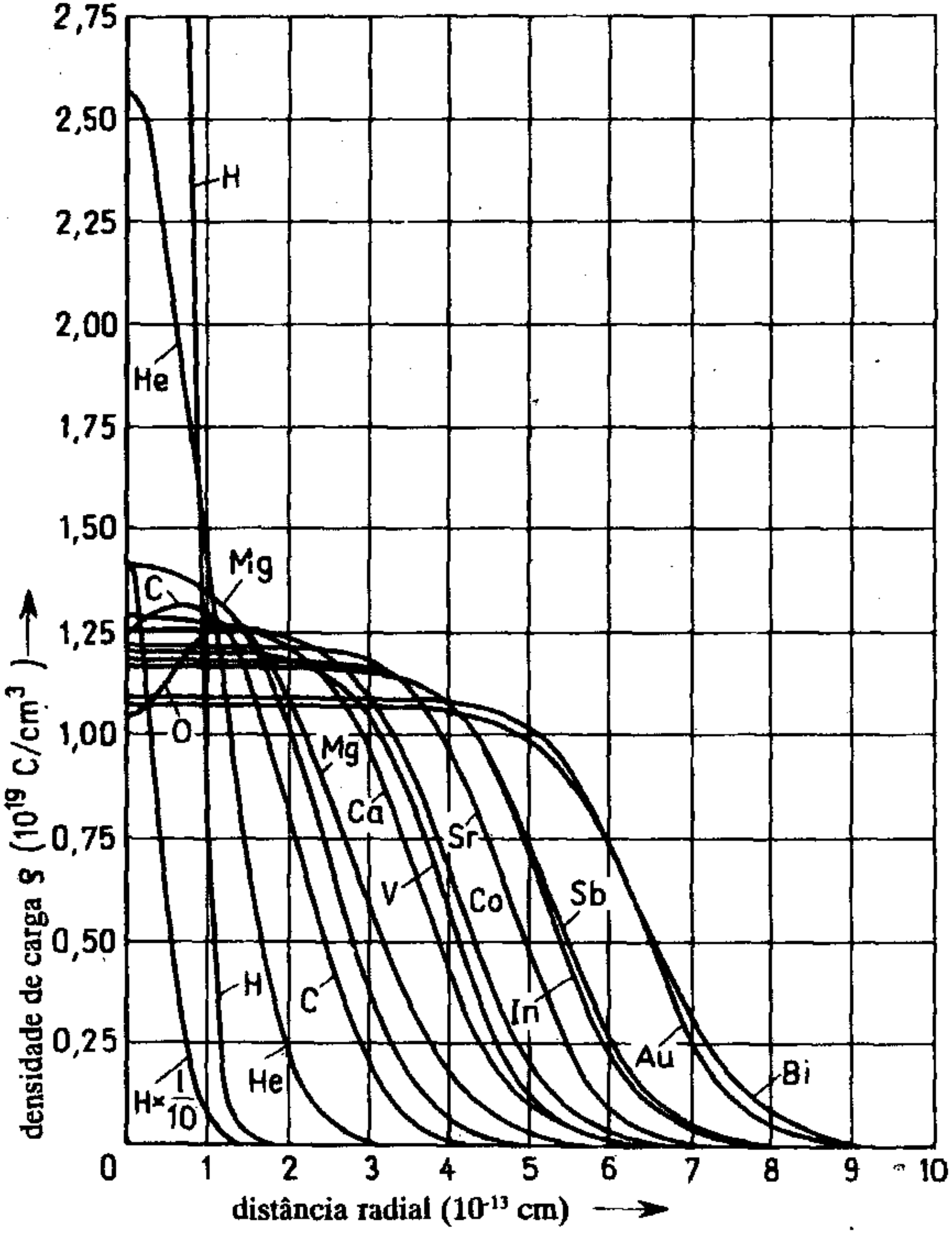
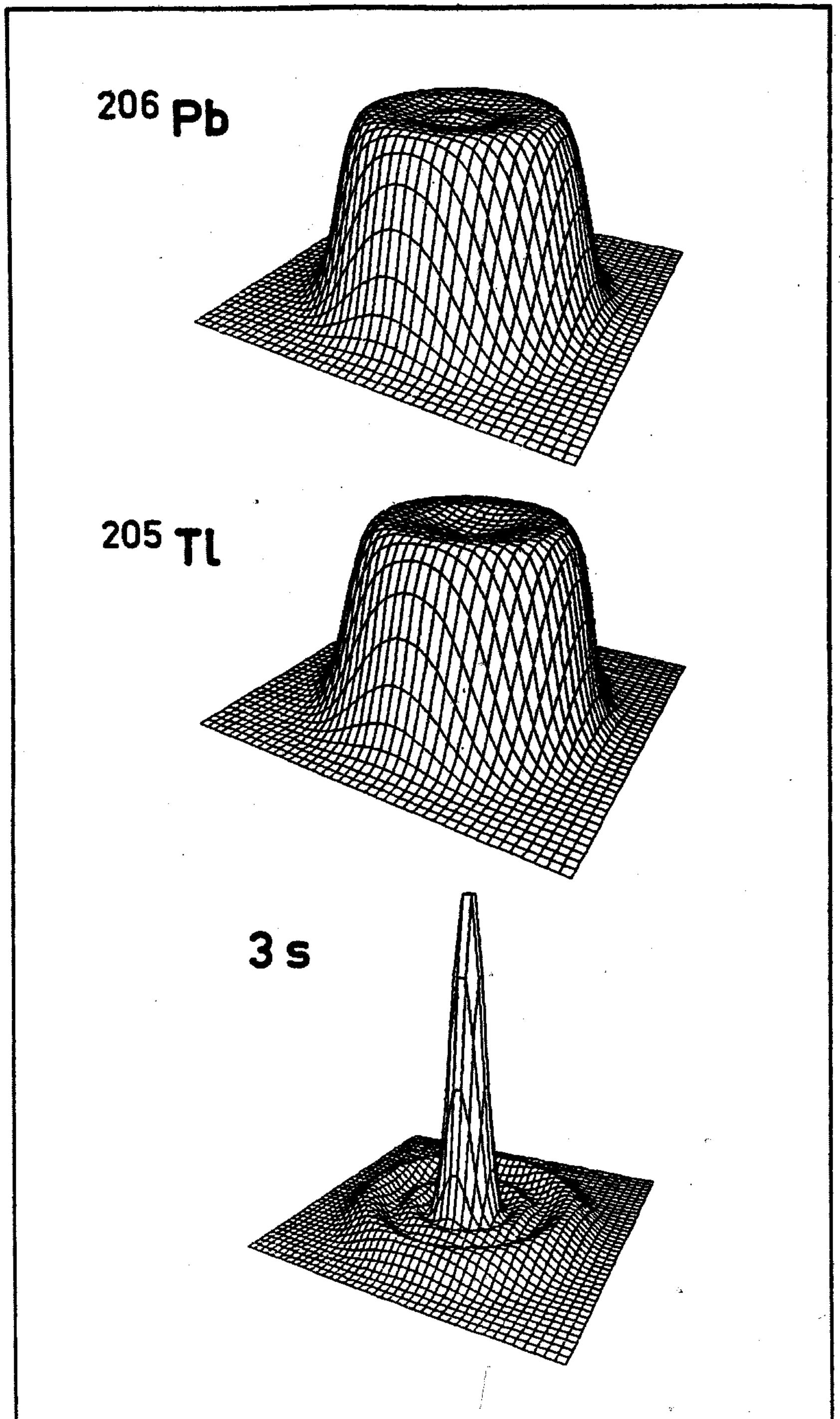


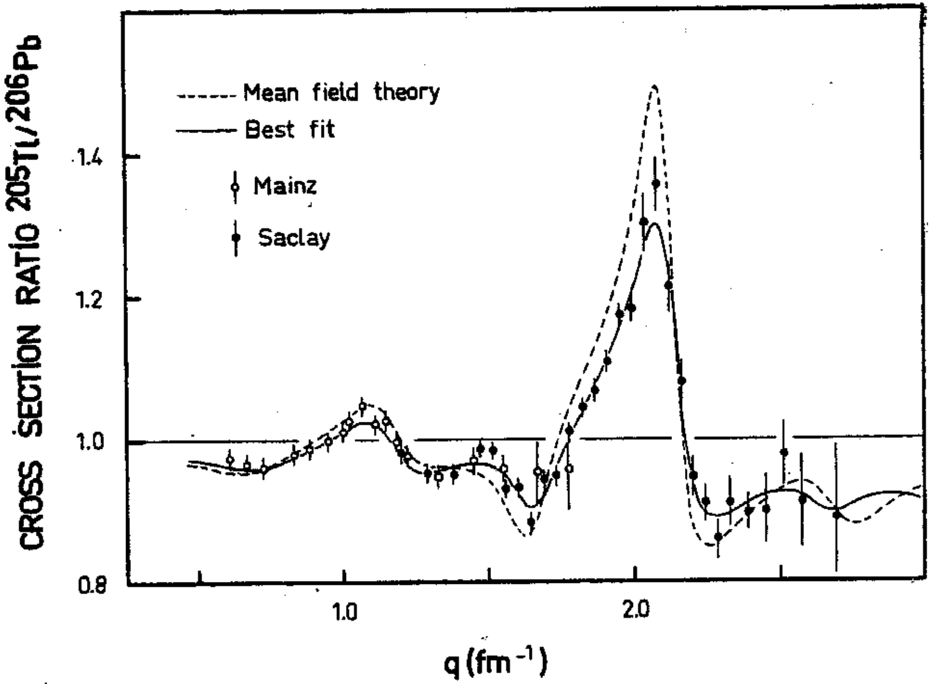
Figure 2.17 Dipole (M1) and octupole (M3) magnetic form factors for ^{11}B deduced from electron-scattering measurements. (After Rand *et al.*, 1966)



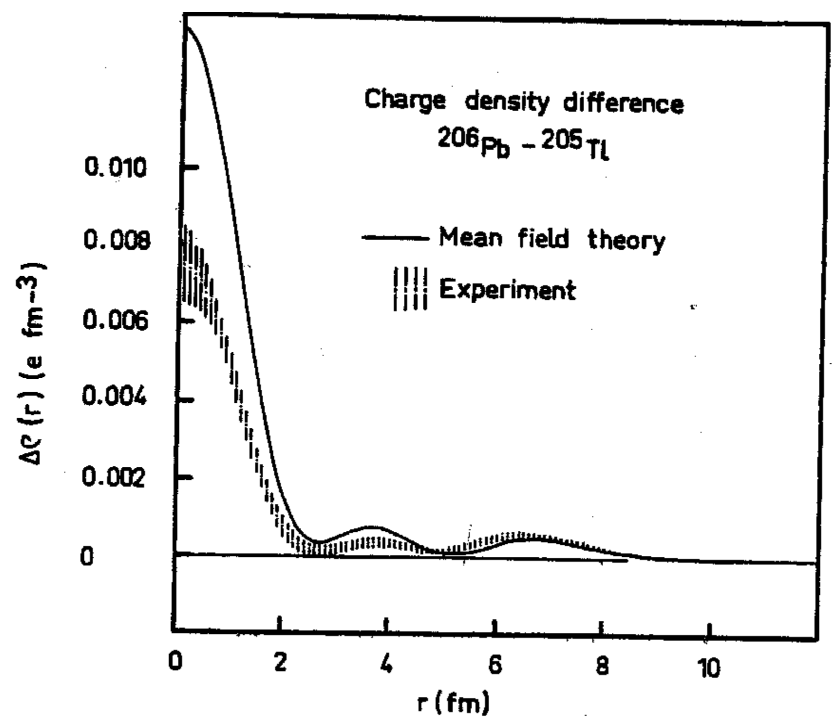
2,75
2,50
2,25
2,00
1,75
1,50
1,25
1,00
0,75
0,50
0,25
0
1
2
3
4
5
6
7
8
9
10
densidade de carga ϱ (10^19 C/cm^3) ⟶
distância radial (10^-13 cm) ⟶
H
He
Mg
C
O
Mg
Ca
Sr
V
Co
Sb
In
Au
Bi
H
C
He
H× 1/10



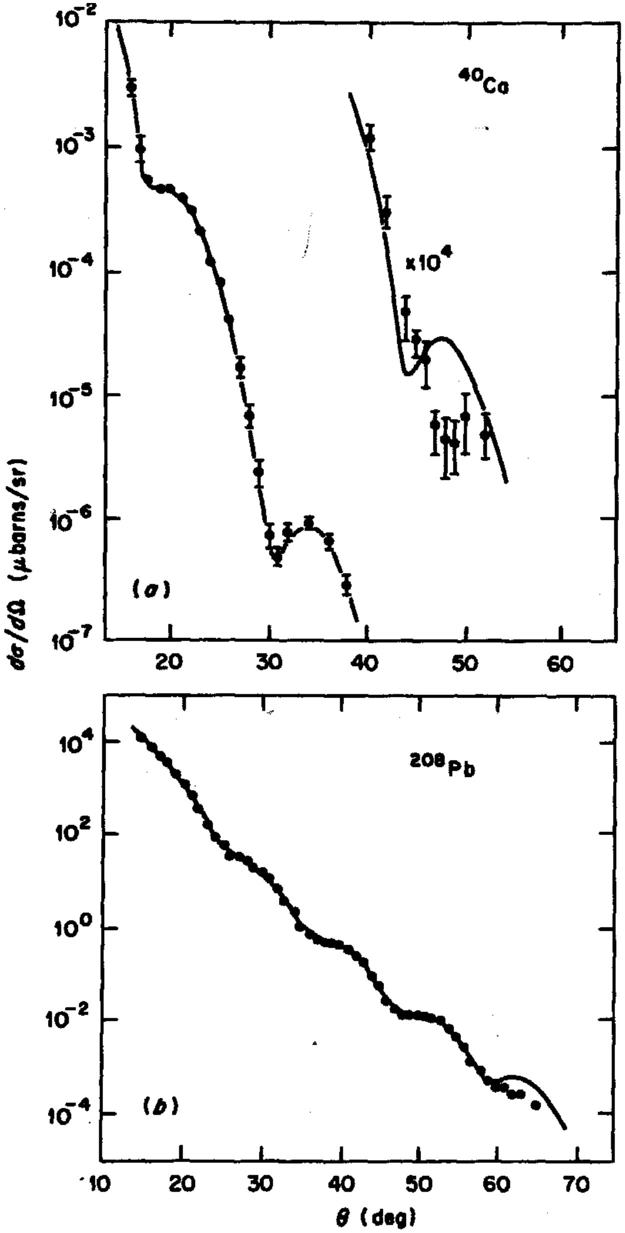
^{206}Pb
^{205}Tl
3 s

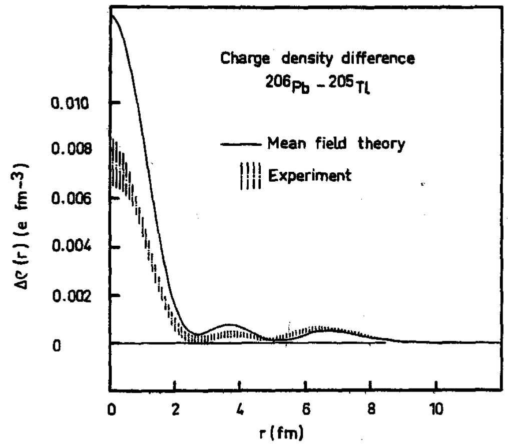


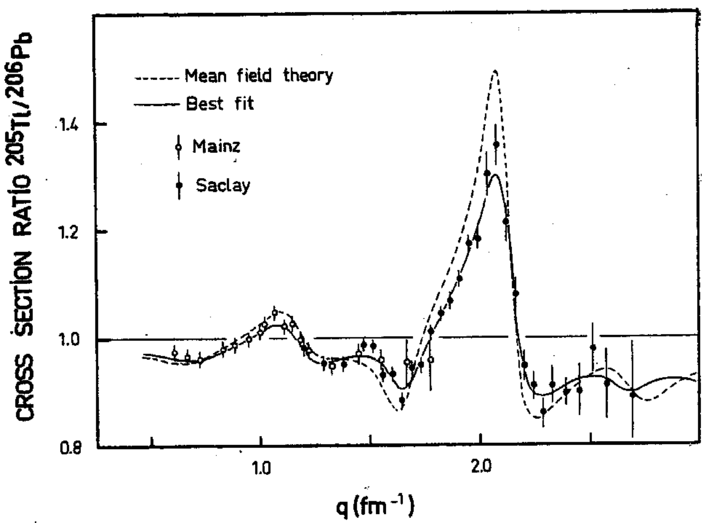
Mean field theory
Best fit
Mainz
Saclay
CROSS SECTION RATIO ^{205}Tl/^{206}Pb
1.4
1.2
1.0
0.8
1.0
2.0
q (fm^{-1})



Charge density difference
^{206}Pb – ^{205}Tl
Mean field theory
Experiment
$\Delta\varrho$ (r) (e fm^{-3})
0.010
0.008
0.006
0.004
0.002
0
0
2
4
6
8
10
r (fm)


^{40}Ca
x10^4
(a)
$d\sigma/d\Omega$ (μbarns/sr)
^{208}Pb
(b)
θ (deg)


Charge density difference
$^{206}Pb - ^{205}Tl$
Mean field theory
Experiment
$\Delta\rho(r)$ (e fm-3)
r (fm)


Mean field theory
Best fit
Mainz
Saclay
CROSS SECTION RATIO $^{205}Tl/^{206}Pb$
q (fm^{-1})

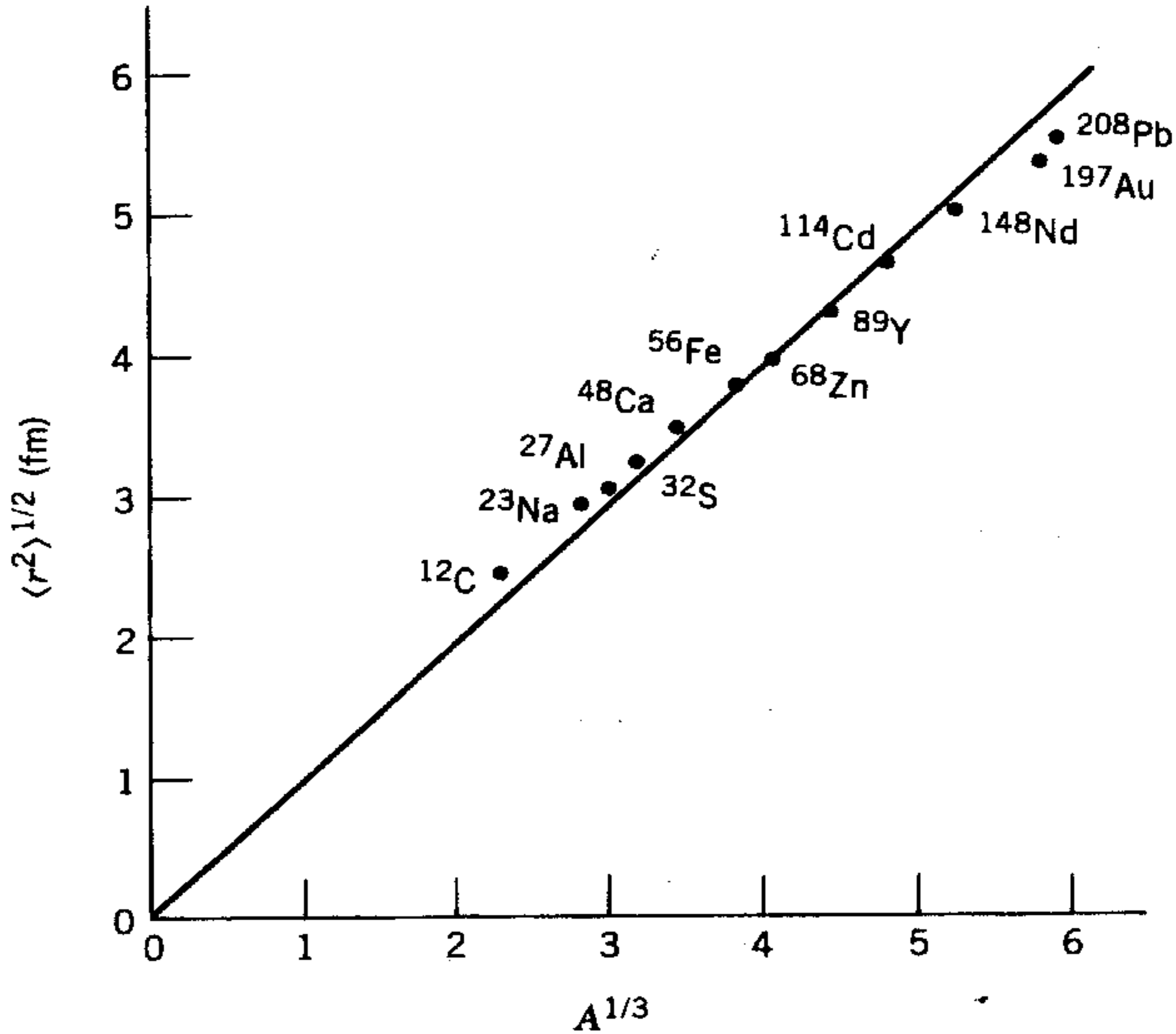
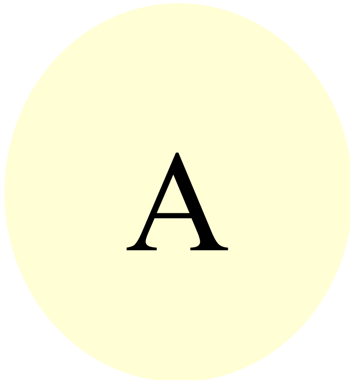
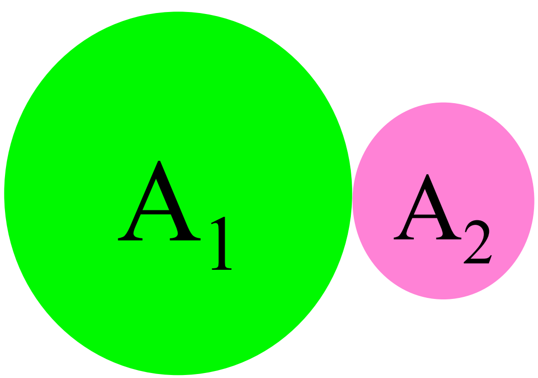


Figure 3.5 The rms nuclear radius determined from electron scattering experiments. The slope of the straight line gives $R_0 = 1.23$ fm. (The line is not a true fit to the data points, but is forced to go through the origin to satisfy the equation $R = R_0 A^{1/3}$.) The error bars are typically smaller than the size of the points (± 0.01 fm). More complete listings of data and references can be found in the review of C. W. de Jager et al., *Atomic Data and Nuclear Data Tables* **14**, 479 (1974).

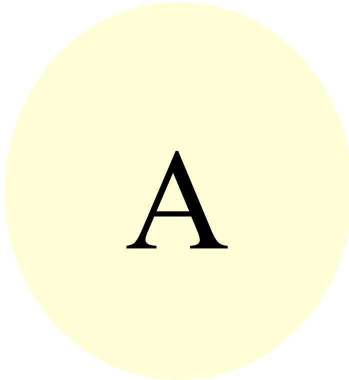


$$R = r_o A^{1/3}$$

$r_0 \sim 1.25$ **fm**

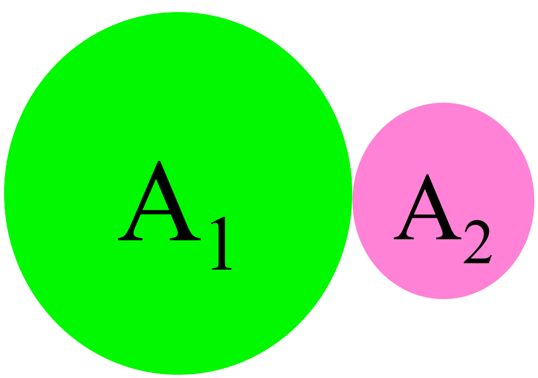


$$R = r_o (A_1^{1/3} + A_2^{1/3})$$

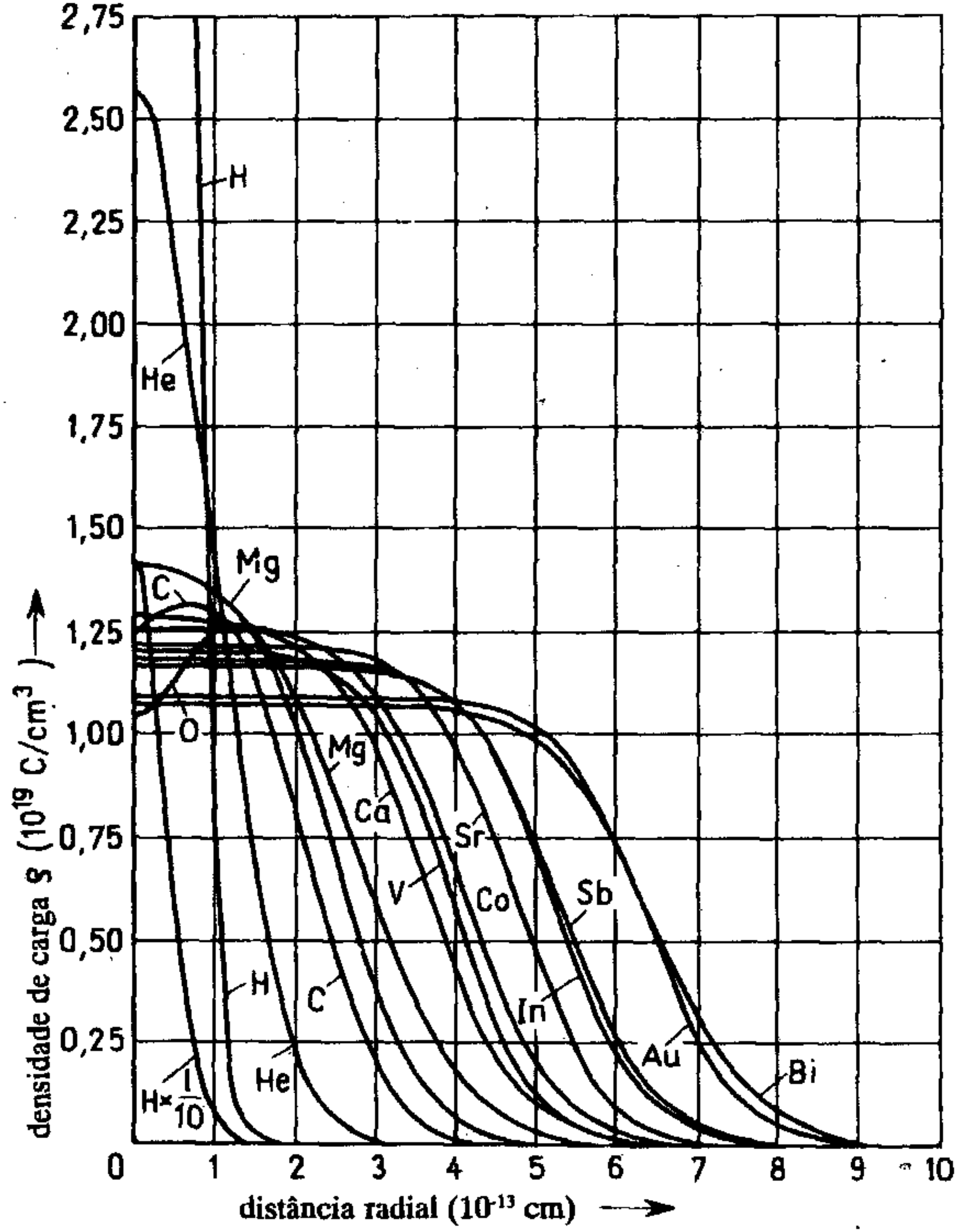


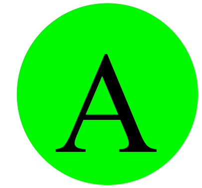
$$R = r_o A^{1/3}$$

$r_0 \sim 1.25\text{fm}$



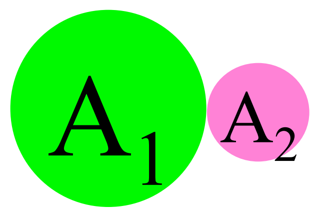
$$R = r_o (A_1^{1/3} + A_2^{1/3})$$





$$R = r_o A^{1/3}$$

$r_0 \sim 1.25$fm



$$R = r_o (A_1^{1/3} + A_2^{1/3})$$

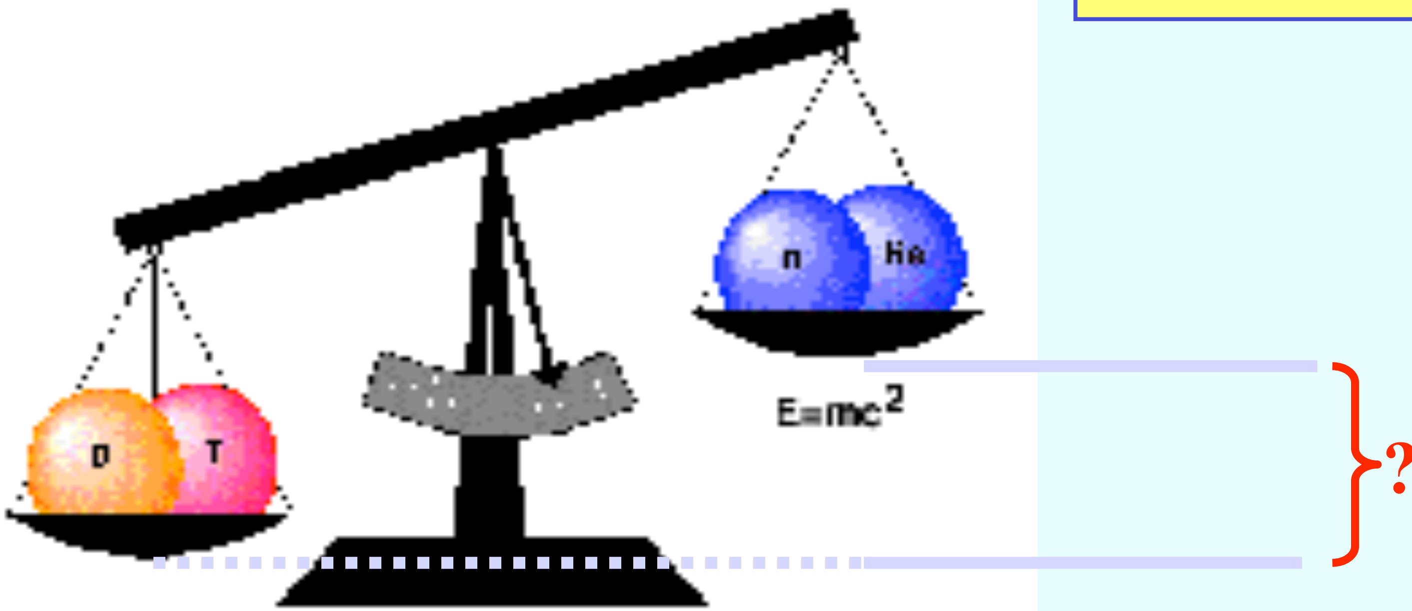
massa dos nucleos

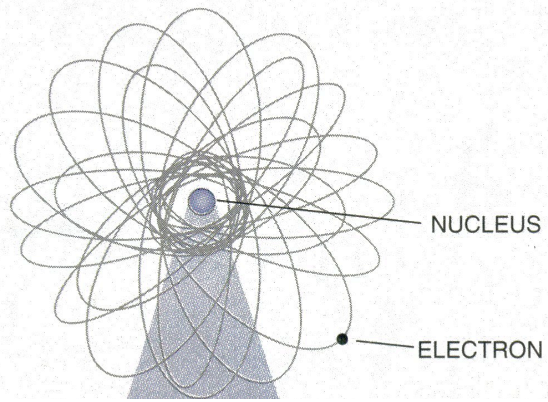
$d = p + n$

$t = p + n + n$

$4He = p + p + n + n$

n =1.00866 u.m.a.
p = 1.0079 u.m.a.
d = 2.01410 u.m.a.
t = 3.01860 u.m.a.
^{4}He = 4. 00260 u.m.a.
^{6}Li = 6. 01512 u.m.a.
^{12}C = 0.00000 u.m.a.





$M_{(proton)} = M_p = 938.27$ MeV

$M_{(neutron)} = M_n = 939.56$ MeV

$M_{(eletron)} = M_e = 0.511$ MeV

$M_{(\text{átomo de Hidrogênio})} = M_H = 938.58$ MeV

$M_{(16O)} = 14\ 899.17$ MeV

$M_p + M_e = 938.27 + 0.511 = 938.78$ MeV

$8M_p + 8M_n = 15\ 021.64$ MeV

$p = (u + u + d)$

$M_{(u)} \approx 3$ MeV

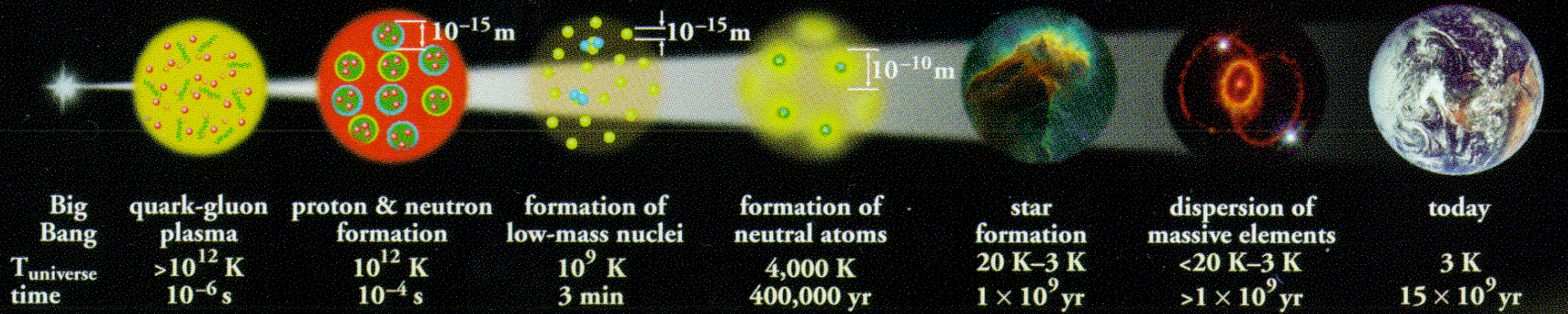
$M_{(d)} \approx 6$ MeV

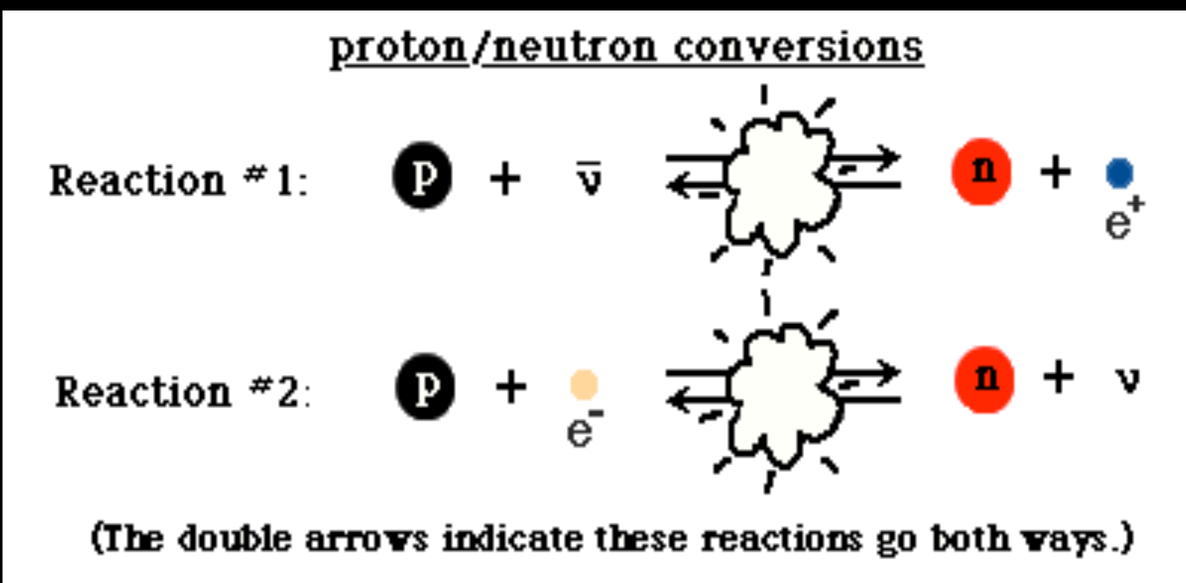
$\Delta M = M_H = 938.58$ MeV

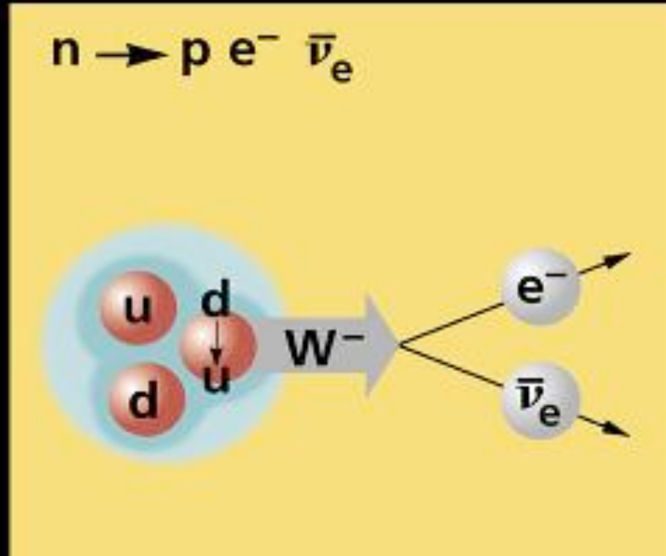
$\Delta M = 122.47$ MeV

Expansion of the Universe

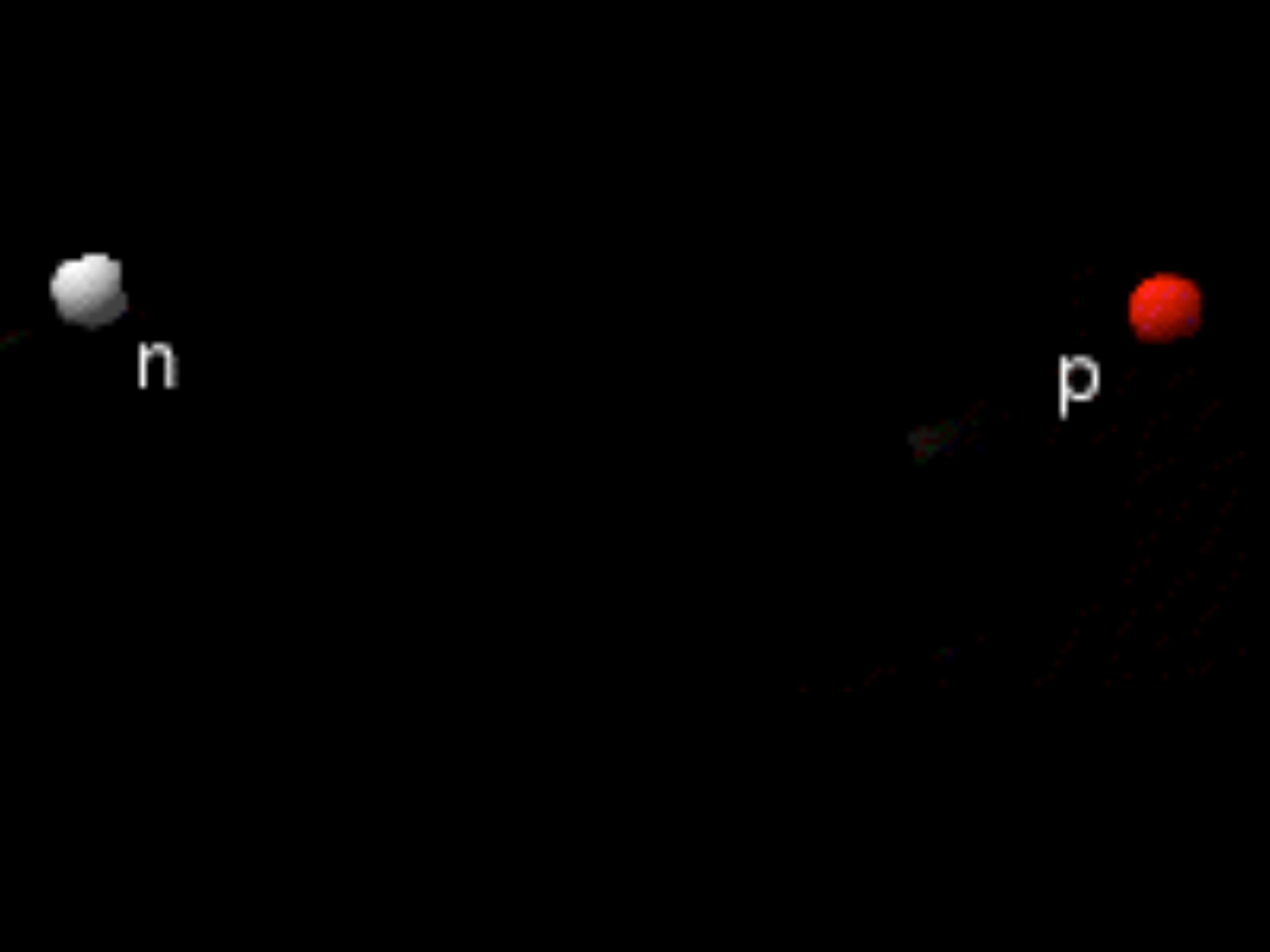
After the Big Bang, the universe expanded and cooled. At about 10^{-6} second, the universe consisted of a soup of quarks, gluons, electrons, and neutrinos. When the temperature of the Universe, $T_{universe}$, cooled to about 10^{12} K, this soup coalesced into protons, neutrons, and electrons. As time progressed, some of the protons and neutrons formed deuterium, helium, and lithium nuclei. Still later, electrons combined with protons and these low-mass nuclei to form neutral atoms. Due to gravity, clouds of atoms contracted into stars, where hydrogen and helium fused into more massive chemical elements. Exploding stars (supernovae) form the most massive elements and disperse them into space. Our earth was formed from supernova debris.



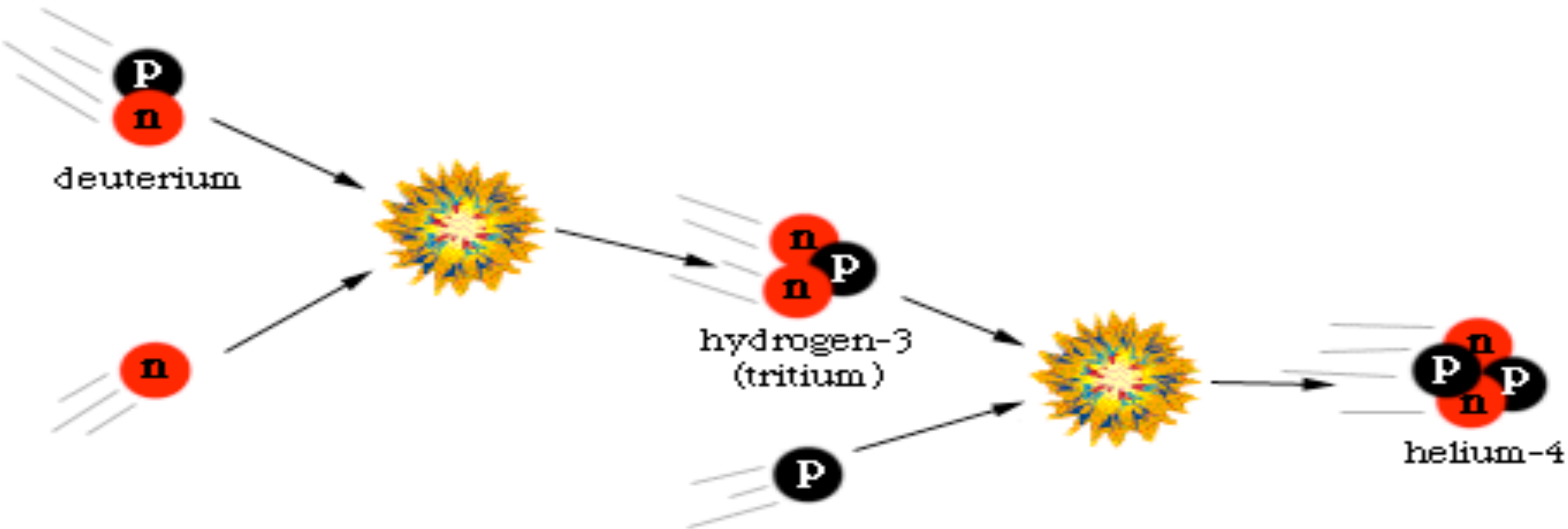




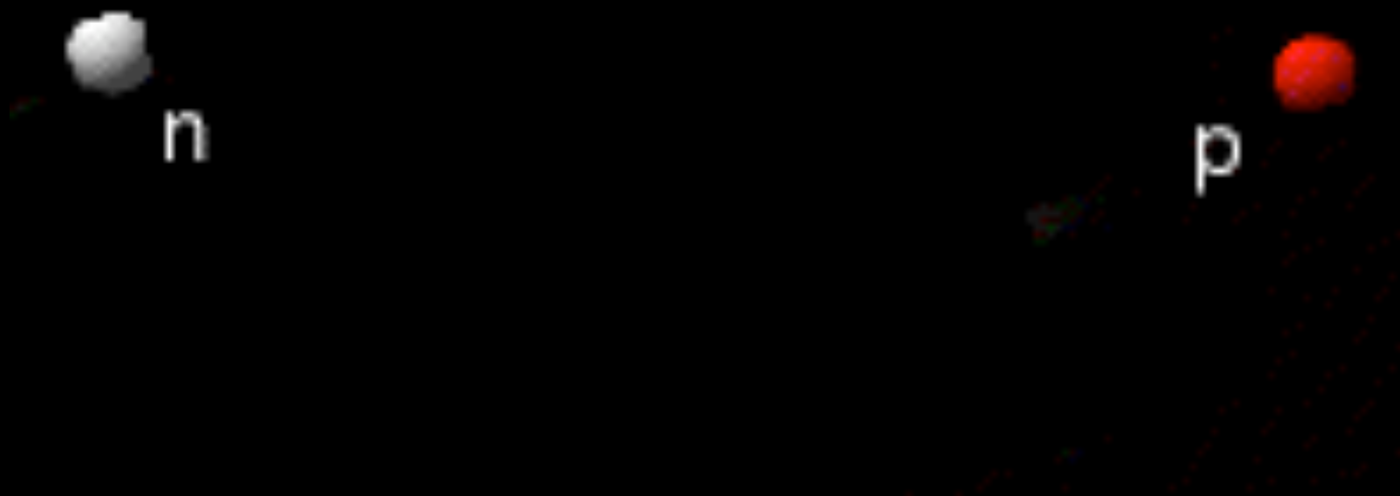
A neutron decays to a proton, an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β decay.


n
p

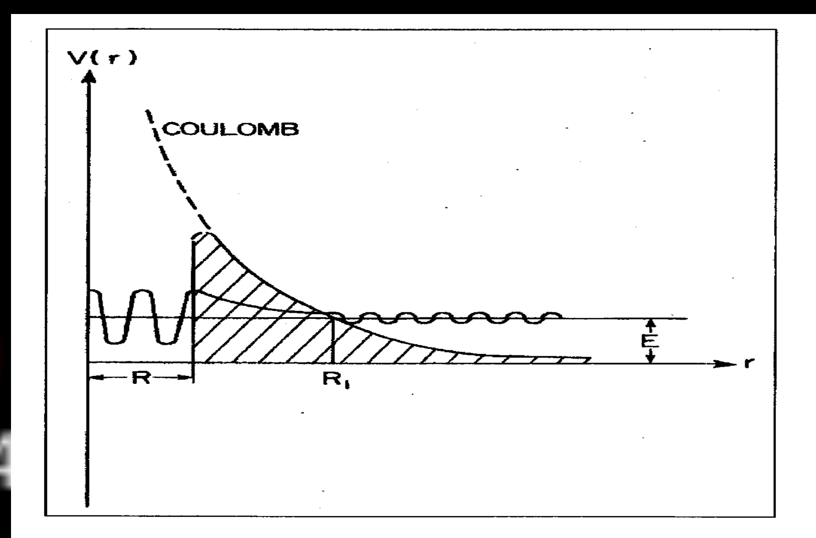

p
^2H

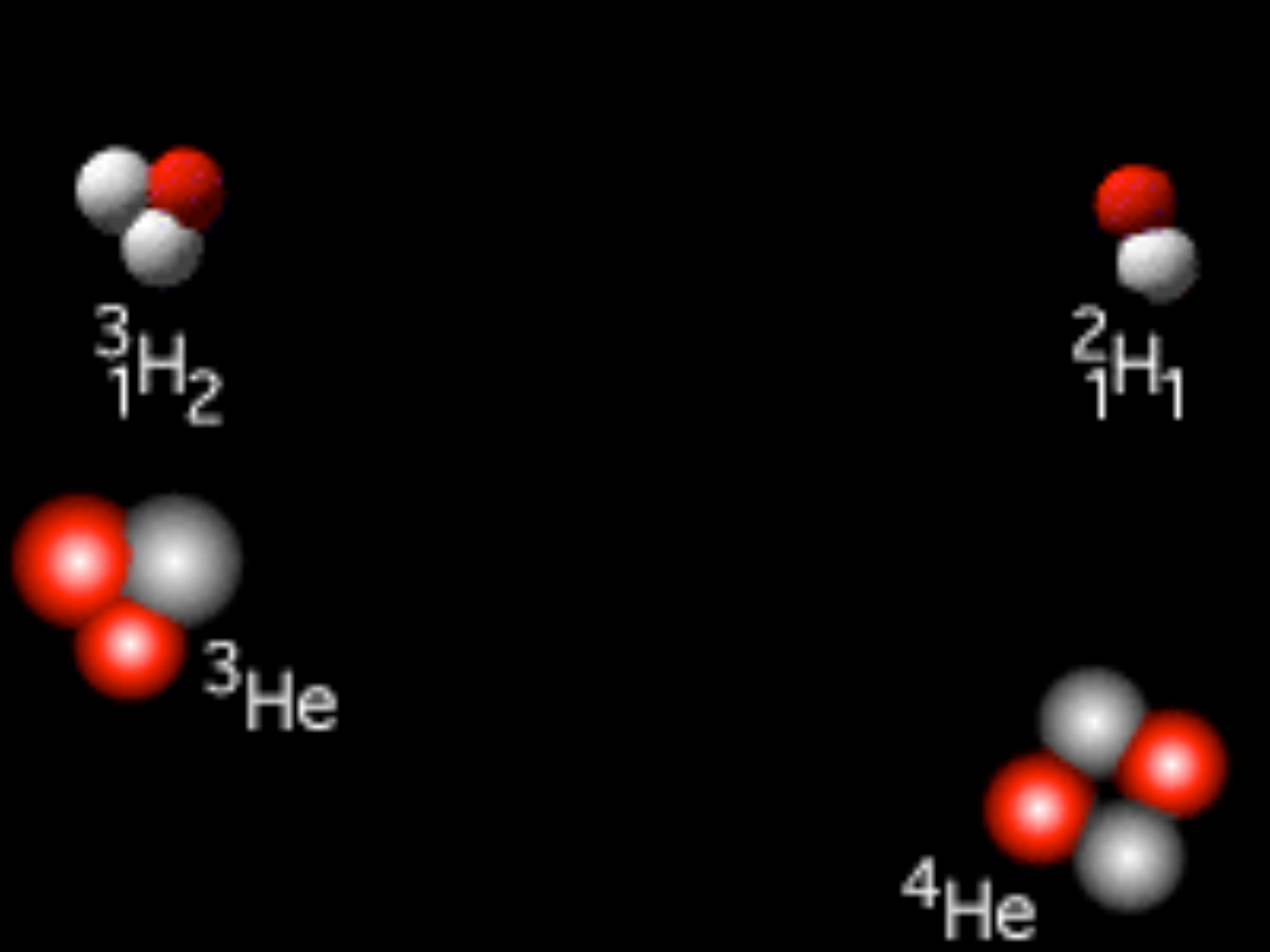

p
n
deuterium
n
n
p
n
hydrogen-3
(tritium)
p
n
p
p
n
helium-4

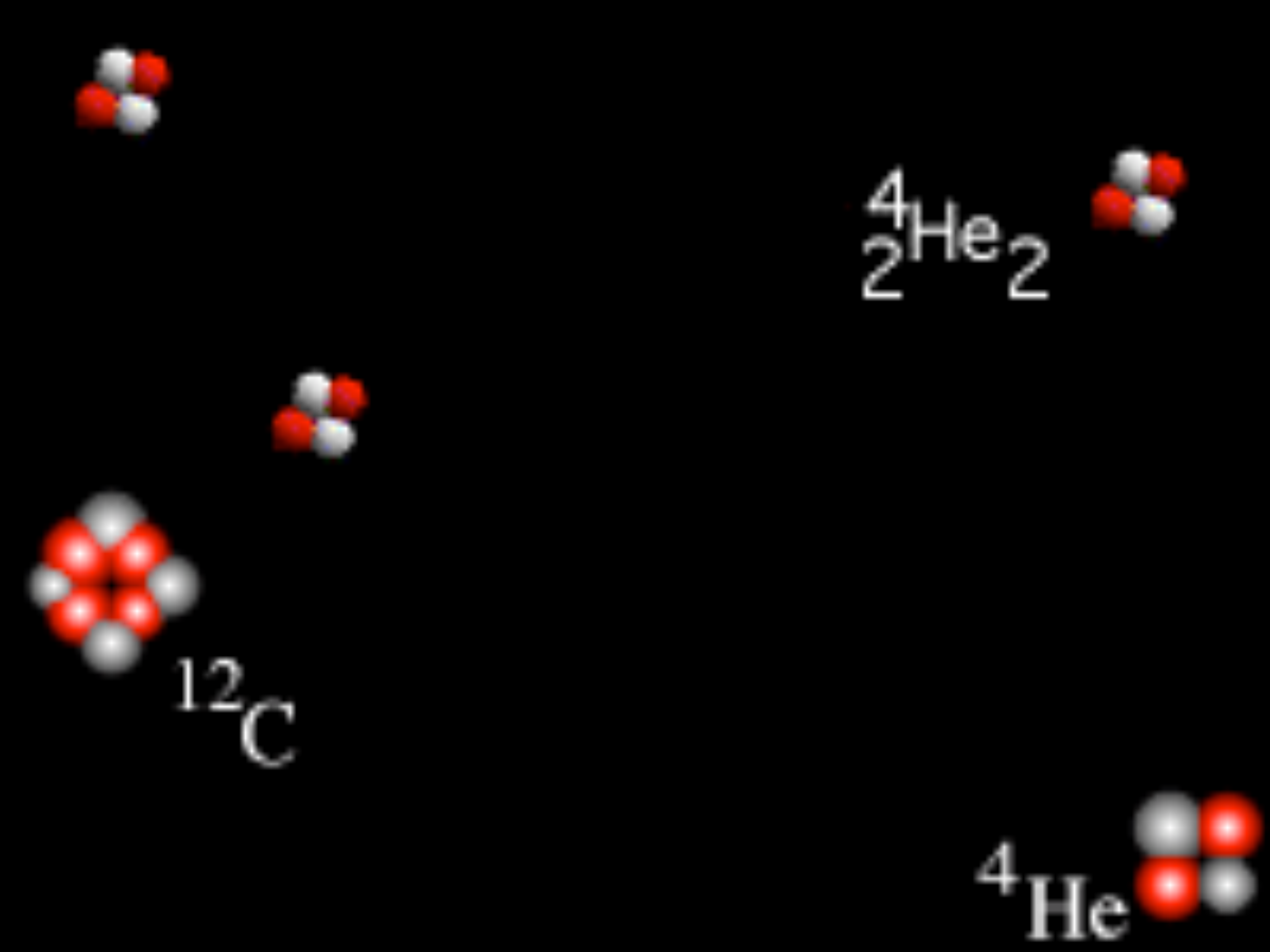
ESPALHAMENTO

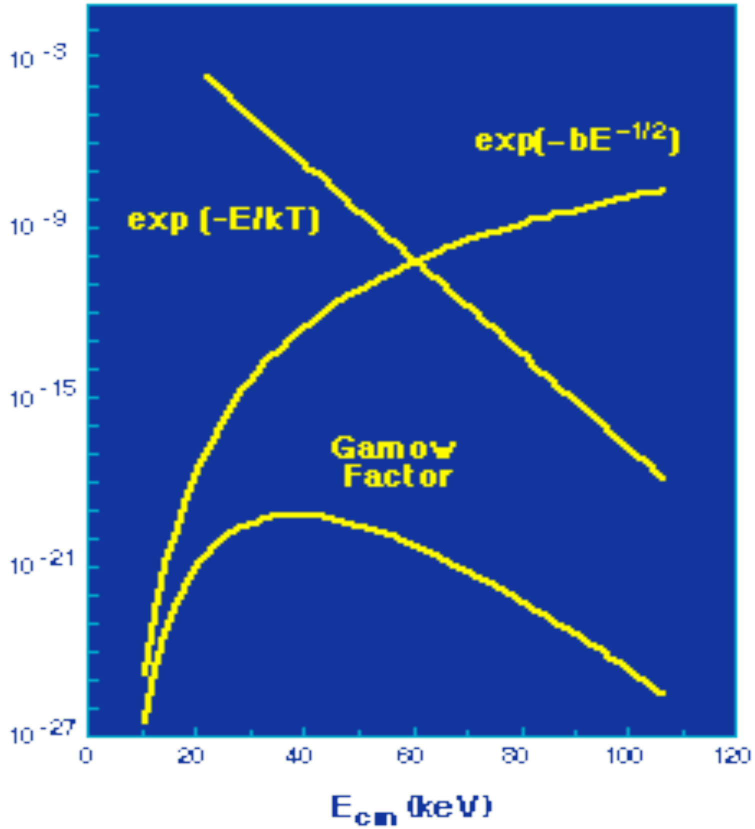


TUNELAMENTO

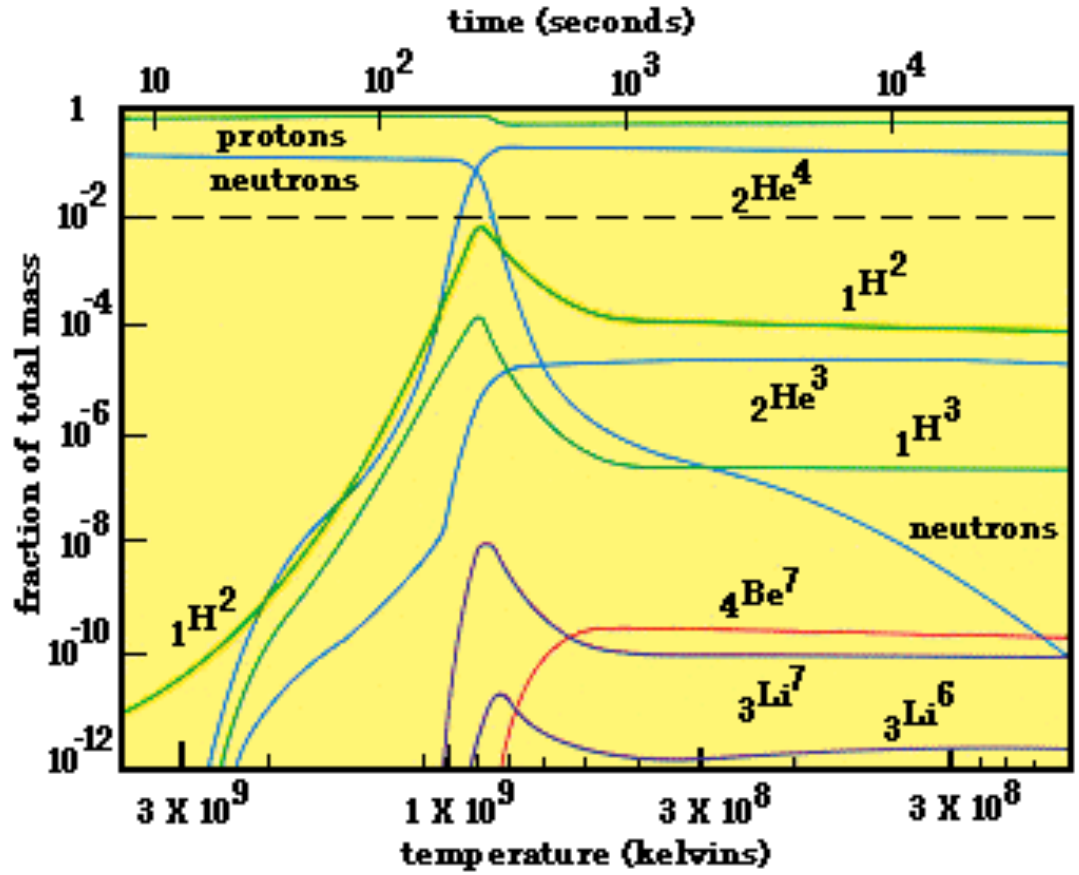



$^{3}_{1}H_{2}$
$^{2}_{1}H_{1}$
^{3}He
^{4}He

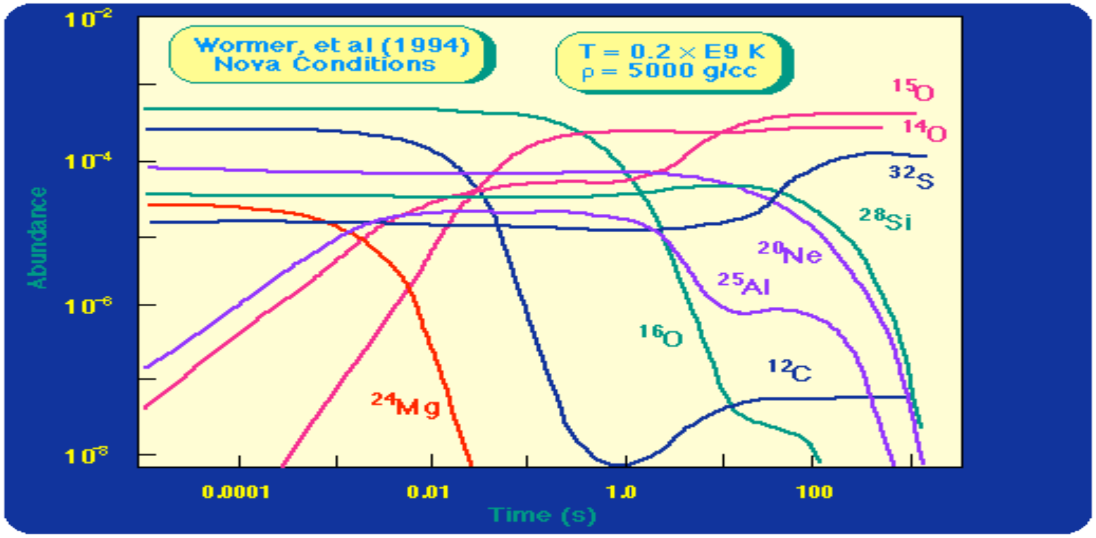

$^{4}_{2}He_{2}$
^{12}C
^{4}He



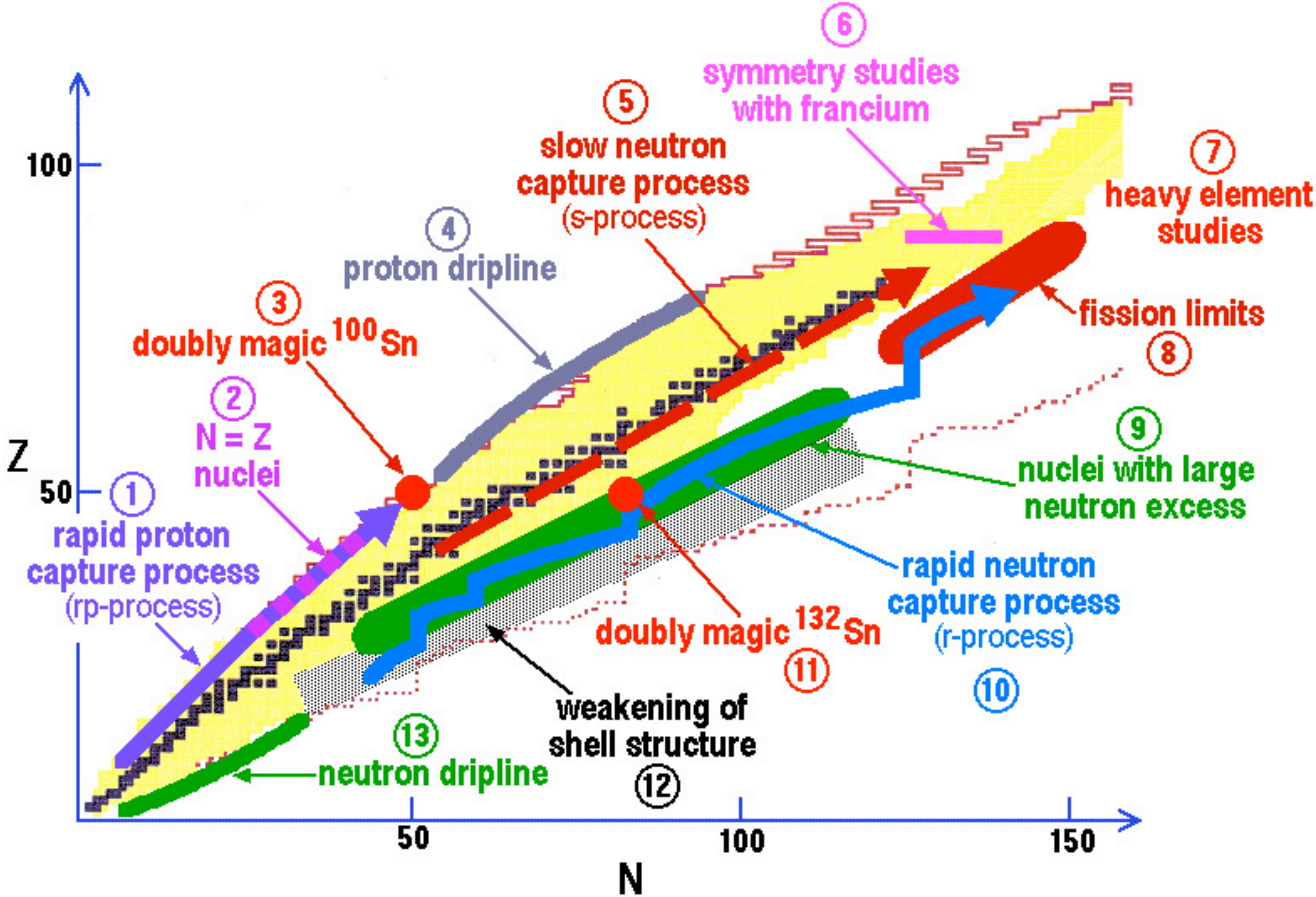
exp[-bE^-1/2]
exp [-E/kT]
Gamow Factor
E_cm (keV)



time (seconds)
protons
neutrons
fraction of total mass
temperature (kelvins)



Wormer, et al (1994)
Nova Conditions
T = 0.2 × E9 K
ρ = 5000 g/cc
Abundance
Time (s)



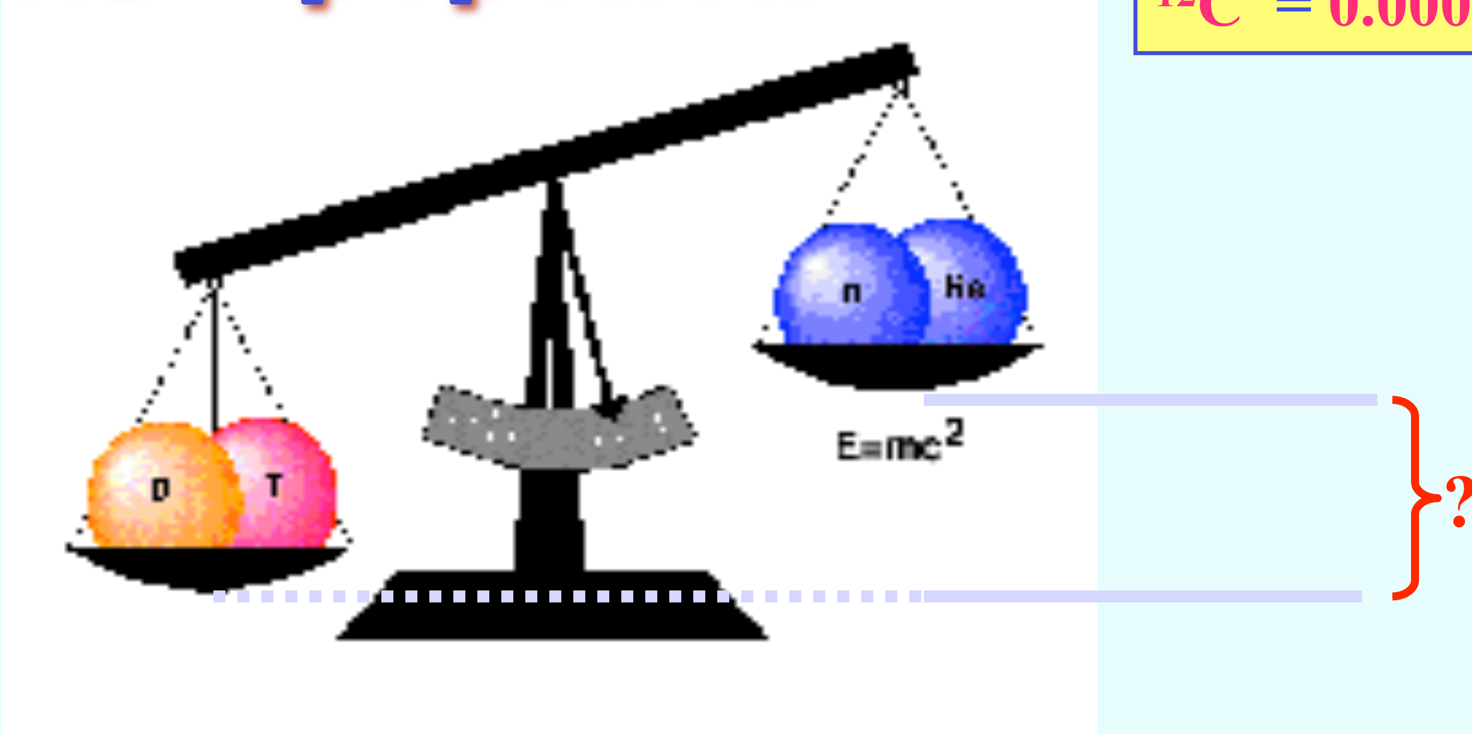
1
rapid proton
capture process
(rp-process)
2
N = Z
nuclei
3
doubly magic ^{100}Sn
4
proton dripline
5
slow neutron
capture process
(s-process)
6
symmetry studies
with francium
7
heavy element
studies
fission limits
8
9
nuclei with large
neutron excess
rapid neutron
capture process
(r-process)
10
doubly magic ^{132}Sn
11
weakening of
shell structure
12
13
neutron dripline
Z
100
50
N
50
100
150


n
235U
92
143

d = p+n

t = p+n+n

4He = p+p+n+n



n =1.00866 u.m.a.

p = 1.0079 u.m.a.

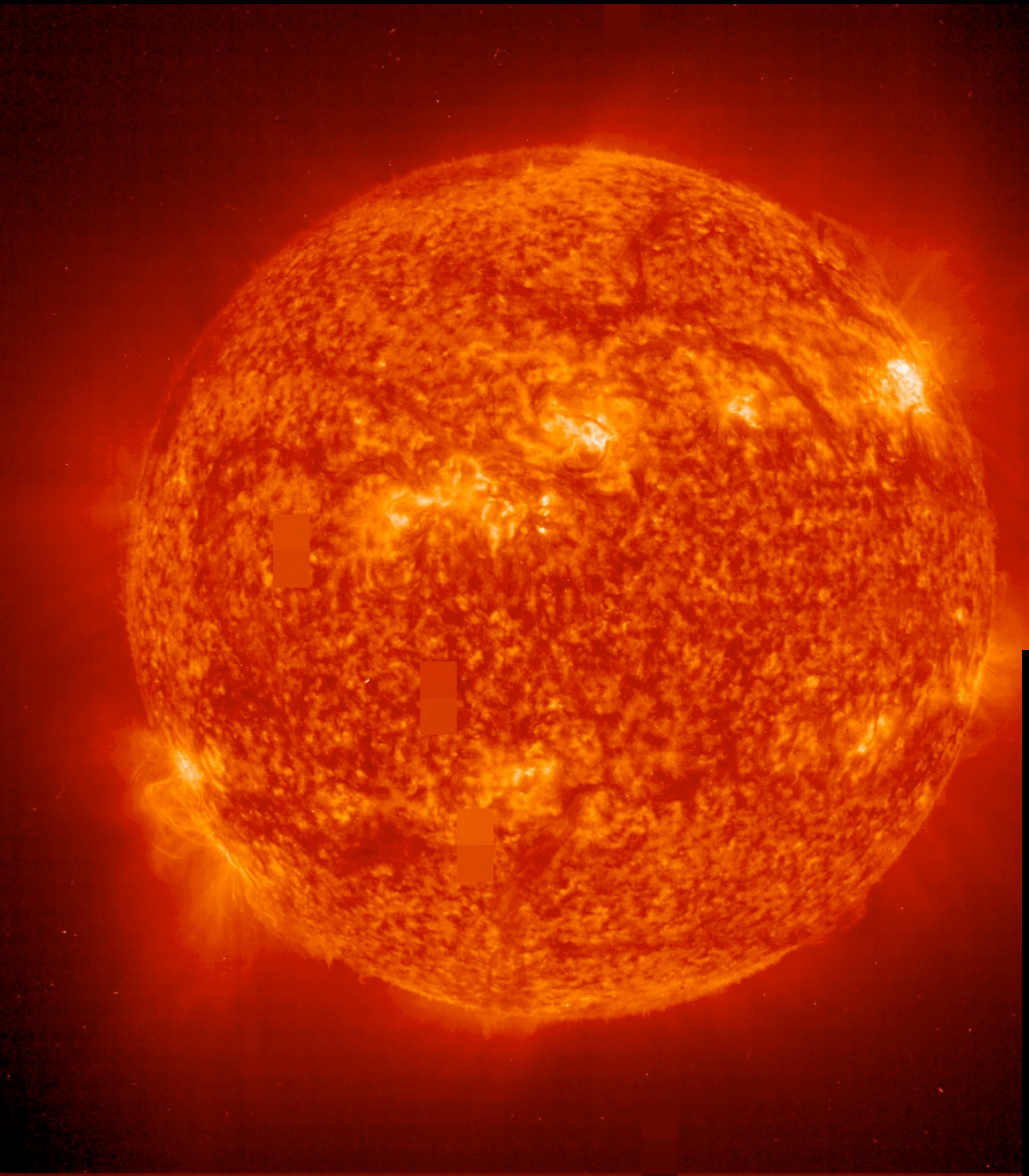
d = 2.01410 u.m.a.

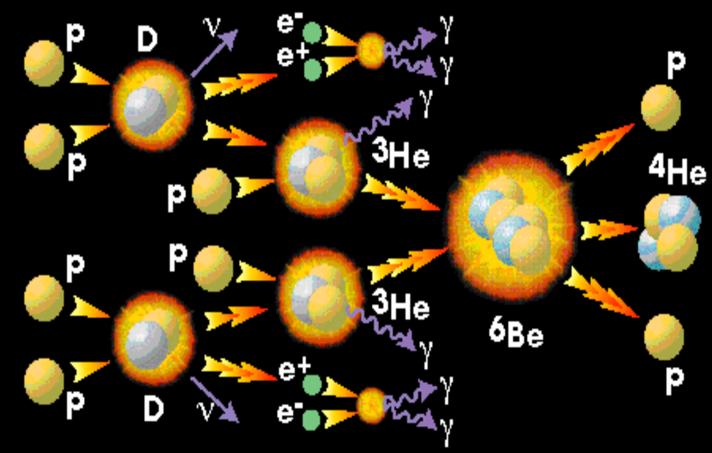
t = 3.01860 u.m.a.

^{4}He = 4. 00260 u.m.a.

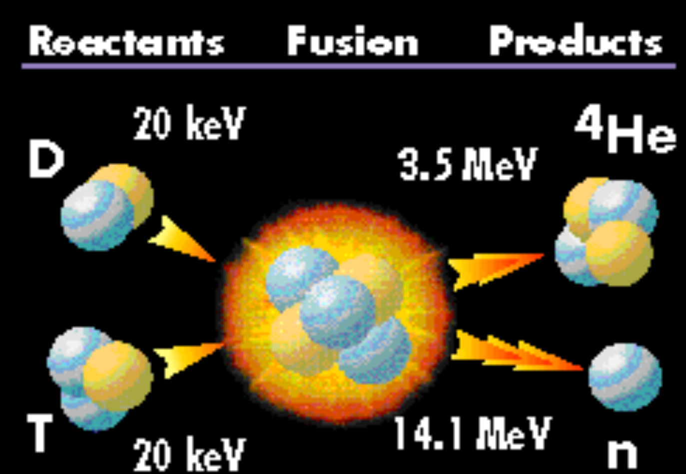
^{6}Li = 6. 01512 u.m.a.

^{12}C = 0.00000 u.m.a.

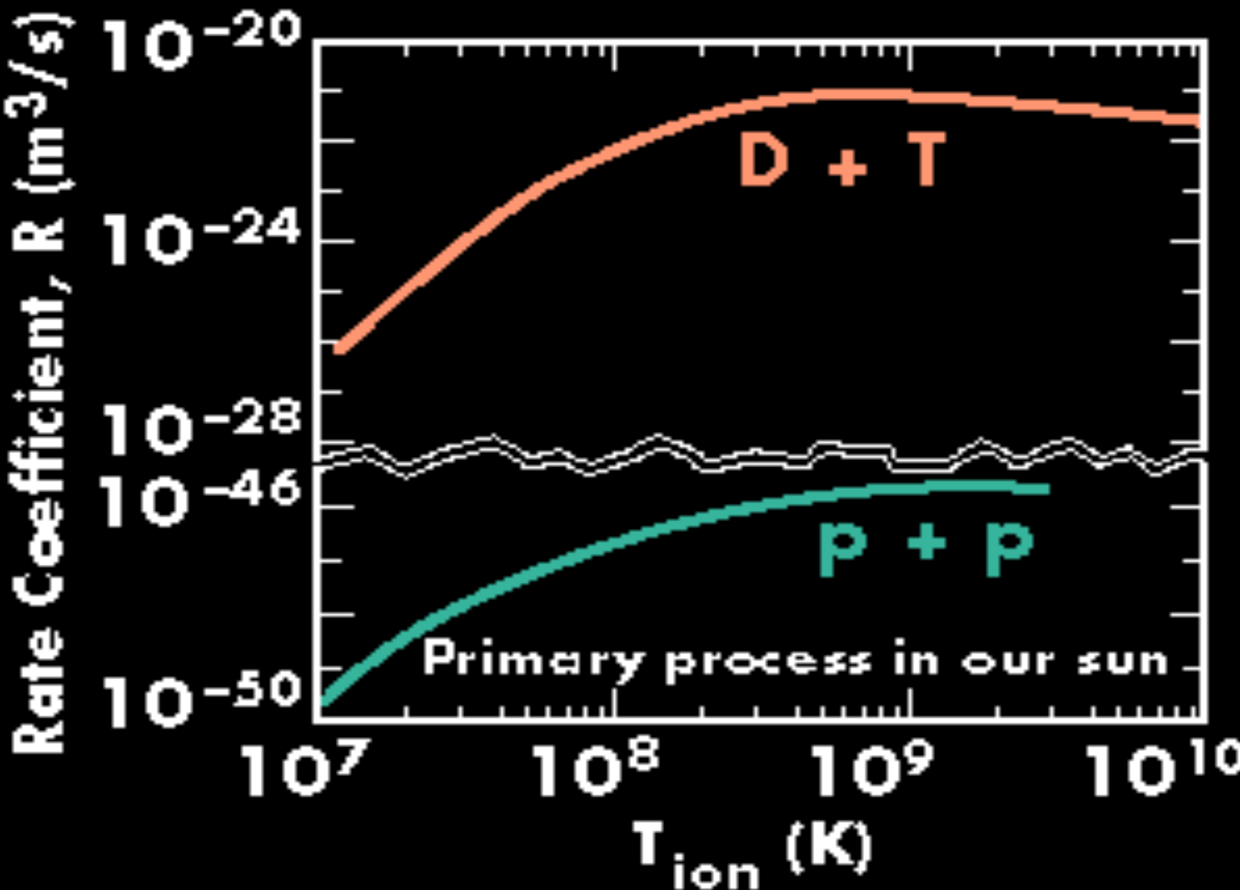




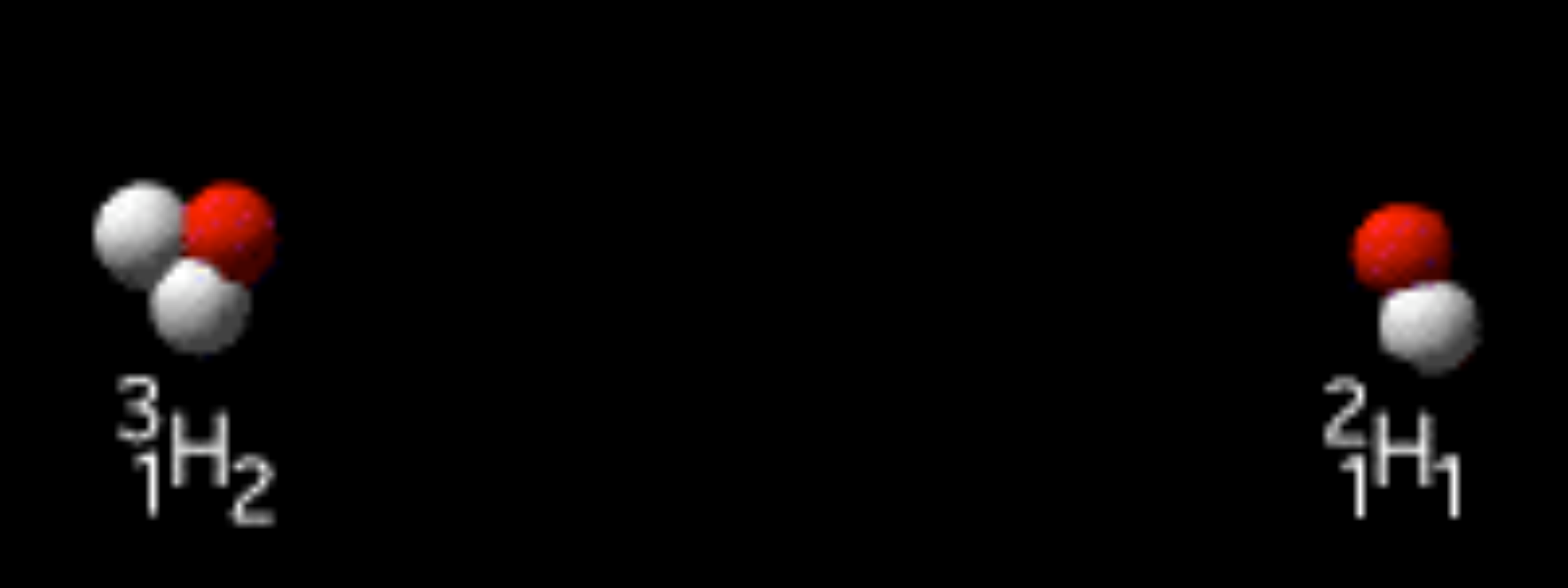
p
D
ν
e-
e+
γ
γ
p
γ
3He
p
4He
p
p
6Be
3He
p
γ
e+
γ
p
D
ν
e-
γ
p

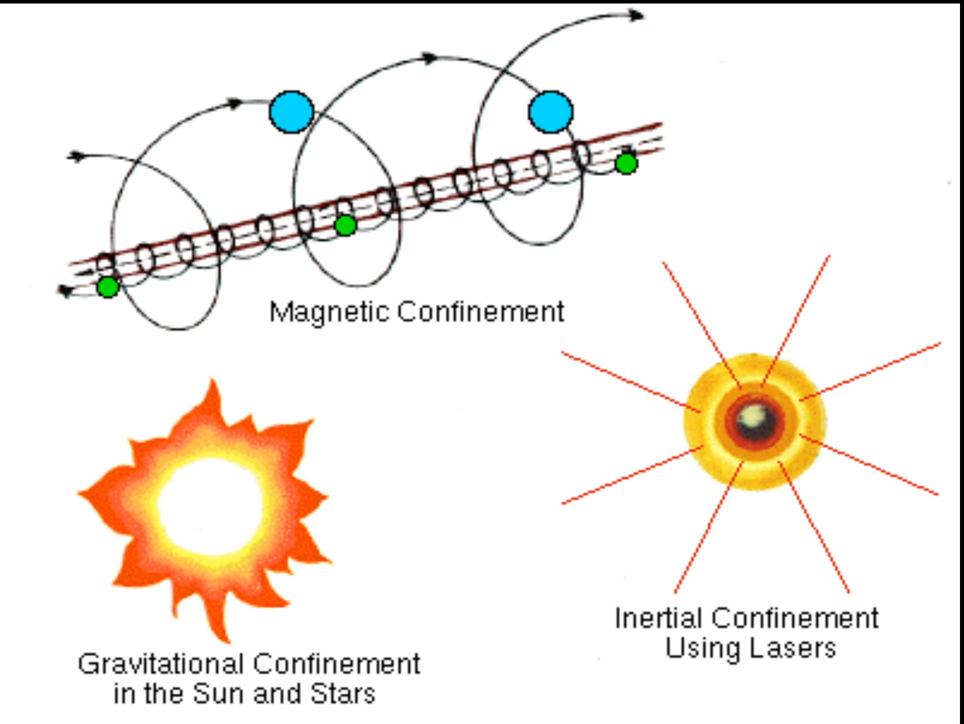


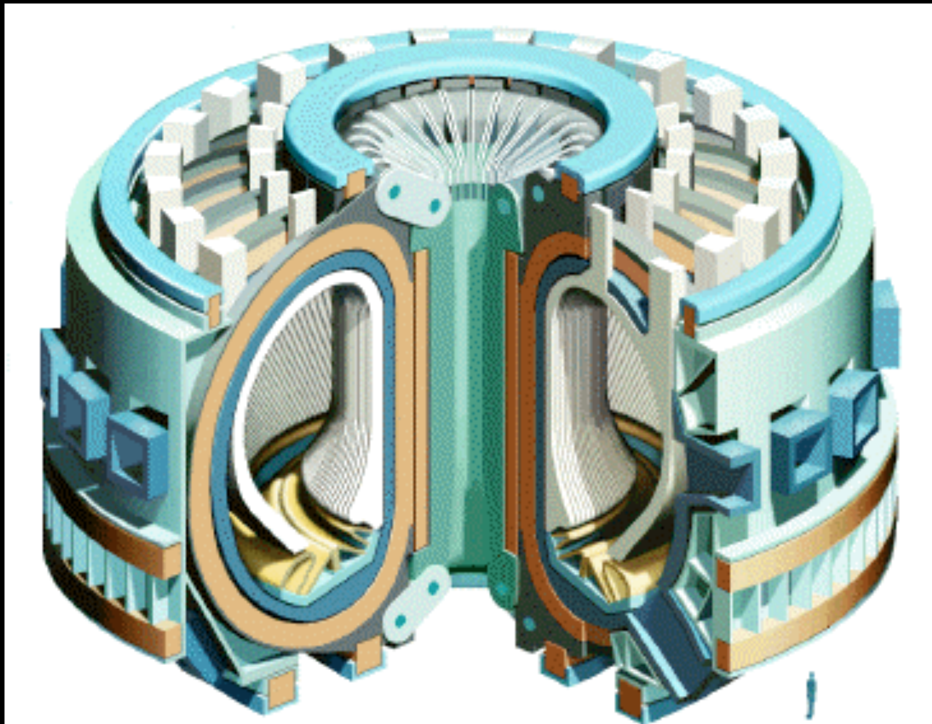
Reactants
Fusion
Products
D
20 keV
3.5 MeV
4He
T
20 keV
14.1 MeV
n



Rate Coefficient, R (m3/s)
10-20
10-24
10-28
10-46
10-50
D + T
p + p
Primary process in our sun
107
108
109
1010
Tion (K)


$^{3}_{1}H_2$
$^{2}_{1}H_1$


Magnetic Confinement
Inertial Confinement
Using Lasers
Gravitational Confinement
in the Sun and Stars

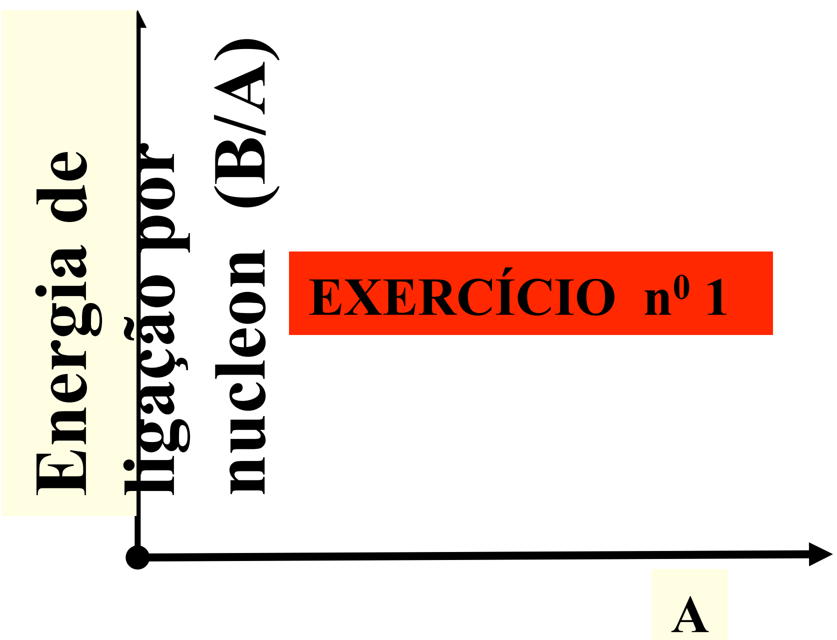


N	Z	A	EL	O	MASS EXCESS (KEV)		B/A	BINDING ENERGY (KEV)		ATOMIC MASS (U)	
1	0	1	N		8071.69	0.1		0.0	0.0	1.00866522	0.00000006
0	1		H		7289.22	0.0		0.0	0.0	1.00782522	0.00000004
1	1	2	H		13136.27	0.1	1.11	2224.64	0.04	2.01410222	0.00000007
2	1	3	H		14950.38	0.2	2.42	8482.22	0.15	3.01604972	0.00000016
1	2		HE		14931.73	0.2	2.57	7718.40	0.14	3.01602970	0.00000016
3	1	4	H	-N	25920	500		5580	500	4.02783	0.00054
2	2		HE		2424.94	0.2	7.07	28296.9	0.4	4.00260326	0.00000027
1	3		LI	+NN	25130	300		4810	300	4.02697	0.00032
4	1	5	H	+	33790	800		5790	800	5.03627	0.00086
3	2		HE	-N	11390	50		27410	50	5.01222	0.00005
2	3		LI	-P	11680	50		26330	50	5.01254	0.00005
4	2	6	HE		17597.3	3.6		29267.9	3.6	6.0188913	0.0000039
3	3		LI		14087.5	0.7	5.33	31995.2	0.8	6.0151234	0.0000008
2	4		BE	-	18375	5		26926	5	6.019726	0.000006
5	2	7	HE	+	26111	30		28826	30	7.028031	0.000032
4	3		LI		14908.6	0.8	5.60	39245.9	0.9	7.0160048	0.0000008
3	4		BE		15770.3	0.8		37601.6	0.9	7.0169299	0.0000008
2	5		B	-	27940	100		24650	100	7.02999	0.00011
6	2	8	HE	+	31650	120		31360	120	8.03397	0.00013
5	3		LI	-N	20947.5	1.0		41278.6	1.2	8.0224879	0.0000011
4	4		BE		4941.8	0.5	7.06	56501.9	0.8	8.0053052	0.0000005
3	5		B	-PP	22922.3	1.2		37738.8	1.3	8.0246079	0.0000013
6	3	9	LI	+	24966	5		45331	5	9.026802	0.000005
5	4		BE		11348.4	0.6	6.46	58167.0	0.9	9.0121828	0.0000006
4	5		B	-	12415.7	0.9		56317.1	1.1	9.0133287	0.0000010
3	6		C		28912	5		39038	5	9.031038	0.000006
7	3	10	LI	-N	35340	SYST		43030	SYST	10.03794	SYST
6	4		BE		12608.1	0.7		64978.9	1.0	10.0135352	0.0000008
5	5		B		12052.3	0.4	6.47	64752.3	0.9	10.0129385	0.0000004
4	6		C		15702.7	1.8		60319.4	2.0	10.0168573	0.0000020
8	3	11	LI	-N	43310	SYST		43130	SYST	11.04649	SYST
7	4		BE		20177	6		65482	6	11.021660	0.000007
6	5		B		8667.95	0.2	6.93	76208.3	1.0	11.00930533	0.00000030
5	6		C	-	10650.2	1.1		73443.6	1.4	11.0114333	0.0000011
4	7		N	-	25450	SYST		57860	SYST	11.02732	SYST
8	4	12	BE	-N	24950	SYST		68780	SYST	12.02678	SYST
[illegible]	[illegible]		[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]
6	6		C		0.0	0.0	7.68	92165.5	1.1	12.00000000	0.0
5	7		N		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]

http://ie.lbl.gov/toimass.html

http://ie.lbl.gov/mass/

http://www.phy.ornl.gov/hribf/calculators/mass-diff.shtml



EXERCÍCIO nº 1

Available online at www.sciencedirect.com
SCIENCE DIRECT
Nuclear Physics A 729 (2003) 337–676
ELSEVIER
NUCLEAR PHYSICS A
www.elsevier.com/locate/npe

The AME2003 atomic mass evaluation *

(II). Tables, graphs and references

G. Audi[a,§], A.H. Wapstra[b] and C. Thibault[a]

[a] *Centre de Spectrometrie Nucleaire et de Spectrometrie de Masse, CSNSM, IN2P3-CNRS&UPS, Batiment 108, F-91405 Orsay Campus, France*

[b] *National Institute of Nuclear Physics and High-Energy Physics, NIKHEF, PO Box 41882, 1009DB Amsterdam, The Netherlands*

Abstract

This paper is the second part of the new evaluation of atomic masses AME2003. From the results of a least-squares calculation described in Part I for all accepted experimental data, we derive here tables and graphs to replace those of 1993. The first table lists atomic masses. It is followed by a table of the influences of data on primary nuclides, a table of separation energies and reaction energies, and finally, a series of graphs of separation and decay energies. The last section in this paper lists all references to the input data used in Part I of this AME2003 and also to the data entering the NUBASE2003 evaluation (first paper in this volume).

AMDC: http://csnwww.in2p3.fr/AMDC/

1. Introduction

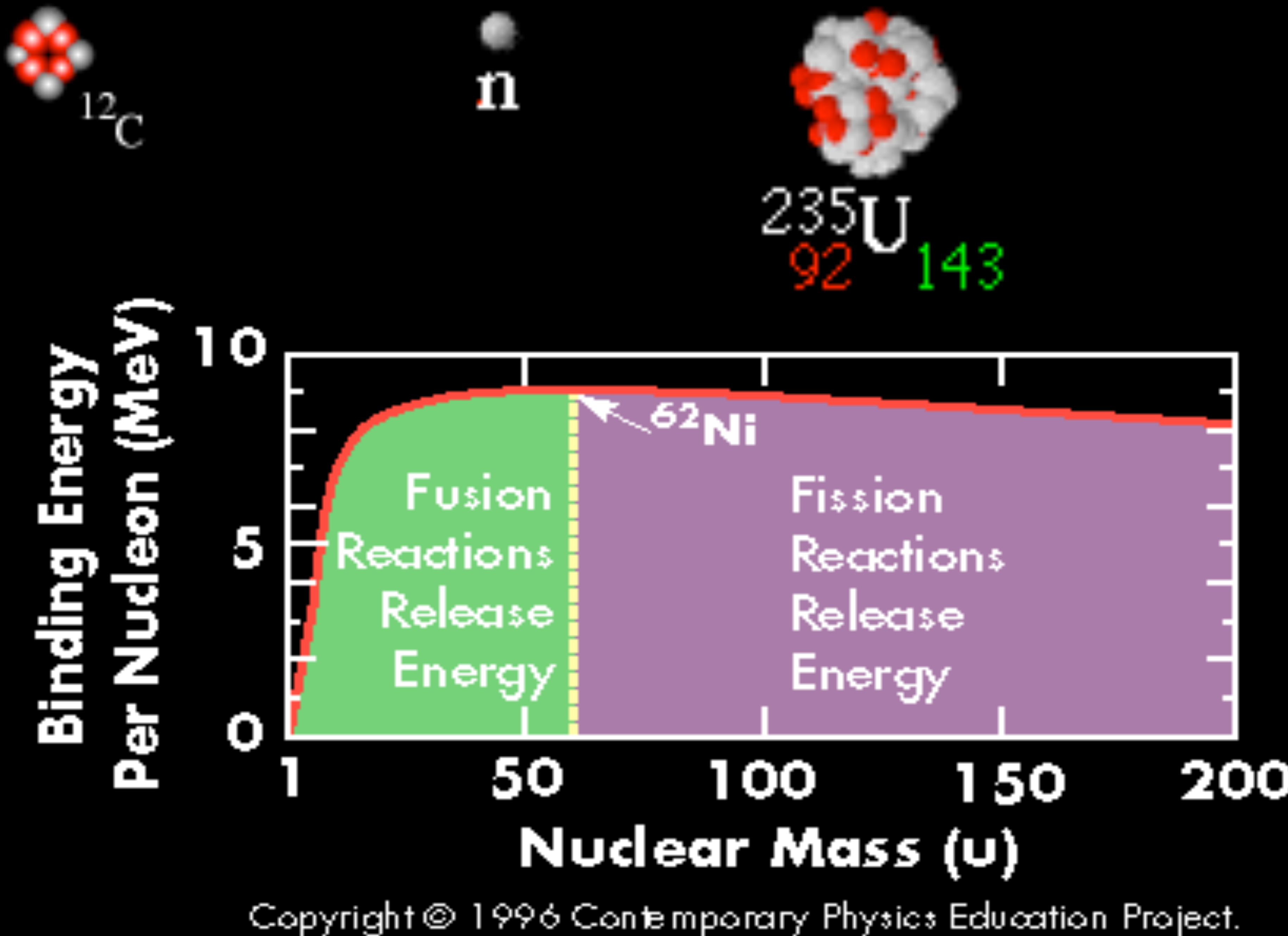
The description of the general procedures and policies are given in Part I of this series of two papers, where the input data used in the evaluation are presented. In this paper we give tables and graphs derived from the evaluation of the input data in Part I.

Firstly, we present the table of atomic masses (Table I) expressed as mass excesses in energy units, together with the binding energy per nucleon, the beta-decay energy and the full atomic mass in mass units.

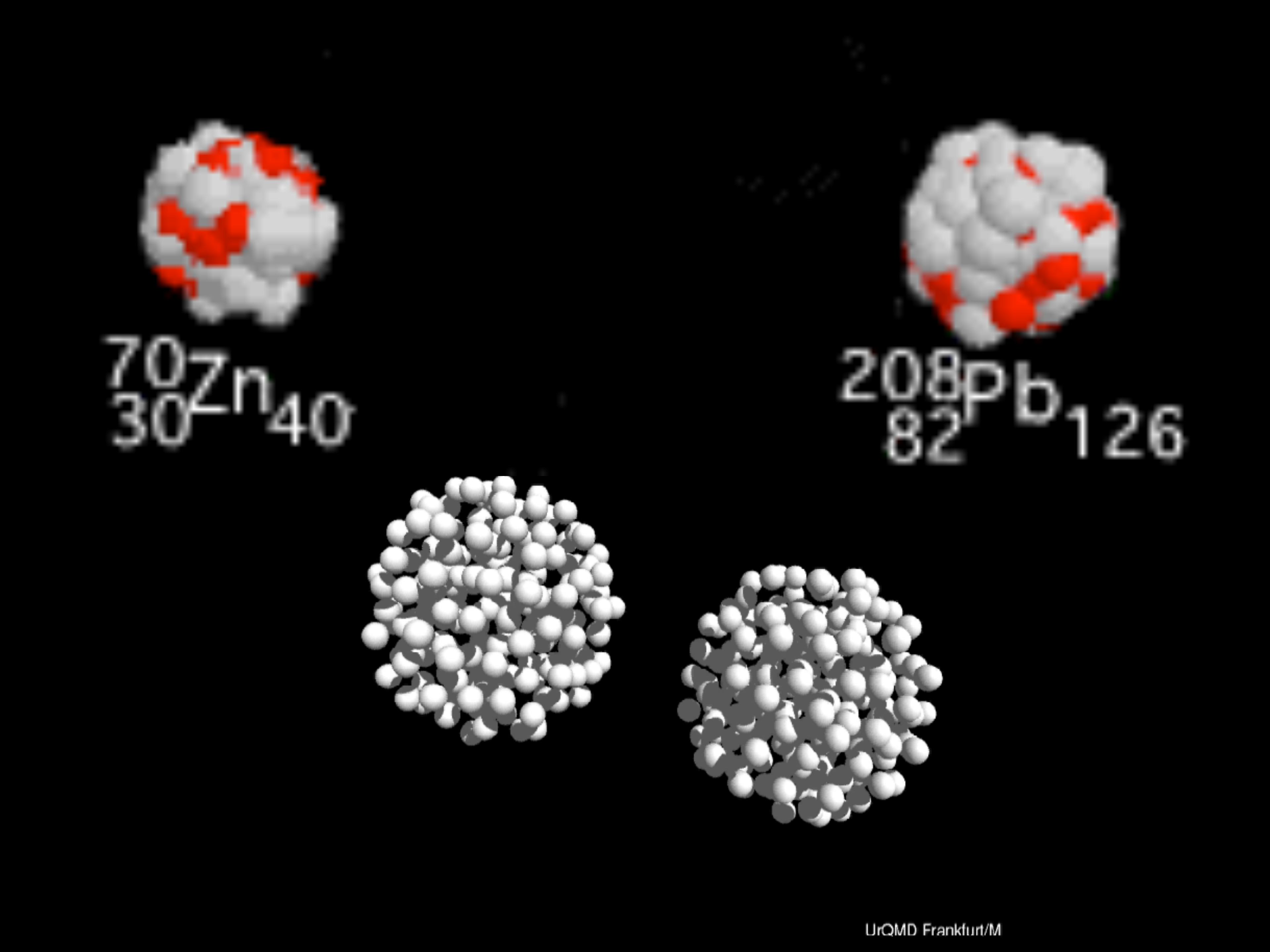
* This work has been undertaken with the encouragement of the IUPAP Commission on Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants (SUN-AMCO).

§ Corresponding author. *E-mail address:* audi@csnsm.in2p3.fr (G. Audi).

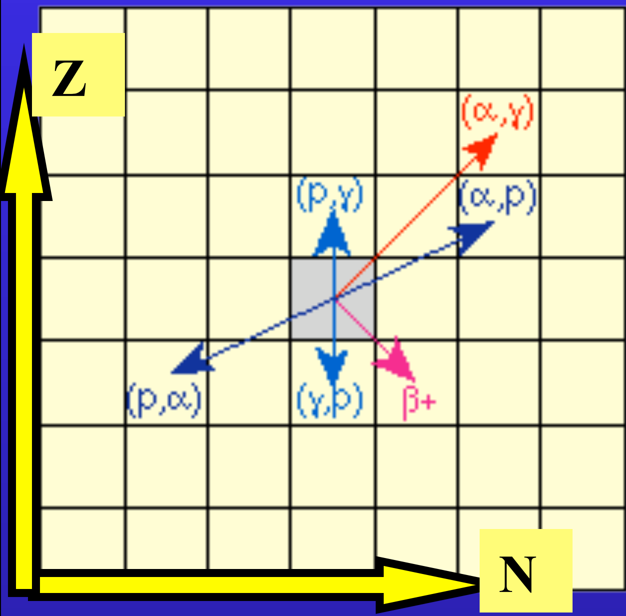
doi:10.1016/j.nuclphysa.2003.11.003



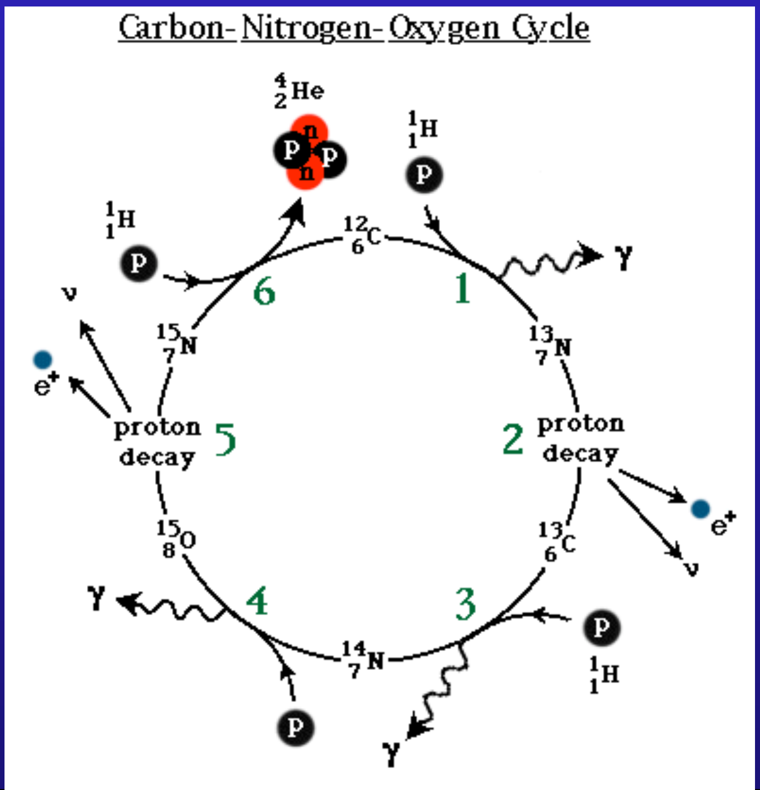
12C
n
235U
92
143
Binding Energy Per Nucleon (MeV)
10
5
0
62Ni
Fusion Reactions Release Energy
Fission Reactions Release Energy
1
50
100
150
200
Nuclear Mass (u)
Copyright © 1996 Contemporary Physics Education Project.



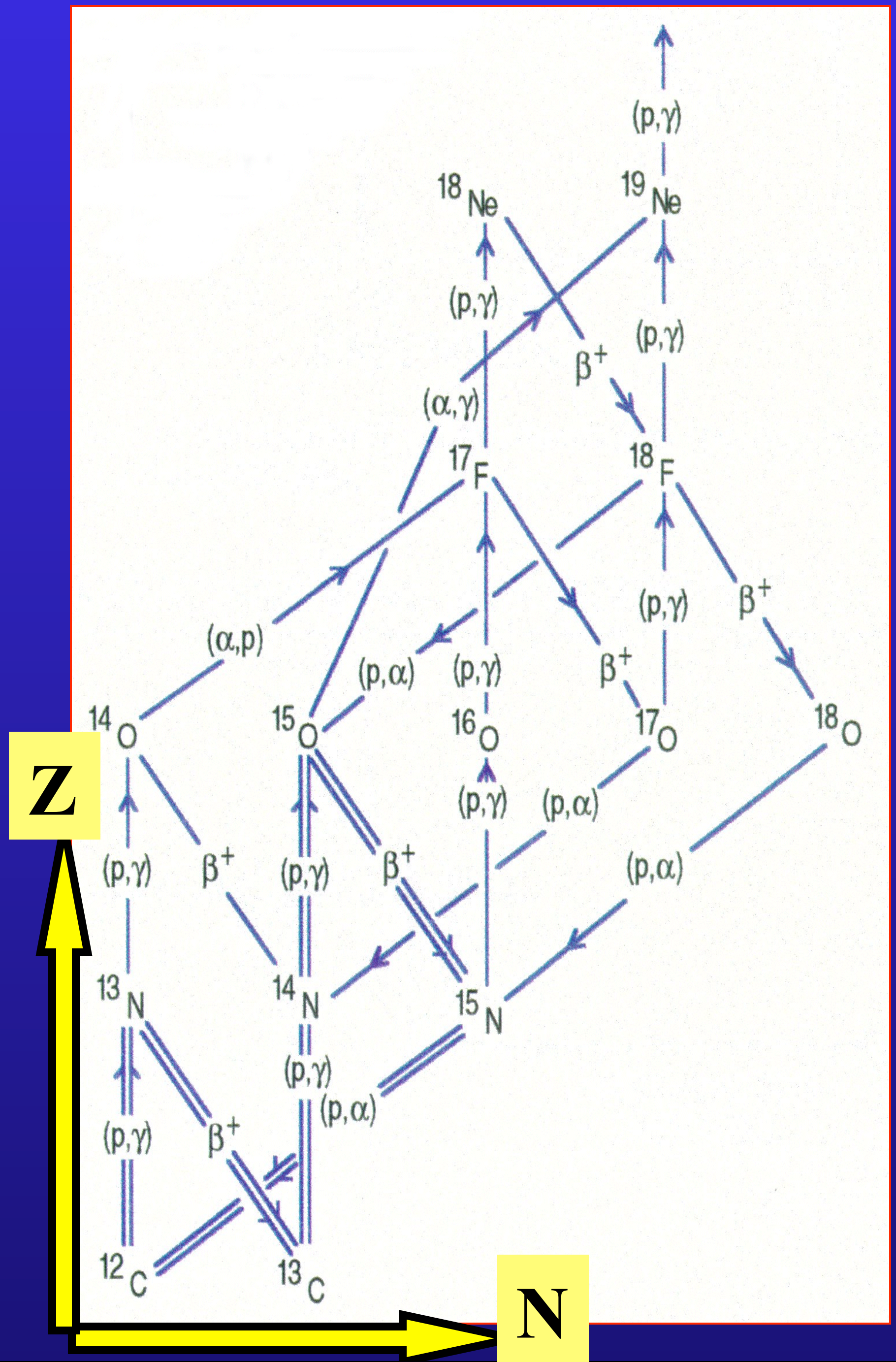
$^{70}_{30}Zn_{40}$
$^{208}_{82}Pb_{126}$
UrQMD Frankfurt/M



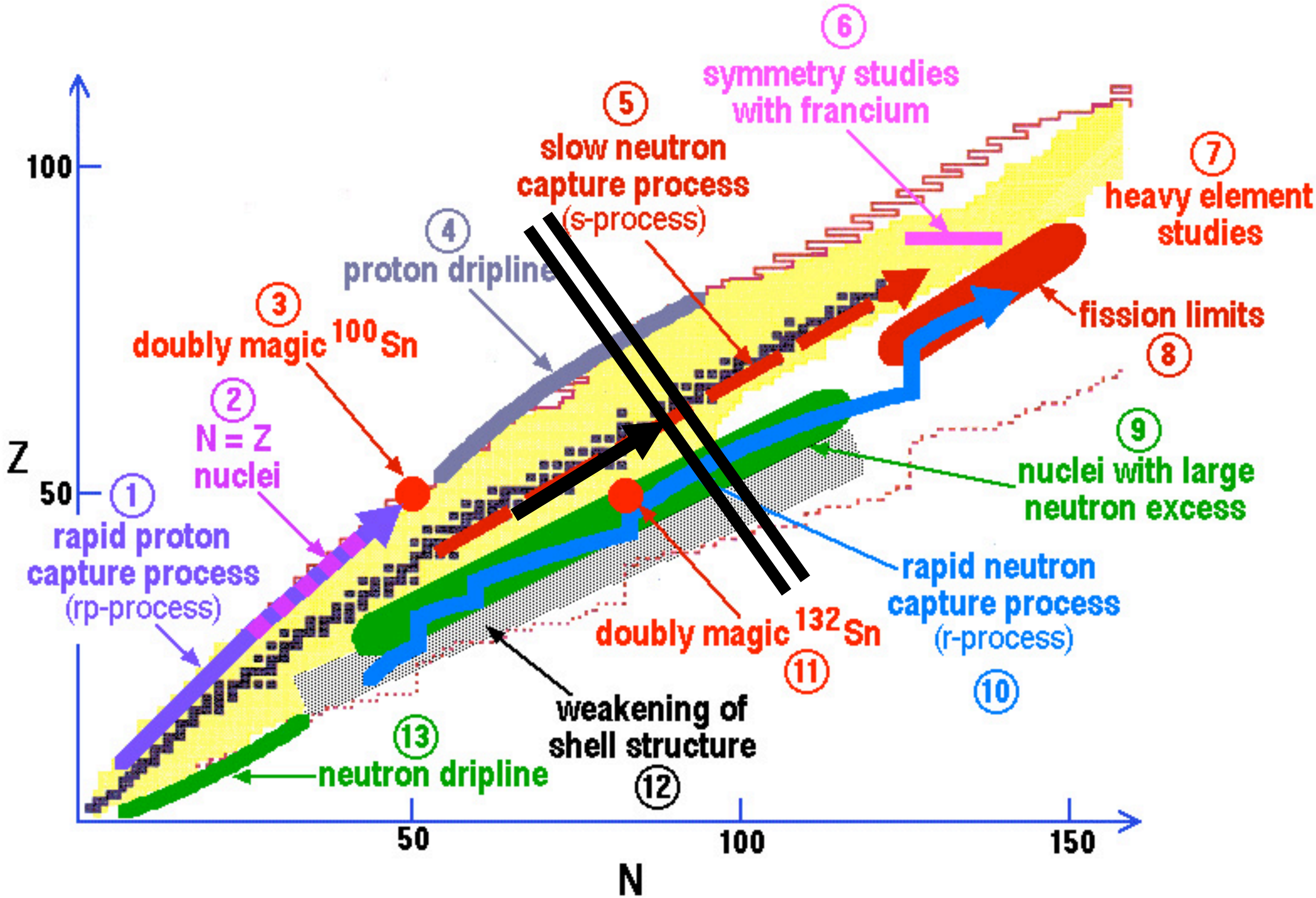
Z
N
(α,γ)
(p,γ)
(α,p)
(p,α)
(γ,p)
β+



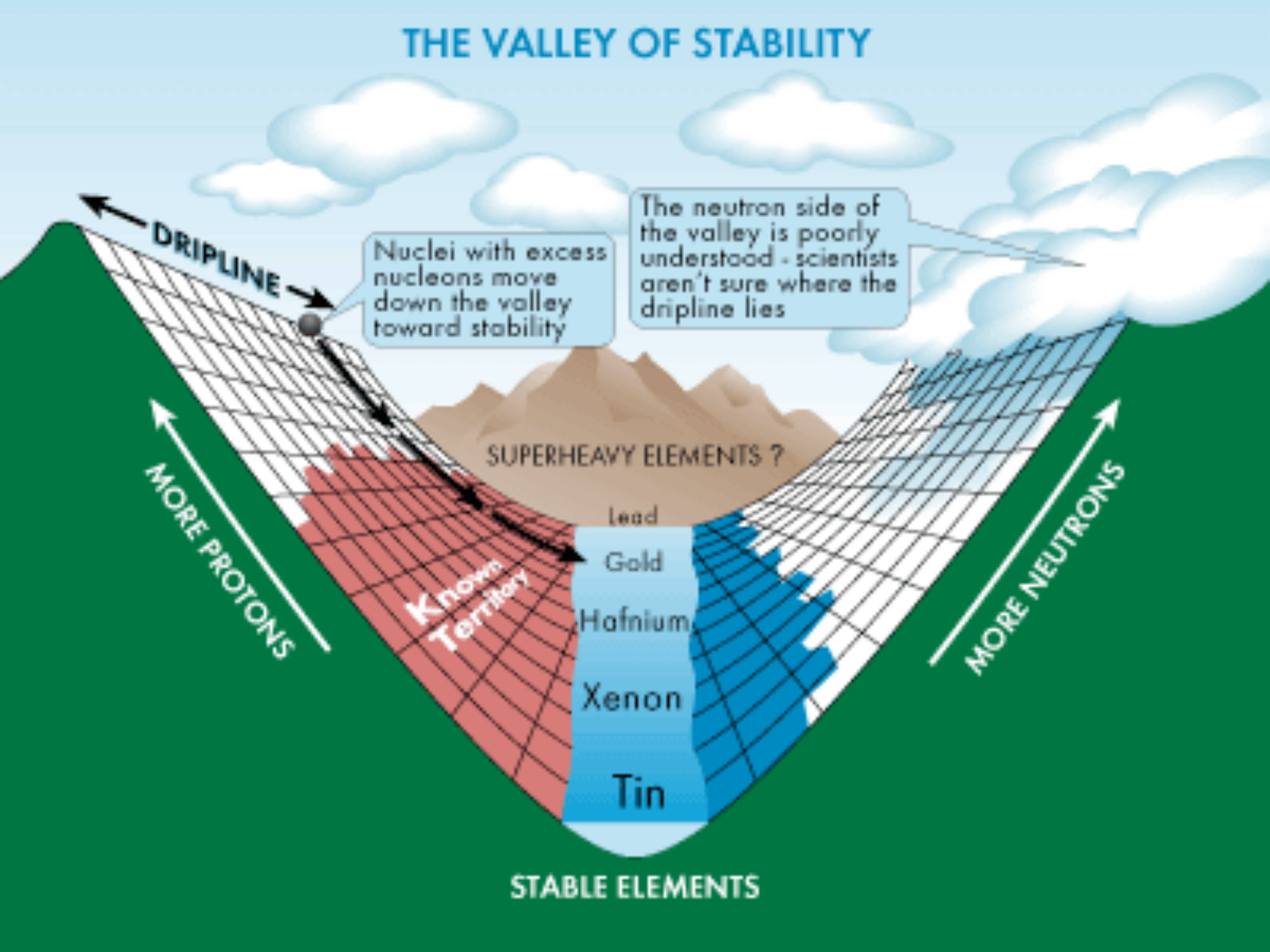
Carbon-Nitrogen-Oxygen Cycle
proton decay
proton decay
1
2
3
4
5
6
γ
ν
e+



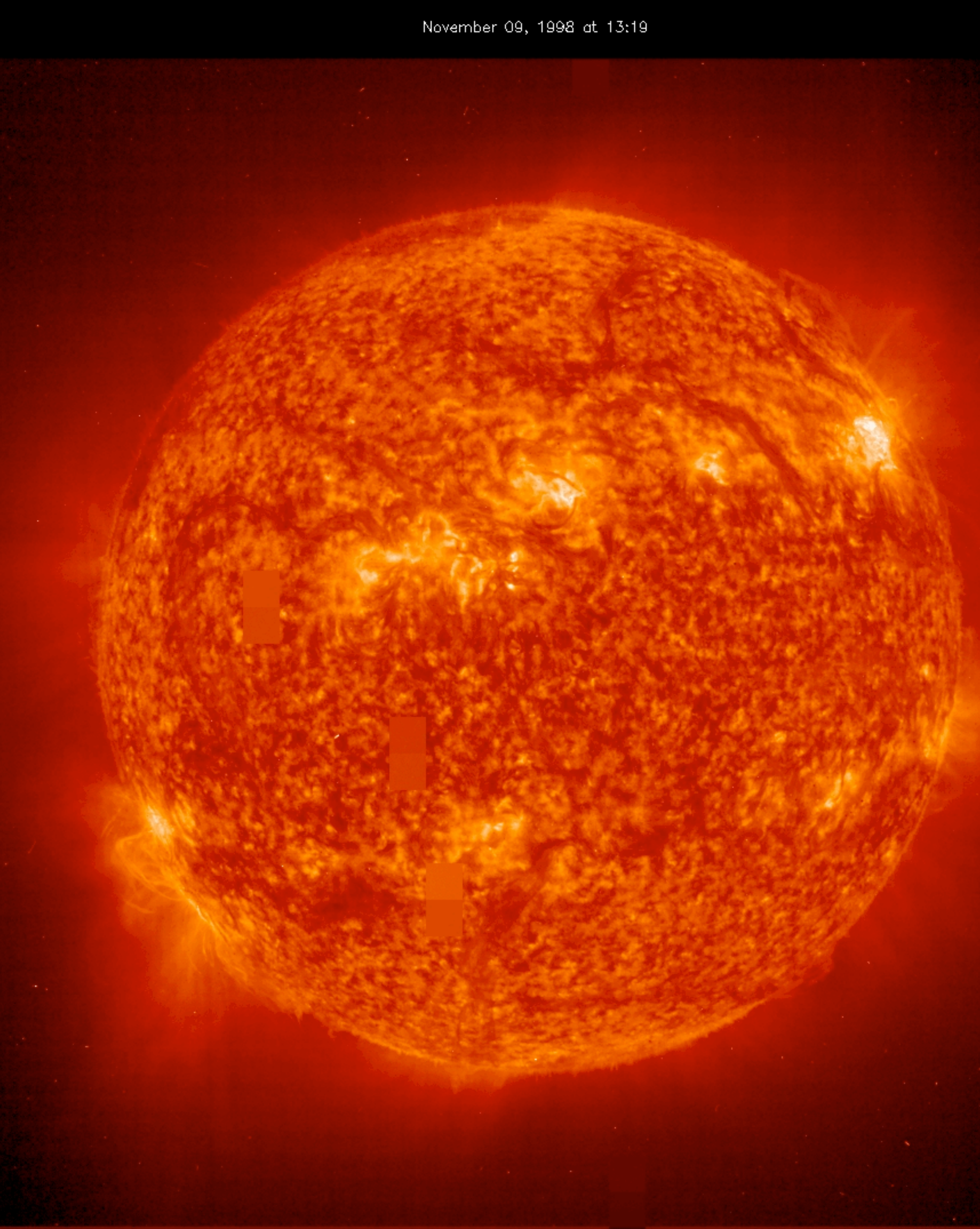
Z
N
(p,γ)
(α,γ)
(α,p)
(p,α)
β+

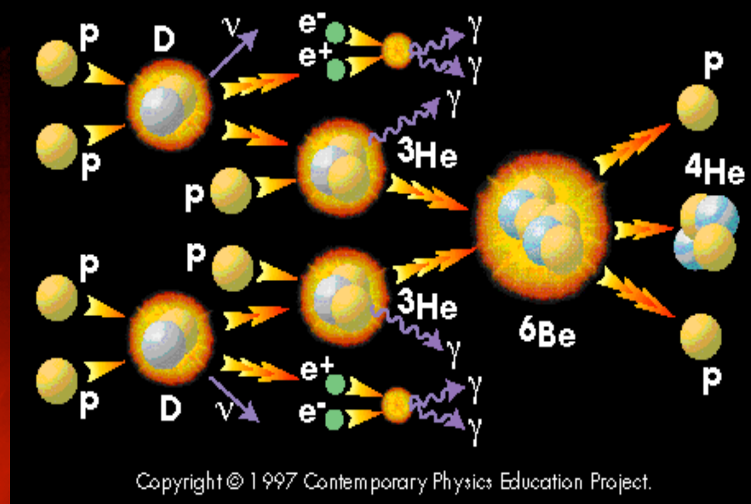


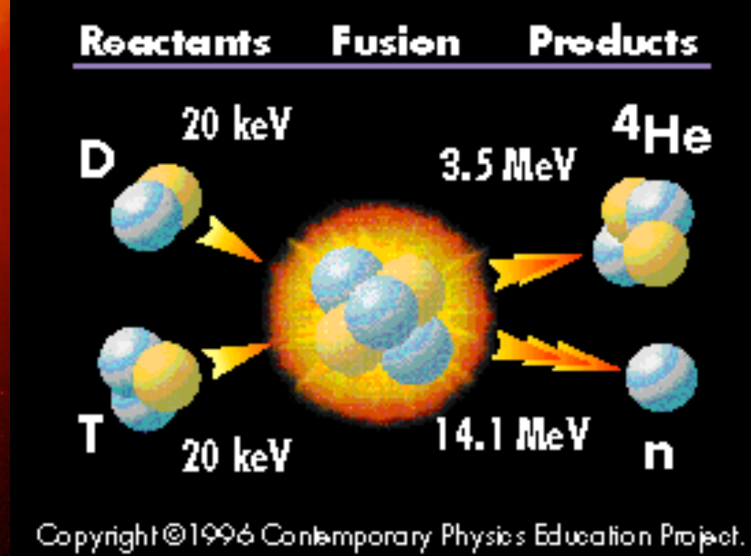
Z
100
50
N
50
100
150
1
rapid proton capture process (rp-process)
2
N = Z nuclei
3
doubly magic ^{100}Sn
4
proton dripline
5
slow neutron capture process (s-process)
6
symmetry studies with francium
7
heavy element studies
fission limits
8
9
nuclei with large neutron excess
rapid neutron capture process (r-process)
10
doubly magic ^{132}Sn
11
weakening of shell structure
12
13
neutron dripline



THE VALLEY OF STABILITY
DRIPLINE
Nuclei with excess nucleons move down the valley toward stability
The neutron side of the valley is poorly understood - scientists aren't sure where the dripline lies
SUPERHEAVY ELEMENTS ?
Lead
Gold
Hafnium
Xenon
Tin
Known Territory
MORE PROTONS
MORE NEUTRONS
STABLE ELEMENTS

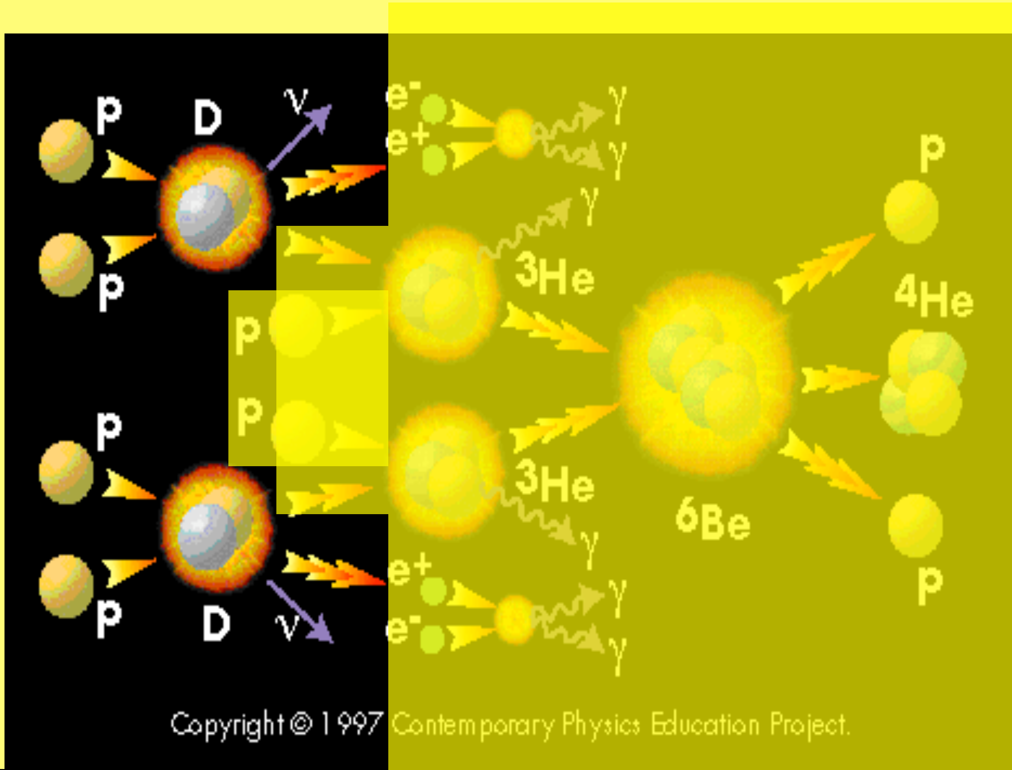

November 09, 1998 at 13:19


p
p
D
ν
e-
e+
γ
γ
γ
3He
p
p
4He
p
p
p
D
ν
3He
γ
6Be
e+
e-
γ
γ
p
Copyright © 1997 Contemporary Physics Education Project.


Reactants
Fusion
Products
D
20 keV
3.5 MeV
4He
T
20 keV
14.1 MeV
n
Copyright ©1996 Contemporary Physics Education Project.

$p + p \rightarrow d + e^{+} + \nu$

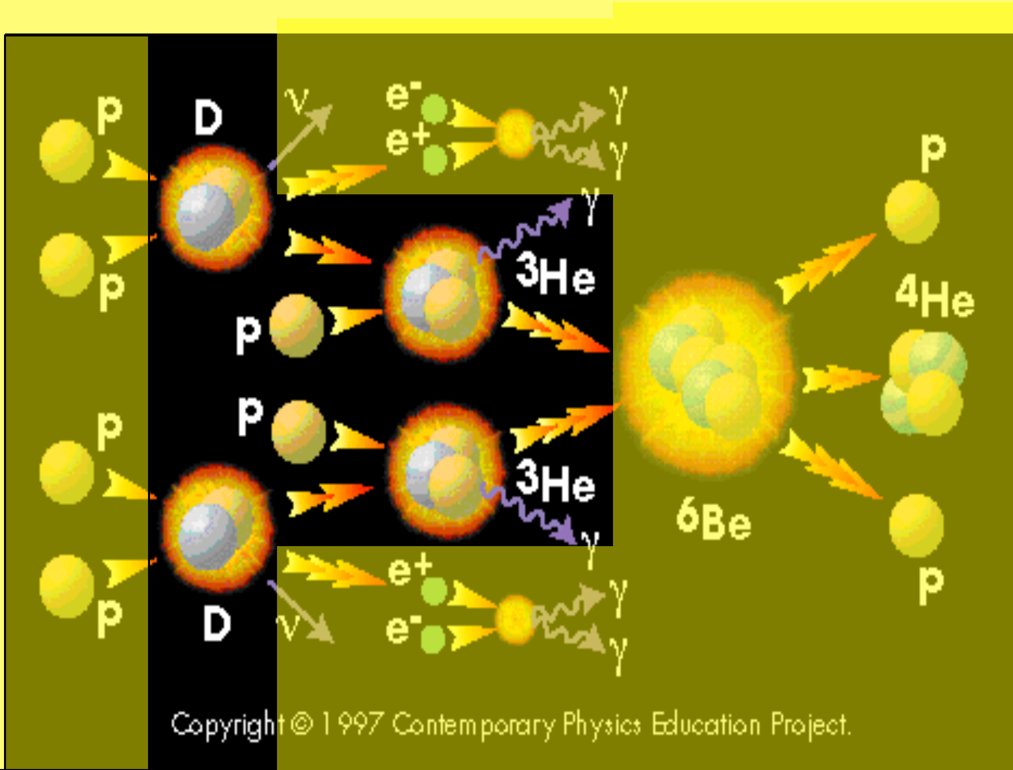
n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He=	14.931
^{4}He=	2.425
^{6}Be=	18.375



$p + p \rightarrow d + e^{+} + \nu$

$p + d \rightarrow {}^{3}He + \gamma$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He= 14.931
^{4}He= 2.425
^{6}Be= 18.375

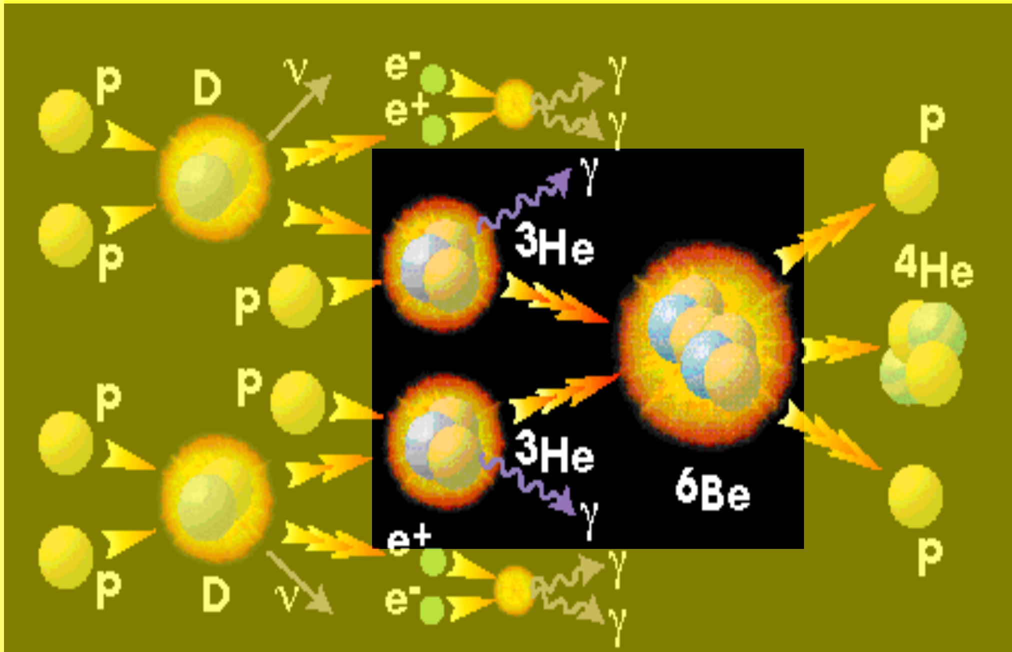


$$p + p \rightarrow d + e^{+} + \nu$$

$$p + d \rightarrow {}^{3}He + \gamma$$

$${}^{3}He + {}^{3}He \rightarrow {}^{6}Be + \gamma$$

n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He=	14.931
^{4}He=	2.425
^{6}Be=	18.375



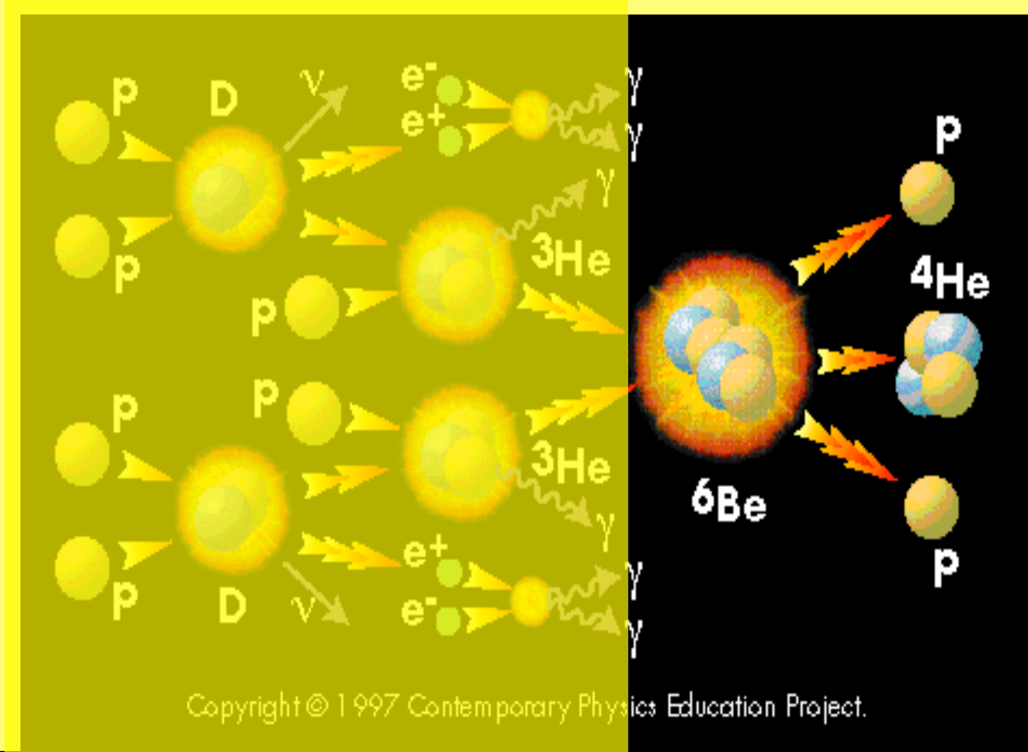
$p + p \rightarrow d + e^{+} + \nu$

$p + d \rightarrow {}^{3}He + \gamma$

${}^{3}He + {}^{3}He \rightarrow {}^{6}Be + \gamma$

${}^{6}Be \rightarrow {}^{4}He + p + p$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He= 14.931
^{4}He= 2.425
^{6}Be= 18.375



$p + p \rightarrow d + e^+ + \nu$

$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931\ \text{MeV}$

$p + d \rightarrow {}^3He + \gamma$

$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494\ \text{MeV}$

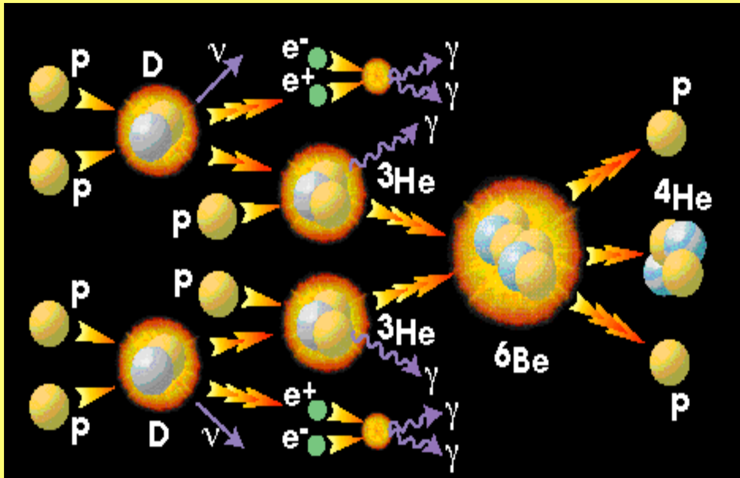
${}^3He + {}^3He \rightarrow {}^6Be + \gamma$

$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487\ \text{MeV}$

${}^6Be \rightarrow {}^4He + p + p$

$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372\ \text{MeV}$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He = 14.931
^{4}He = 2.425
^{6}Be = 18.375



$$p + p \rightarrow d + e^+ + \nu$$

$$7.289 + 7.289 \rightarrow 13.136 + 0.511 + Q \Rightarrow Q = 0.931 \text{ MeV}$$

$$p + d \rightarrow {}^3He + \gamma$$

$$7.289 + 13.136 \rightarrow 14.931 + Q \Rightarrow Q = 5.494 \text{ MeV}$$

$${}^3He + {}^3He \rightarrow {}^6Be + \gamma$$

$$14.931 + 14.931 \rightarrow 18.375 + Q \Rightarrow Q = 11.487 \text{ MeV}$$

$${}^6Be \rightarrow {}^4He + p + p$$

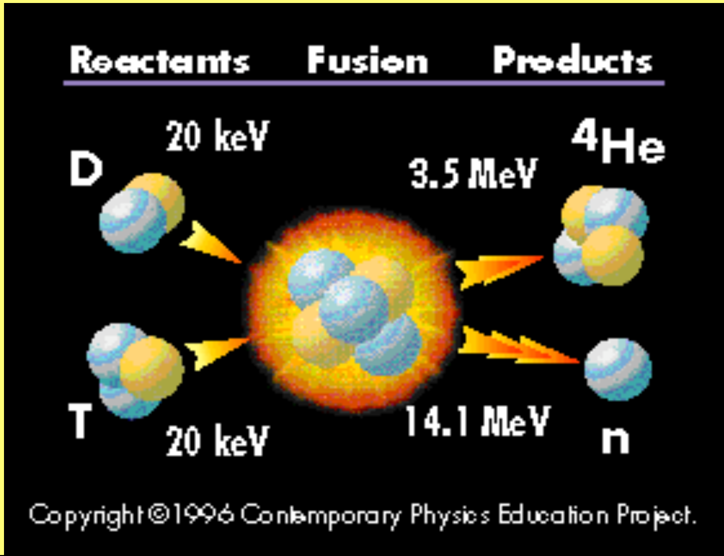
$$18.375 \rightarrow 2.425 + 7.289 + 7.289 + Q \Rightarrow Q = 1.372 \text{ MeV}$$

n = 8.071
p = 7.289
d = 13.136
t = 14.950
^{3}He = 14.931
^{4}He = 2.425
^{6}Be = 18.375

$$d + t \rightarrow {}^4He + n$$

$$13.136 + 14.950 \rightarrow 2.425 + 8.071 + Q$$

$$Q = 17.59 \text{ MeV}$$



EXERCÍCIOS

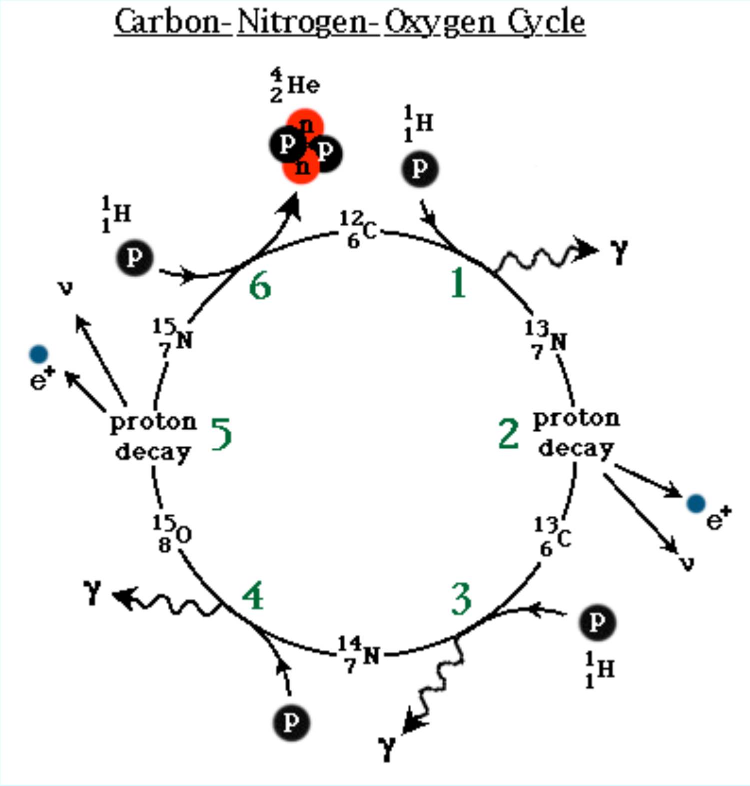
CALCULAR O BALANÇO ENERGÉTICO NAS SEGUINTES REAÇÕES

$\Delta = (M-A)c^2$ (MeV)

Exemplo== $^{12}C + p \Rightarrow {}^{13}N + \gamma$

n =	8.071
p =	7.289
d =	13.136
t =	14.950
^{3}He =	14.931
^{4}He =	2.425
^{6}Li =	14.086
^{7}Li =	14.908
^{6}Be =	18.375

^{12}C =	0.00
^{13}C =	3.125
^{13}N =	5.345
^{14}N =	2.863
^{15}N =	0.011
^{15}O =	2.855
^{16}O =	-4.737
^{17}O =	-0.809
^{18}O =	-0.782



$^{12}C + p \rightarrow {}^{13}N + \gamma$

$0 + 7.289 \rightarrow 5.345 + Q \Rightarrow Q = 1.944$ MeV

$^{13}N \rightarrow {}^{13}C + e^+ + \nu$

$5.345 \rightarrow 3.125 + 0.511 + Q \Rightarrow Q = 1.709$ MeV

$^{13}C + p \rightarrow {}^{14}N + \gamma$

$3.125 + 7.289 \rightarrow 2.863 + Q \Rightarrow Q = 7.551$ MeV

$^{14}N + p \rightarrow {}^{15}O + \gamma$

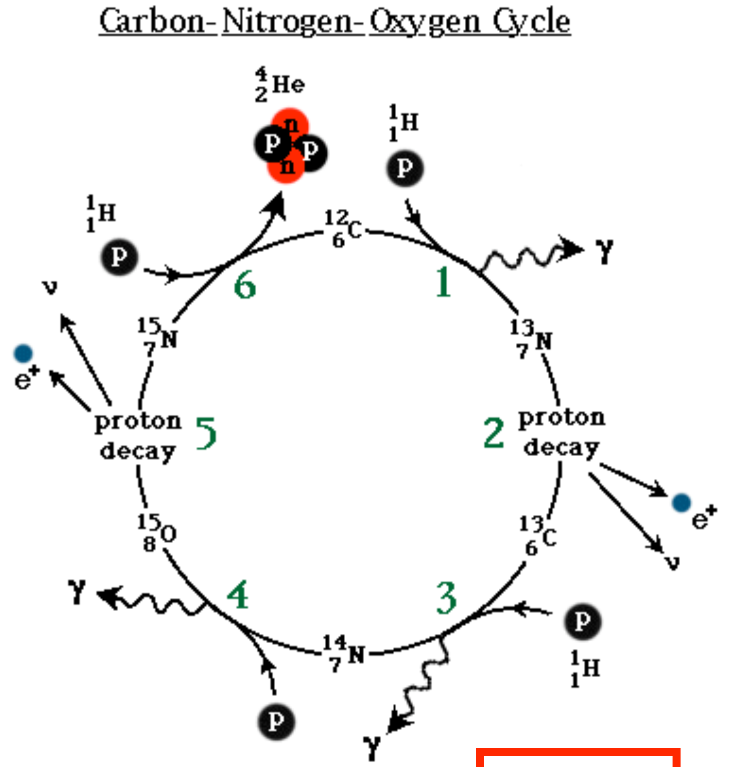
$2.863 + 7.289 \rightarrow 2.855 + Q \Rightarrow Q = 7.297$ MeV

$^{15}O \rightarrow {}^{15}N + e^+ + \nu$

$2.855 \rightarrow 0.101 + 0.511 + Q \Rightarrow Q = 2.243$ MeV

$^{15}N + p \rightarrow {}^{12}C + \alpha + \gamma$

$0.101 + 7.289 \rightarrow 0 + 2.425 + Q \Rightarrow Q = 4.965$ MeV



$$\begin{array}{r} 1.944 \\ +1.709 \\ +7.551 \\ +7.297 \\ +2.243 \\ +4.965 \\ \hline 25.709 \end{array}$$

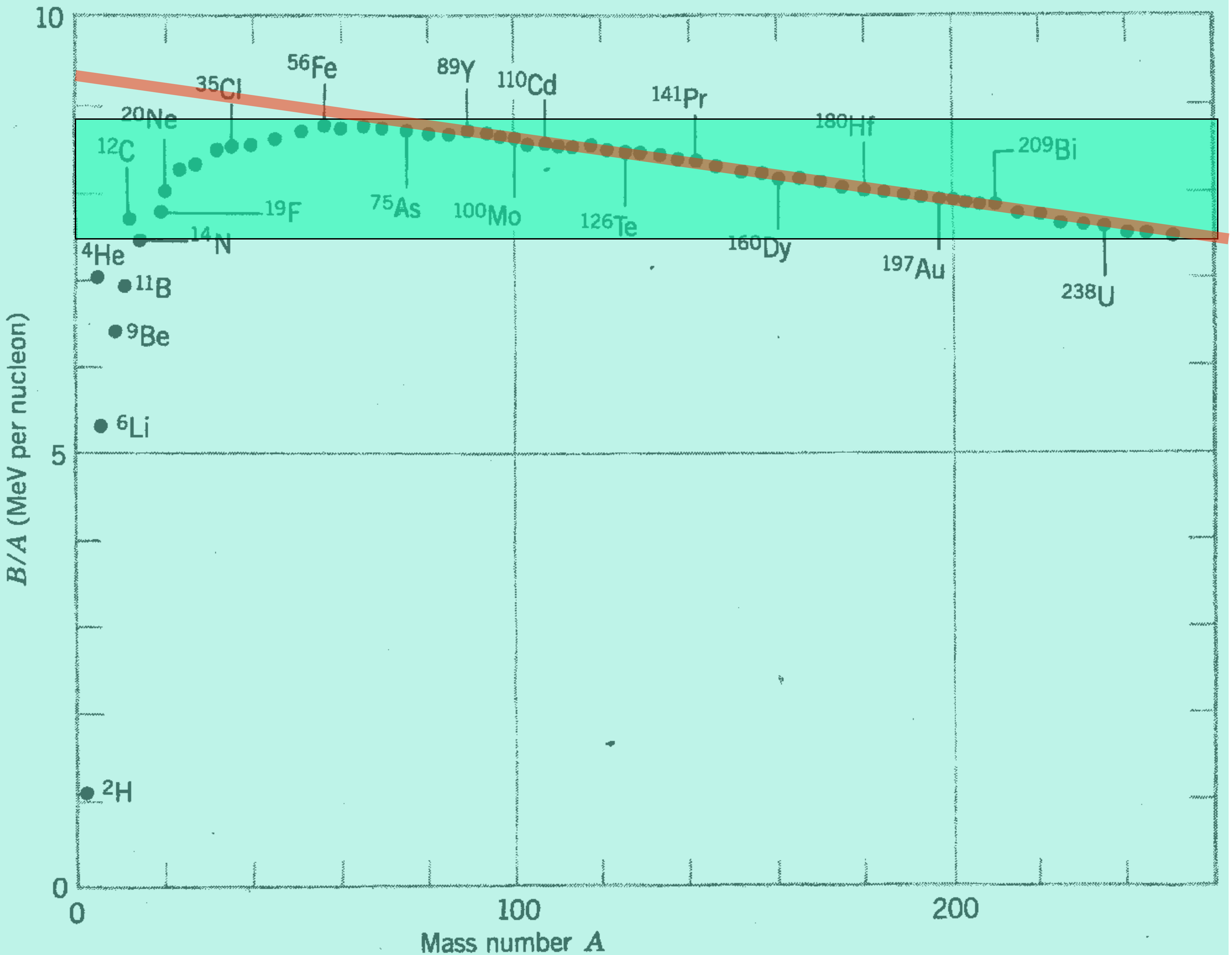
$^{12}C + p + p + p + p \rightarrow {}^{12}C + {}^{4}He + 2e^+ + 2\nu$

$0 + 4x\,(7.289) \rightarrow 0 + 2.425 + 1.022 + Q$

Q = 25.709 MeV

modêlos macroscópicos

N	Z	A	EL	O	MASS EXCESS (KEV)		B/A	BINDING ENERGY (KEV)		BETA-DECAY ENERGY (KEV)		ATOMIC MASS (U)	
1	0	1	N		8071.69	0.10		0.0	0.0	782.47	0.05	1.00866522	0.00000006
0	1		H		7289.22	0.09		0.0	0.0	*		1.00782522	0.00000004
1	1	2	H		13136.27	0.16	1.11	2224.64	0.04	*		2.01410222	0.00000007
2	1	3	H		14950.38	0.22	2.42	8482.22	0.15	18.65	0.04	3.01604972	0.00000016
1	2		HE		14931.73	0.22	2.57	7718.40	0.14	*		3.01602970	0.00000016
3	1	4	H	-N	25920	500		5580	500	23500	500	4.02783	0.00054
2	2		HE		2424.94	0.25	7.07	28296.9	0.4	-22700	300	4.00260326	0.0000027
1	3		LI	+NN	25130	300		4810	300	*		4.02697	0.00032
4	1	5	H	+	33790	800		5790	800	22400	800	5.03627	0.00086
3	2		HE	-N	11390	50		27410	50	-290	70	5.01222	0.00005
2	3		LI	-P	11680	50		26330	50	*		5.01254	0.00005
4	2	6	HE		17597.3	3.6		29267.9	3.6	3509.8	3.6	6.0188913	0.0000039
3	3		LI		14087.5	0.7	5.33	31995.2	0.8	-4287	5	6.0151234	0.0000008
2	4		BE	-	18375	5		26926	5	*		6.019726	0.000006
5	2	7	HE	+	26111	30		28826	30	11203	30	7.028031	0.000032
4	3		LI		14908.6	0.8	5.60	39245.9	0.9	-861.75	0.09	7.0160048	0.0000008
3	4		BE		15770.3	0.8		37601.6	0.9	-12170	100	7.0169299	0.0000008
2	5		B	-	27940	100		24650	100	*		7.02999	0.00011
6	2	8	HE	+	31650	120		31360	120	10700	120	8.03397	0.00013
5	3		LI	-N	20947.5	1.0		41278.6	1.2	16005.8	1.1	8.0224879	0.0000011
4	4		BE		4941.8	0.5	7.06	56501.9	0.8	-17980.5	1.3	8.0053052	0.0000005
3	5		B	-PP	22922.3	1.2		37738.8	1.3	*		8.0246079	0.0000013
6	3	9	LI	+	24966	5		45331	5	13618	5	9.026802	0.000005
5	4		BE		11348.4	0.6	6.46	58167.0	0.9	-1067.3	0.7	9.0121828	0.0000006
4	5		B	-	12415.7	0.9		56317.1	1.1	-16497	5	9.0133287	0.0000010
3	6		C		28912	5		39038	5	*		9.031038	0.000006
7	3	10	LI	-N	35340	SYST		43030	SYST	22730	SYST	10.03794	SYST
6	4		BE		12608.1	0.7		64978.9	1.0	555.9	0.8	10.0135352	0.0000008
5	5		B		12052.3	0.4	6.47	64752.3	0.9	-3650.4	1.8	10.0129385	0.0000004
4	6		C		15702.7	1.8		60319.4	2.0	*		10.0168573	0.0000020
8	3	11	LI	-N	43310	SYST		43130	SYST	23130	SYST	11.04649	SYST
7	4		BE		20177	6		65482	6	11509	6	11.021660	0.000007
6	5		B		8667.95	0.29	6.93	76208.3	1.0	-1982.2	1.0	11.00930533	0.00000030
5	6		C	-	10650.2	1.1		73443.6	1.4	-14800	SYST	11.0114333	0.0000011
4	7		N	-	25450	SYST		57860	SYST	*		11.02732	SYST
8	4	12	BE	-N	24950	SYST		68780	SYST	11580	SYST	12.02678	SYST
[illegible]	[illegible]		[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	6		C		0.0	0.0	7.68	92165.5	1.1	-17344	5	12.00000000	0.0
[illegible]	[illegible]		N		[illegible]	[illegible]		[illegible]	[illegible]			[illegible]	[illegible]



10
5
0
0
100
200
B/A (MeV per nucleon)
Mass number A
56Fe
89Y
110Cd
35Cl
141Pr
20Ne
180Hf
12C
209Bi
19F
75As
100Mo
126Te
14N
160Dy
4He
197Au
11B
238U
9Be
6Li
2H

2.4 Modelo da gota líquida e limites de estabilidade

Na secção anterior ficaram claros os limites da analogia entre o comportamento da matéria nuclear e o de um líquido. Nesta secção, pretende-se desenvolver um modelo nuclear simples, em que apenas se faz uso de propriedades do núcleo análogas às de um líquido. Este *modelo da gota líquida* permite compreender o comportamento das energias de ligação e, por meio delas, as massas nucleares, não conseguindo contudo explicar outros tipos de propriedades.

No que se segue, considera-se o núcleo como uma gota de um líquido incompressível que se mantém coesa sob a acção de forças de alcance curto. A energia de ligação do núcleo, B, obtém-se pela soma de várias parcelas

$$B = B_1 + B_2 + B_3 + B_4 + B_5 \qquad (2.47)$$

correspondentes a outras tantas contribuições que se discutem de seguida, nas alíneas 1) a 5), sendo a energia B expressa como função de Z e de A. Interessa apenas obter, para cada contribuição, a relação funcional com aquelas grandezas e determinar depois, empiricamente, os valores das constantes necessárias.

1) A principal contribuição para a energia de ligação é a "energia de condensação", libertada no momento em que os nucleões se reunem para formar o núcleo. Ela deve ser proporcional ao número de partículas ligadas, de acordo com o valor aproximadamente constante de B/A (Fig.10). Se a_V for a constante de proporcionalidade, tem-se, portanto,

$$B_1 = a_V A \qquad (2.48)$$

Como A é proporcional ao volume do núcleo chama-se a este termo *energia de volume.*

2) Os nucleões que se encontram à superfície do núcleo têm menor número de ligações com os vizinhos do que os que estão no interior, ficando por isso menos ligados e contribuindo menos para uma energia de ligação. Introduz-se, portanto, um termo negativo, B_2, proporcional à superfície $4\pi R^2 = 4\pi r_o^2 A^{2/3}$ e, como importa apenas a dependência funcional em A, vem

$$B_2 = -a_S A^{\frac{2}{3}} \quad \text{(Energia de superfície)} \qquad (2.49)$$

3) A energia de ligação é ainda mais reduzida devido à repulsão entre os protões. A energia de Coulomb duma esfera de raio R e carga q, carregada uniformemente, é $(3/5).(q^2/R)$. Para o núcleo de carga Ze e raio $R = r_o A^{1/3}$, a dependência funcional em Z e A conduz a um termo da forma

$$B_3 = -a_C Z^2 A^{-\frac{1}{3}} \quad \text{(Energia de Colomb)} \qquad (2.50)$$

4) Ao considerar a dependência de B em A e Z, deve-se também considerar que o excesso de neutrões é acompanhado por uma diminuição da energia de ligação em relação à situação simétrica (N = Z). De acordo com (2.46), esta diferença de energia depende do excesso de neutrões, sendo o termo correspondente dado por

$$B_4 = -a_A \frac{T_z^2}{A} = -a_A \frac{(Z - A/2)^2}{A} \quad \text{(Energia de assimetria)} \qquad (2.51)$$

5) Sabe-se, com base na sistemática das energias de separação, que os nucleões do mesmo tipo produzem uma ligação particularmente forte quando surgem aos pares. A *energia de emparelhamento* não pode ser explicada com base na analogia com a gota líquida, sendo necessário, neste contexto, introduzi-la como correcção empírica. Se tanto Z como N são números pares (núcleos par-par) esta energia é particularmente elevada, sendo pelo contrário particularmente baixa para núcleos em que Z e N são ímpares (núcleos ímpar-ímpar). Introduz-se pois da seguinte maneira a contribuição B_5:

$$B_5 = \begin{cases} +\delta & \text{núcleos par-par} \\ 0 & \text{núcleos par-ímpar ou ímpar-par} \\ -\delta & \text{núcleos impar-ímpar} \end{cases} \qquad (2.52)$$

Uma fórmula empírica, válida em boa aproximação, é

$$\delta \approx a_p A^{-\frac{1}{2}} \qquad (2.53)$$

A energia de emparelhamento não pode ser explicada facilmente. Isso torna-se evidente se se pensar que um par de nucleões idênticos não está ligado. De facto nem o "diprotão" nem o "dineutrão" existem como sistemas ligados. Se esses núcleos existissem os seus nucleões apresentariam spins

desemparelhados no estado fundamental, de acordo com o princípio de Pauli. Pelo contrário, o spin do deuterão no estado fundamental é 1, ou seja, o protão e o neutrão têm spins paralelos. Isto significa que a estrutura das forças nucleares é tal que a energia de ligação é maior no caso de spins paralelos. Não se pode portanto compreender a energia de emparelhamento a partir do potencial da ligação entre pares de nucleões. Trata-se efectivamente dum fenómeno que surge somente nos sistemas de muitas partículas e cuja origem será discutida no secção 6.5.

Veja-se agora de que modo as contribuições 1) a 5) se adicionam para dar a energia de ligação. De acordo com (2.10) a massa nuclear m(Z,A) exprime-se por

$$m(Z, A) = Zm_H + (A - Z)m_n - B/c^2 .$$

Introduzindo aqui a expressão de B dada em (2.47) e fazendo uso das relações (2.48) a (2.52), resulta

$$\begin{aligned} m(Z, A) = Zm_H + (A - Z)m_n - a_V A + a_S A^{\frac{2}{3}} \\ + a_C Z^2 A^{-\frac{1}{3}} + a_A (Z - A/2)^2 A^{-1} \pm \delta \end{aligned} \tag{2.54}$$

onde as constantes a_V até a_p contêm agora um factor $1/c^2$. Esta expressão é conhecida por *fórmula de Weizsaecker* (1935). Para determinar os valores das constantes serão precisas em princípio cinco massas nucleares. Contudo, o ajuste é muito melhor se forem consideradas tantas mais massas quantas for possível, uma vez que a fórmula apenas descreve um comportamento médio. Um conjunto de valores para aquelas constantes é o seguinte [Wap 58]:

$$a_V = 17{,}011 \text{ mu} = 15{,}85 \text{ MeV/c}^2$$

$$a_S = 19{,}691 \text{ mu} = 18{,}34 \text{ MeV/c}^2$$

$$a_C = 0{,}767 \text{ mu} = 0{,}71 \text{ MeV/c}^2$$

$$a_A = 99{,}692 \text{ mu} = 92{,}86 \text{ MeV/c}^2$$

$$a_p = \pm 12{,}3 \text{ mu} = 11{,}46 \text{ MeV/c}^2$$

A contribuição dos termos individuais da expressão (2.54) para a energia de ligação por nucleão está representada na Fig.19. A figura mostra como o decréscimo da energia de superfície e do crescimento da energia de Coulomb, conduz a um máximo de B/A para $A \approx 60$. É claro que a fórmula de Weizsaecker exprime apenas o comportamento médio dos núcleos, não podendo decerto reproduzir quaisquer efeitos da estrutura em camadas. A expressão só é aplicável para $A > 30$ (ver Fig.10), produzindo para $A > 40$ valores de B/A correctos dentro de $\approx 1\%$. É, de facto, notável que um modelo tão simples seja capaz de descrever tão bem a energia de ligação. Para aplicações práticas existem fórmulas de massa que foram refinadas à custa da inclusão de hipóteses suplementares, e que produzem resultados ainda melhores que a expressão (2.54) (ver, por exemplo, [See 61, Mye 66, Gar 69]).

A constante a_A do termo de assimetria pode calcular-se a partir do modelo do gás de Fermi (v. (2.45)), mas o valor assim determinado representa apenas cerca de metade do valor determinado empiricamente a partir das massas nucleares. Existe porém uma outra contribuição para o termo de assimetria, que tem a ver com a já referida dependência que apresentam as forças nucleares relativamente ao spin. A ligação entre um neutrão e um protão que estejam alinhados paralelamente é maior que entre dois neutrões, os quais, devido ao princípio de Pauli, só podem ter orientação anti-paralela. Os núcleos com excesso de neutrões apresentam por isso uma energia de ligação menor. Verifica-se que esta contribuição é proporcional a T_z/A.

A fórmula de Weizsaecker permite deduzir um certo número de regularidades importantes. Repare-se na variação da massa nuclear ao longo duma série de isóbaros, i.e., tome-se A = const. e faça-

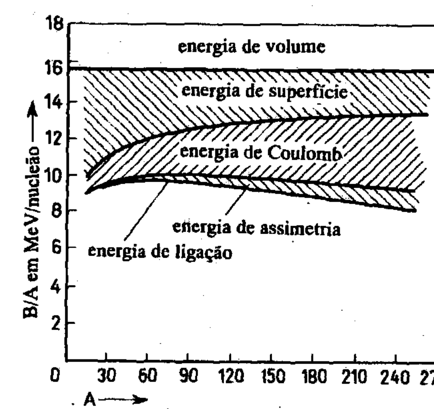


Fig.19
Contribuição dos diferentes termos da fórmula das massas nucleares para a energia de ligação média por nucleão [Eva 55].

-se variar Z em (2.54). Olhando para a expressão vê-se que ela é quadrática em Z. Para A ímpar obtém-se pois uma parábola como a representada na Fig.20a. Nos casos de A par surgem duas parábolas diferentes, devido à energia de emparelhamento ±δ. O núcleo está numa ou noutra parábola conforme seja do tipo par-par ou ímpar-ímpar (Fig.20b). Como se vê na Fig.20, nuclidos de Z vizinho podem transformar-se uns nos outros por emissão duma partícula β^+ ou β^-. Na Fig.20 lê-se também a regra segundo a qual para A ímpar apenas existe um isóbaro estável, enquanto para A par se têm vários isóbaros estáveis possíveis.

O número de protões, Z_o, para a qual a massa nuclear duma série de isóbaros é mínima ocorre para

$$\left(\frac{\partial m(Z, A)}{\partial Z}\right)_{A = \text{const}} = 0$$

Introduzindo aqui (2.54), resulta

$$-m_n + 2Z_0 a_C A^{-1/3} + 2a_A(Z_0 - A/2)A^{-1} = 0$$

e, resolvendo esta equação em ordem a Z_o, tem-se

$$Z_0 = \frac{A}{2}\left[\frac{m_n - m_H + a_A}{a_C A^{2/3} + a_A}\right] = \frac{A}{1{,}98 + 0{,}015\,A^{2/3}} \qquad (2.55)$$

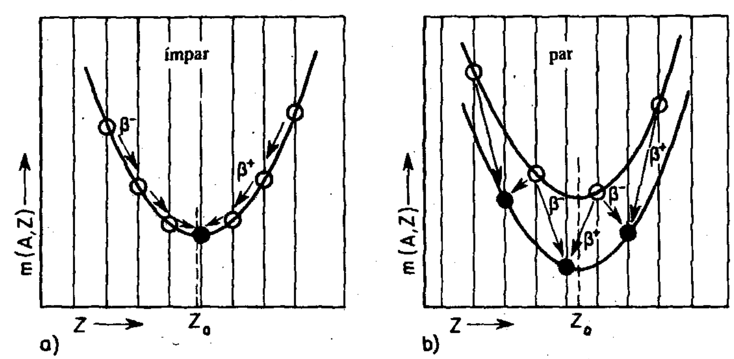


Fig.20 As energias dos núcleos com um mesmo A. Os núcleos estáveis são indicados pelos círculos a cheio.

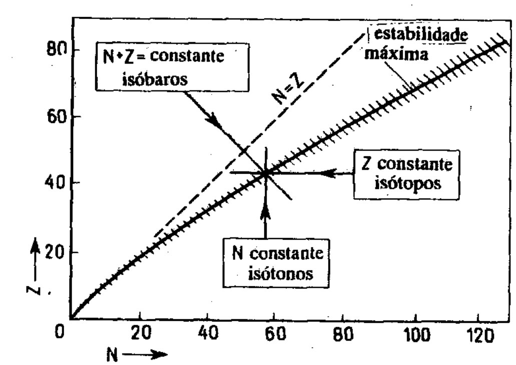
Representando estes valores num diagrama de N em função de Z, obtém-se a Fig.21. Se, além disso, as massas nucleares forem representadas segundo o eixo perpendicular ao plano NZ, a linha a cheio na Fig.21 corresponde à localização aproximada dos núcleos estáveis, i.e., que não estão sujeitos ao decaimento β. Esses núcleos são os que se encontram no fundo do "vale" das massas nucleares.

Nos processos de transmutação por decaimento β o número de massa não é alterado. Pode igualmente usar-se a fórmula de Weizsaecker para saber se um dado processo de separação de nucleões pode libertar energia. É de esperar que, frequentemente, se ganhe energia na separação duma partícula α, em particular, devido à sua elevada energia de ligação. É realmente o que acontece sempre que a soma das massas da partícula, m_α, e do núcleo resultante, m(Z-2, A-4), seja inferior à massa do núcleo original. A energia cinética libertada será

$$E_\alpha = [m(Z, A) - m(Z-2, A-4) - m_\alpha]c^2 \qquad (2.56)$$

A comparação com (2.19) mostra que isto apenas significa uma energia de separação negativa. Em princípio, ganha-se energia pela separação duma partícula α sempre que $E_\alpha>0$. Com a ajuda da fórmula de Weizsaecker é possível determinar as regiões do plano NZ que correspondem a $E_\alpha>0$, >2, >4, >6 MeV, etc. Na Fig.22 representam-se as fronteiras dessas regiões para diferentes valores de E_α. Para maior clareza, não se representa o plano NZ, mas sim N/Z em função de A. Representam-se igualmente na figura os limites das regiões de instabilidade para a separação de neutrões e de protões. Como se vê, esses limites afastam-se bastante da linha dos núcleos estáveis, que aliás nunca cruzam. Resulta daí que a emissão de neutrões ou de

Fig.21
Localização dos núcleos estáveis no plano NZ.



actually observed, it must take this effect into account. (Otherwise it would allow stable isotopes of hydrogen with hundreds of neutrons!) This term is very important for light nuclei, for which $Z \simeq A/2$ is more strictly observed. For heavy nuclei, this term becomes less important, because the rapid increase in the Coulomb repulsion term requires additional neutrons for nuclear stability. A possible form for this term, called the symmetry term because it tends to make the nucleus symmetric in protons and neutrons, is $-a_{\text{sym}}(A-2Z)^2/A$ which has the correct form of favoring nuclei with $Z = A/2$ and reducing in importance for large A.

Finally, we must include another term that accounts for the tendency of like nucleons to couple pairwise to especially stable configurations. When we have an odd number of nucleons (odd Z and even N, or even Z and odd N), this term does not contribute. However, when both Z and N are odd, we gain binding energy by converting one of the odd protons into a neutron (or vice versa) so that it can now form a pair with its formerly odd partner. We find evidence for this *pairing force* simply by looking at the stable nuclei found in nature—there are only four nuclei with odd N and Z (^{2}H, ^{6}Li, ^{10}B, ^{14}N), but 167 with even N and Z. This pairing energy δ is usually expressed as $+a_p A^{-3/4}$ for Z and N even, $-a_p A^{-3/4}$ for Z and N odd, and zero for A odd.

Combining these five terms we get the complete binding energy:

$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{\text{sym}}\frac{(A-2Z)^2}{A} + \delta \tag{3.28}$$

and using this expression for B we have the *semiempirical mass formula*:

$$M(Z,A) = Zm(^1\text{H}) + Nm_n - B(Z,A)/c^2 \tag{3.29}$$

The constants must be adjusted to give the best agreement with the experimental curve of Figure 3.16. A particular choice of $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV, $a_{\text{sym}} = 23$ MeV, $a_p = 34$ MeV, gives the result shown in Figure 3.17, which reproduces the observed behavior of B rather well.

The importance of the semiempirical mass formula is not that it allows us to predict any new or exotic phenomena of nuclear physics. Rather, it should be regarded as a first attempt to apply nuclear models to understand the systematic behavior of a nuclear property, in this case the binding energy. It includes several different varieties of nuclear models: the *liquid-drop model*, which treats some of the gross collective features of nuclei in a way similar to the calculation of the properties of a droplet of liquid (indeed, the first three terms of Equation 3.28 would also appear in a calculation of the energy of a charged liquid droplet), and the *shell model*, which deals more with individual nucleons and is responsible for the last two terms of Equation 3.28.

For constant A, Equation 3.29 represents a parabola of M vs. Z. The parabola will be centered about the point where Equation 3.29 reaches a minimum. To compare this result with the behavior of actual nuclei, we must find the minimum, where $\partial M/\partial Z = 0$:

$$Z_{\min} = \frac{[m_n - m(^1\text{H})] + a_c A^{-1/3} + 4a_{\text{sym}}}{2a_c A^{-1/3} + 8a_{\text{sym}}A^{-1}} \tag{3.30}$$

With $a_c = 0.72$ MeV and $a_{\text{sym}} = 23$ MeV, it follows that the first two terms in the numerator are negligible, and so

$$Z_{\min} \simeq \frac{A}{2}\frac{1}{1+\frac{1}{4}A^{2/3}a_c/a_{\text{sym}}} \tag{3.31}$$

For small A, $Z_{\min} \simeq A/2$ as expected, but for large A, $Z_{\min} < A/2$. For heavy nuclei, Equation 3.31 gives $Z/A \simeq 0.41$, consistent with observed values for heavy stable nuclei.

Figure 3.18 shows a typical odd-A decay chain for $A = 125$, leading to the stable nucleus at $Z = 52$. The unstable nuclei approach stability by converting a neutron into a proton or a proton into a neutron by radioactive β decay. Notice how the decay energy (that is, the mass difference between neighboring isobars) increases as we go further from stability. For even A, the pairing term gives two parabolas, displaced by 2δ. This permits two unusual effects, not seen in odd-A decays: (1) some odd-Z, odd-N nuclei can decay in either direction, converting a neutron to a proton or a proton to a neutron; (2) certain *double β decays* can become energetically possible, in which the decay may change 2 protons to 2 neutrons. Both of these effects are discussed in Chapter 9.

3.4 NUCLEAR ANGULAR MOMENTUM AND PARITY

In Section 2.5 we discussed the coupling of orbital angular momentum ℓ and spin s to give total angular momentum j. To the extent that the nuclear potential is central, ℓ and s (and therefore j) will be constants of the motion. In the quantum mechanical sense, we can therefore label every nucleon with the corresponding quantum numbers ℓ, s, and j. The total angular momentum of a nucleus containing A nucleons would then be the vector sum of the angular momenta of all the nucleons. This total angular momentum is usually called the *nuclear spin* and is represented by the symbol I. The angular momentum I has all of the usual properties of quantum mechanical angular momentum vectors: $I^2 = \hbar^2 I(I+1)$ and $I_z = m\hbar$ ($m = -I, \ldots, +I$). For many applications involving angular momentum, the nucleus behaves as if it were a single entity with an intrinsic angular momentum of I. In ordinary magnetic fields, for example, we can observe the nuclear Zeeman effect, as the state I splits up into its $2I+1$ individual substates $m = -I, -I+1, \ldots, I-1, I$. These substates are equally spaced, as in the atomic normal Zeeman effect. If we could apply an incredibly strong magnetic field, so strong that the coupling between the nucleons were broken, we would see each individual j splitting into its $2j+1$ substates. Atomic physics also has an analogy here: when we apply large magnetic fields we can break the coupling between the electronic ℓ and s and separate the $2\ell+1$ components of ℓ and the $2s+1$ components of s. No fields of sufficient strength to break the coupling of the nucleons can be produced. We therefore observe the behavior of I as if the nucleus were only a single "spinning" particle. For this reason, the spin (total angular momentum) I and the corresponding spin quantum number I are used to describe nuclear states.

To avoid confusion, we will always use I to denote the nuclear spin; we will use j to represent the total angular momentum of a single nucleon. It will often

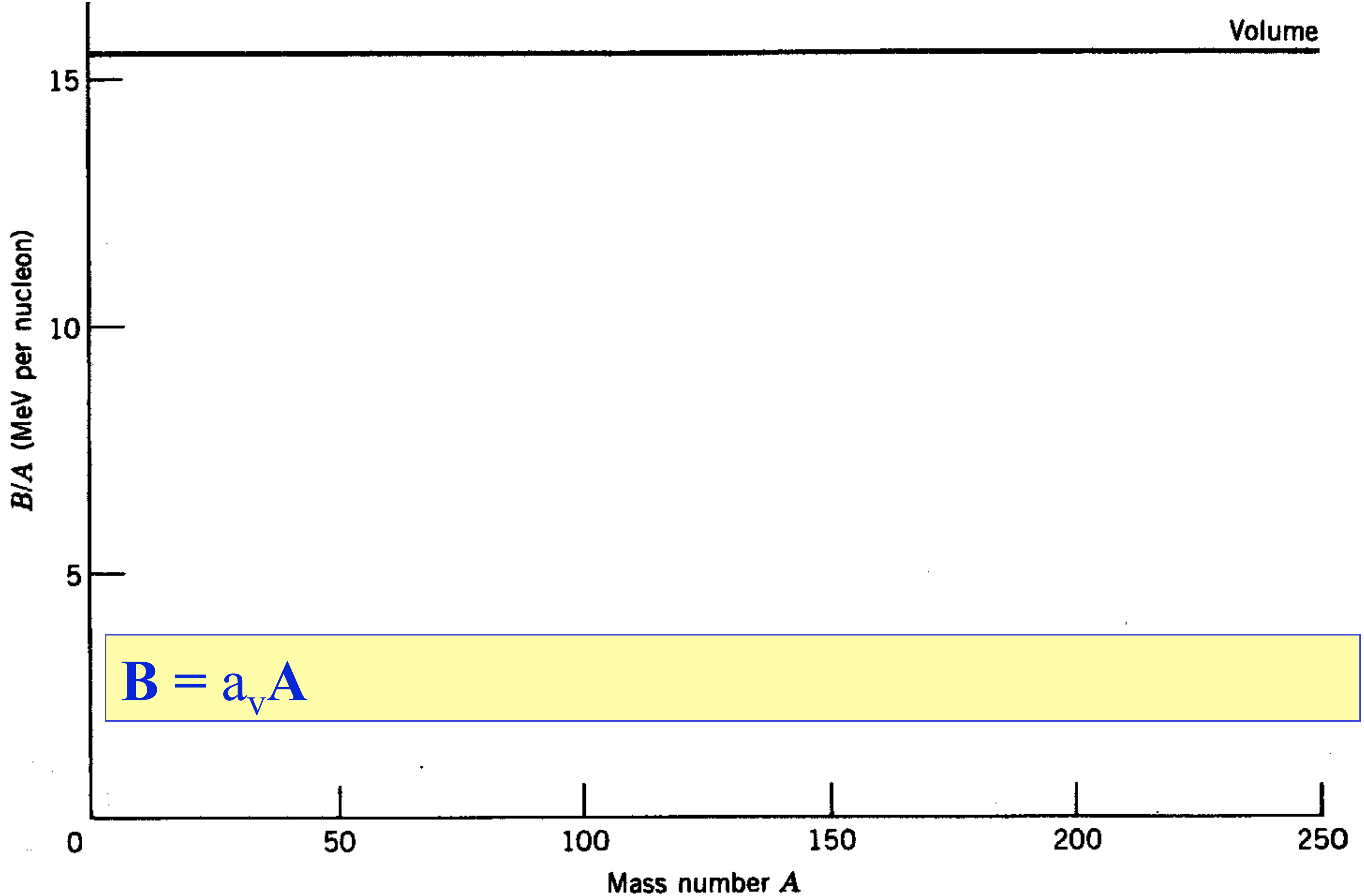


Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

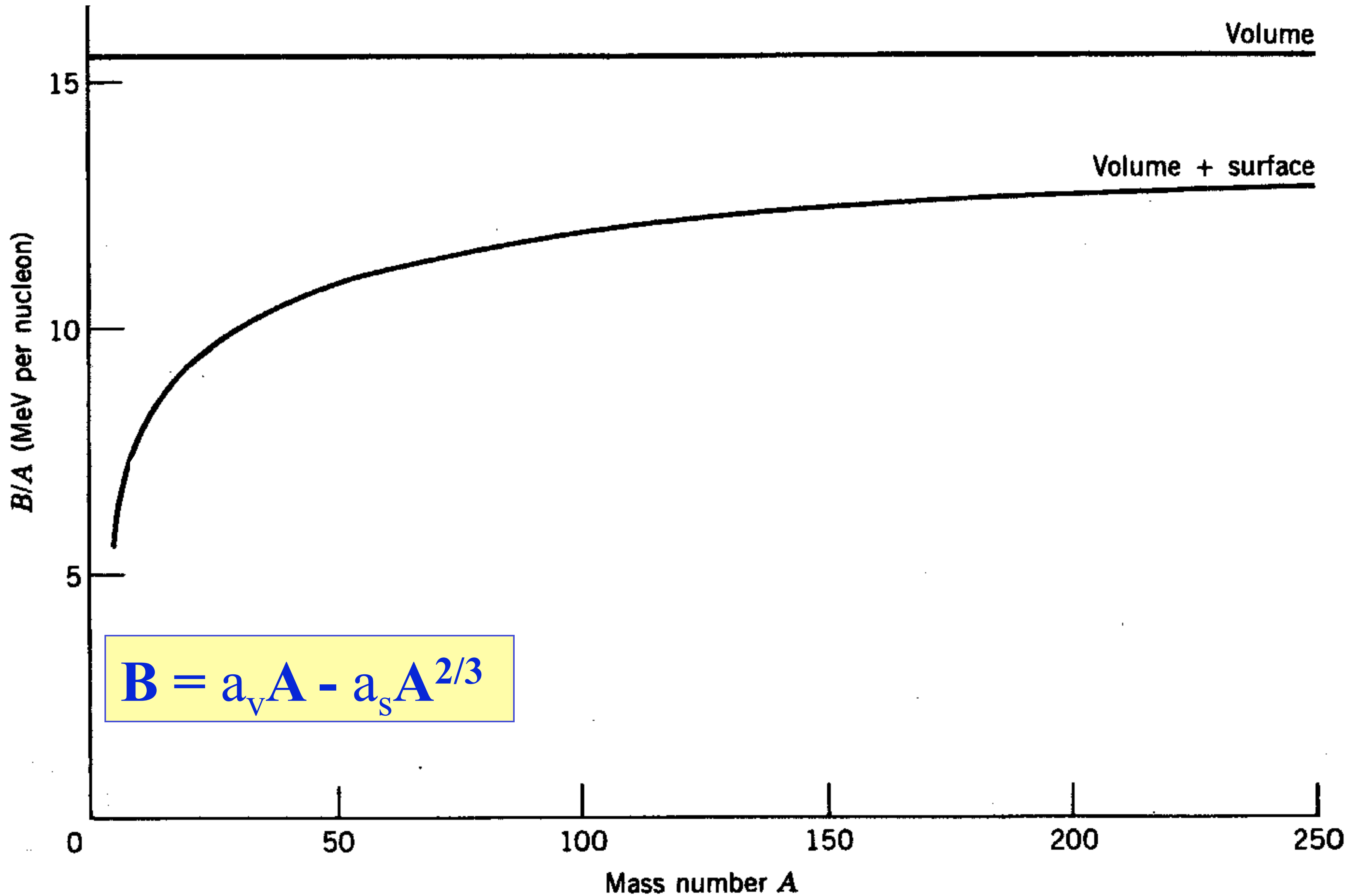
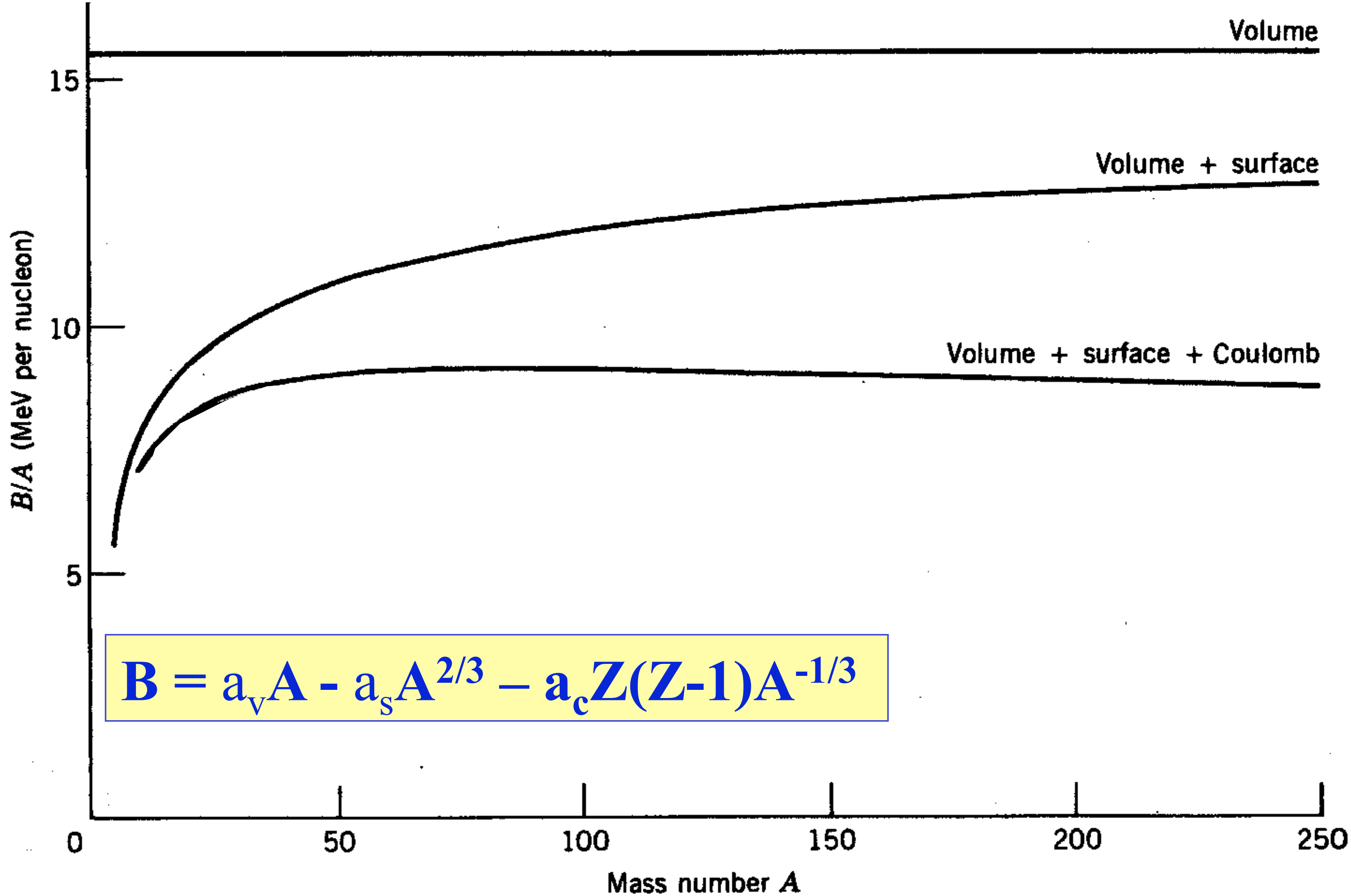


Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.



$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3}$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

TERMO DE SIMETRIA

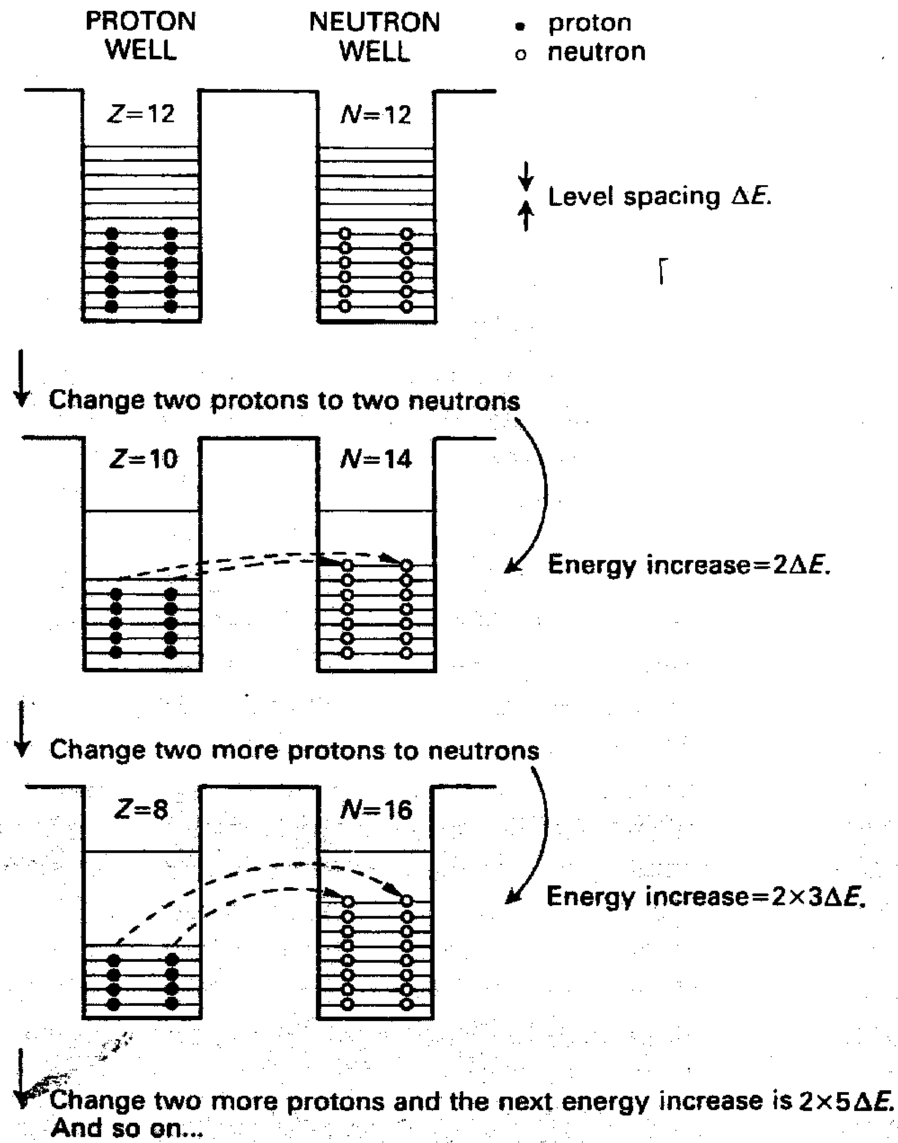
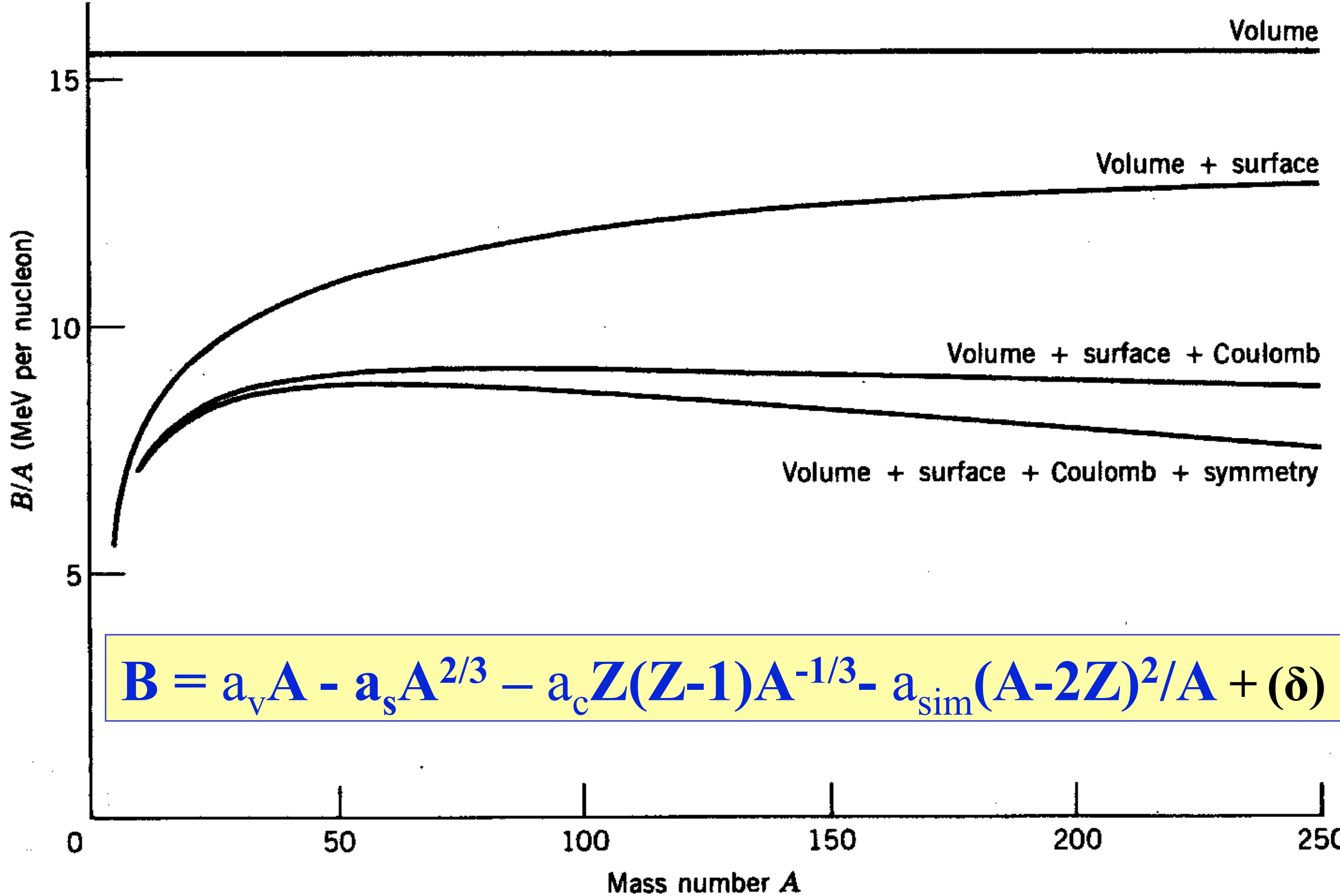


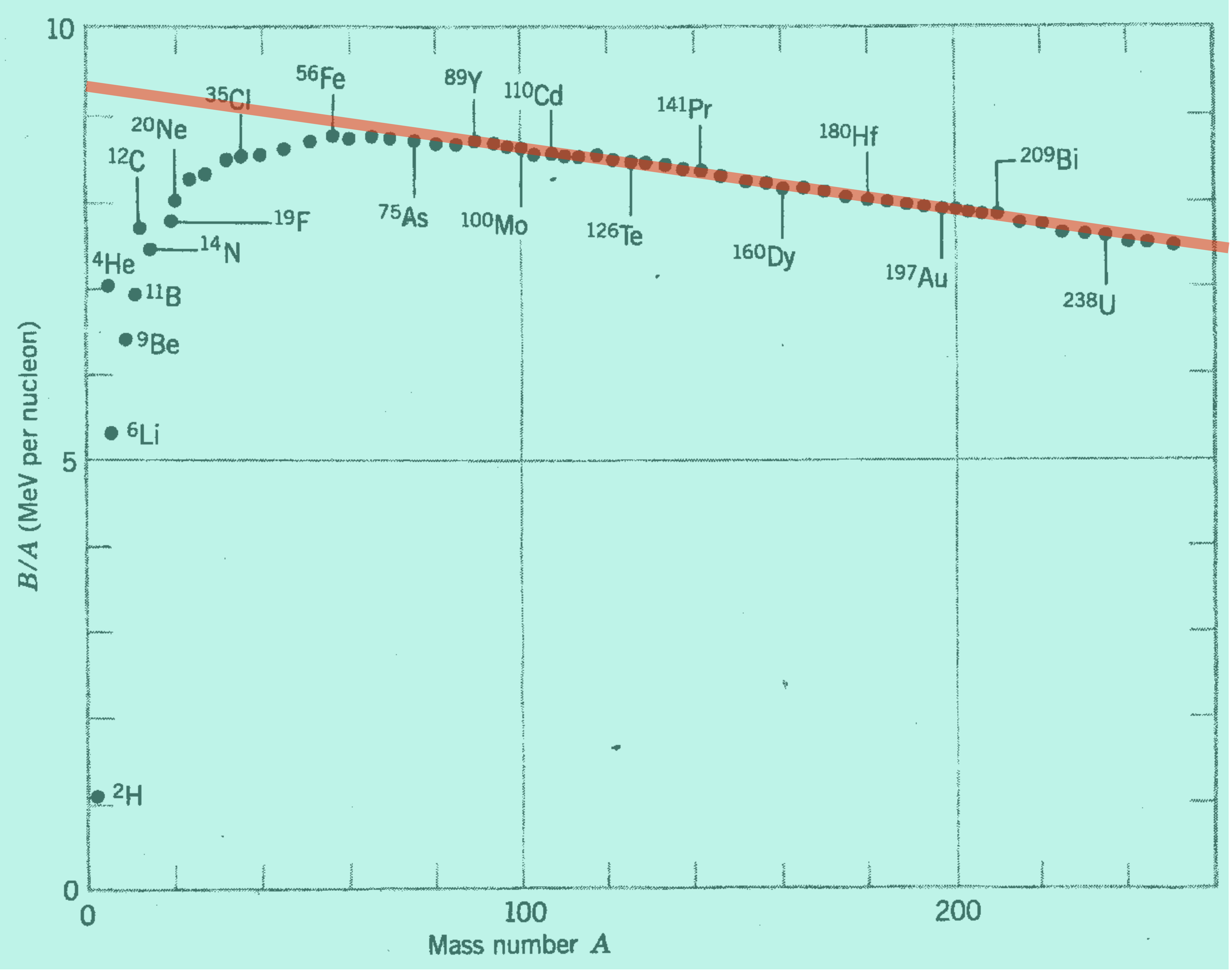
Fig. 4.3 The occupation of energy levels of a nucleus by protons (●) and by neutrons (○) according to the Pauli exclusion principle in a nucleus which is changing from $Z=N$ to $N>Z$, while $A=Z+N$ remains constant. The cost in energy of making the change of two protons into two neutrons and placing the latter in unoccupied neutron levels increases at every change. (The cost or gain in energy due to the neutron–proton mass difference is not included.)



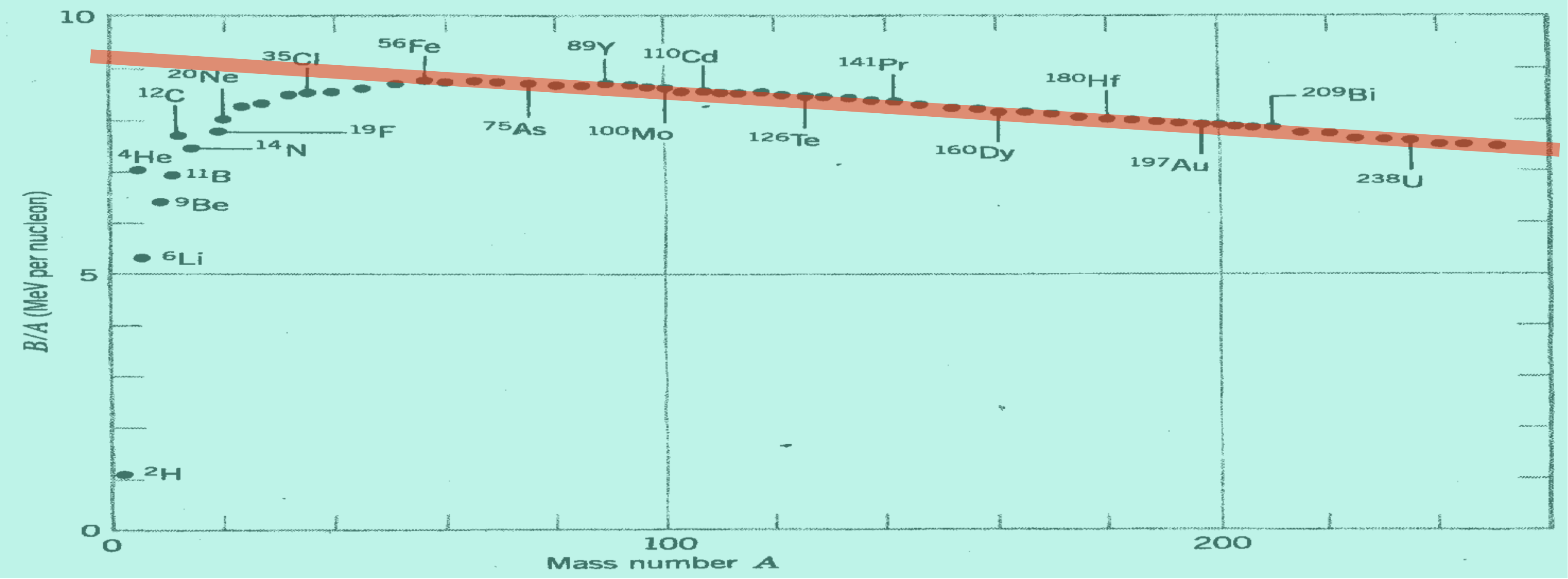
$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim}(A-2Z)^2/A + (\delta)$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

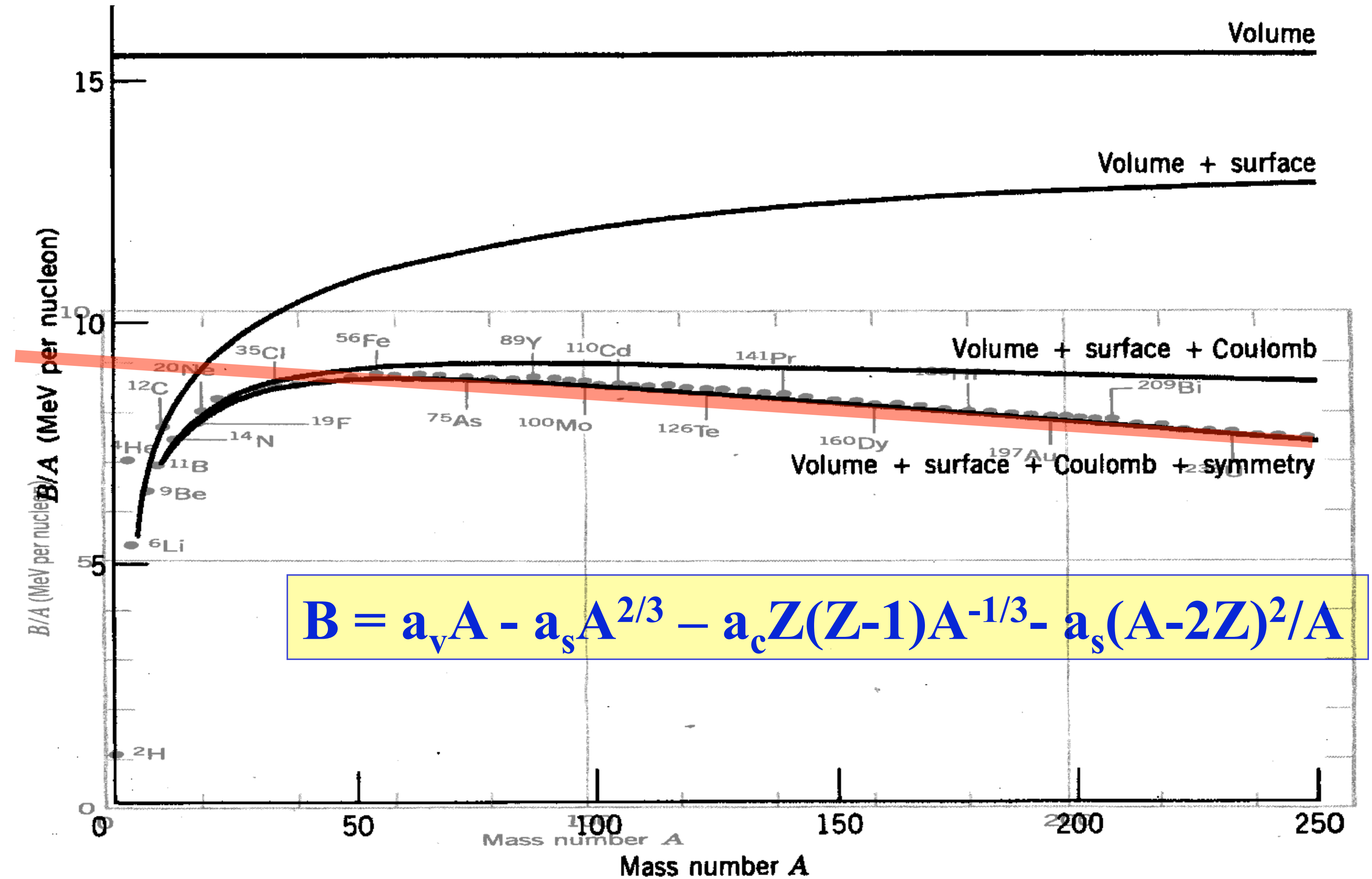
$$T_z = \tfrac{1}{2}(N-Z)$$



10
5
0
0
100
200
Mass number A
B/A (MeV per nucleon)
56Fe
89Y
110Cd
35Cl
141Pr
20Ne
180Hf
12C
209Bi
19F
75As
100Mo
126Te
14N
160Dy
4He
197Au
11B
238U
9Be
6Li
2H



B/A (MeV per nucleon)
Mass number A
10
5
0
0
100
200
2H
6Li
9Be
11B
4He
14N
12C
19F
20Ne
35Cl
56Fe
75As
89Y
100Mo
110Cd
126Te
141Pr
160Dy
180Hf
197Au
209Bi
238U



$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_s (A-2Z)^2/A$$

Figure 3.17 The contributions of the various terms in the semiempirical mass formula to the binding energy per nucleon.

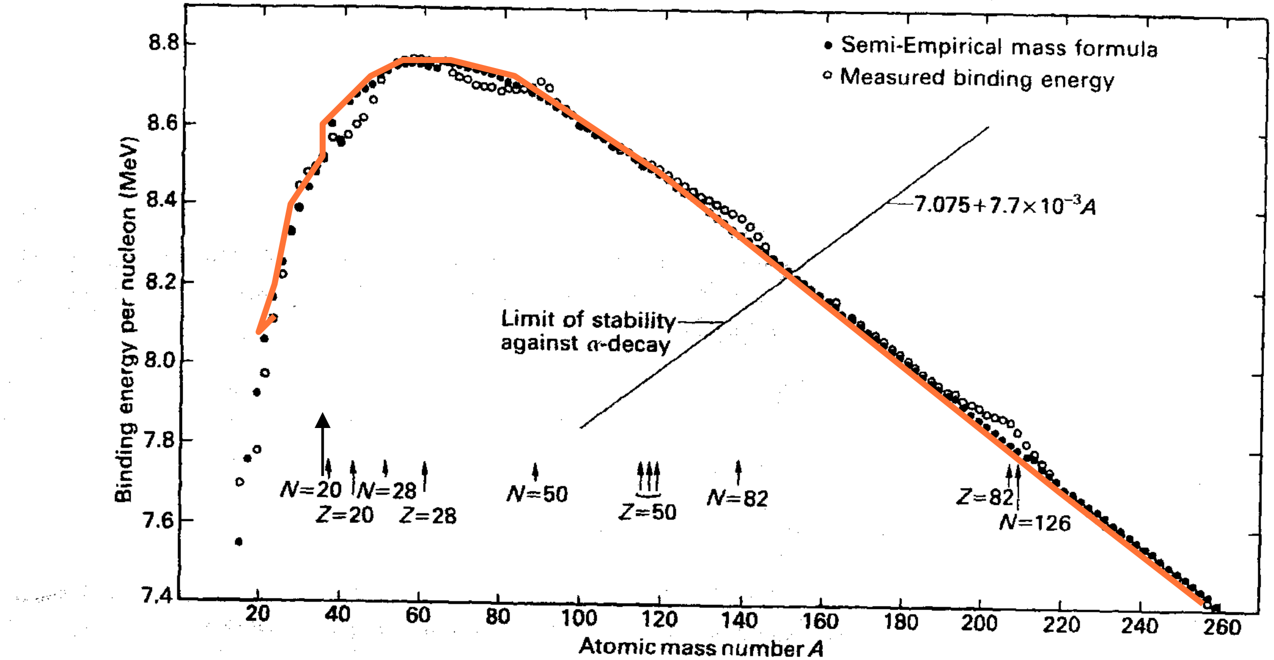
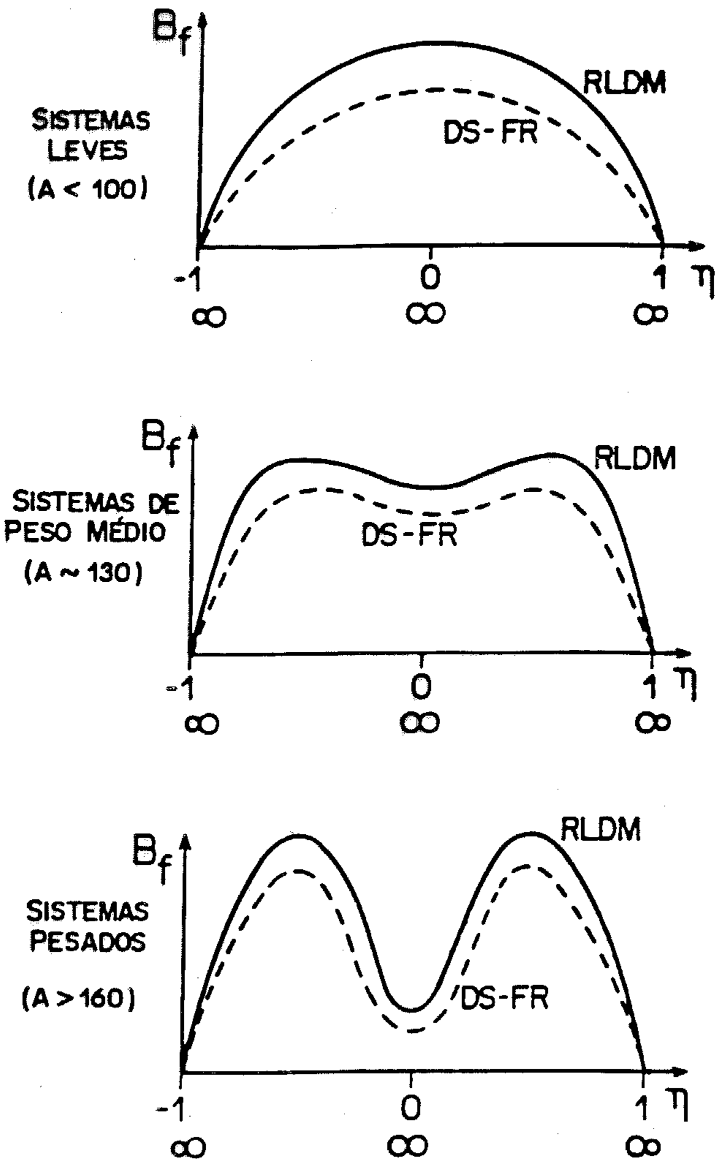


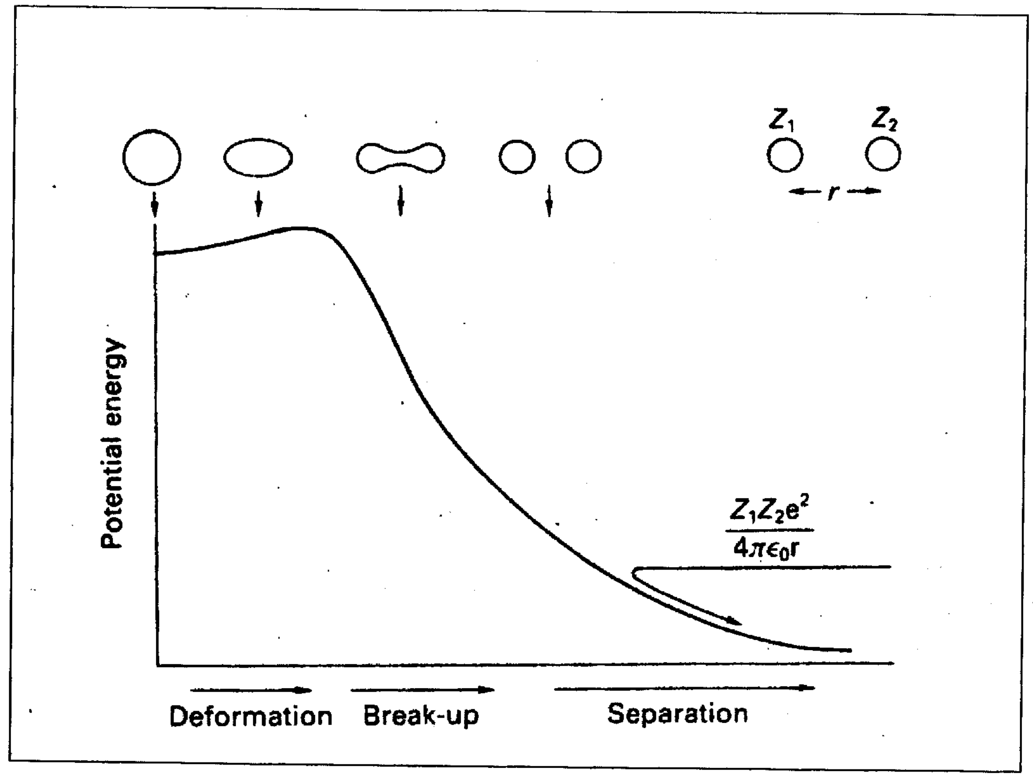
Fig. 4.6 The binding energy as a function of A for the odd-A nuclei from $A = 15$–259. The solid points are the prediction of the semi-empirical mass formula as given in Table 4.1. The open points are the measured values. The points for the formula do not lie on a smooth curve because Z for these nuclei is not a smooth function of A (see Fig. 4.1). Note that the zero of the ordinate is suppressed and its scale is much enlarged. Thus, in spite of the deviations from the formula, it is clear that the formula predicts the binding energy per nucleon for $A > 20$ with a precision which is, for the majority of cases, better than 0.1 MeV. The straight line crossing the curve at $A = 151$ gives the limit of stability of nuclei to α-decay (see Section 5.4).

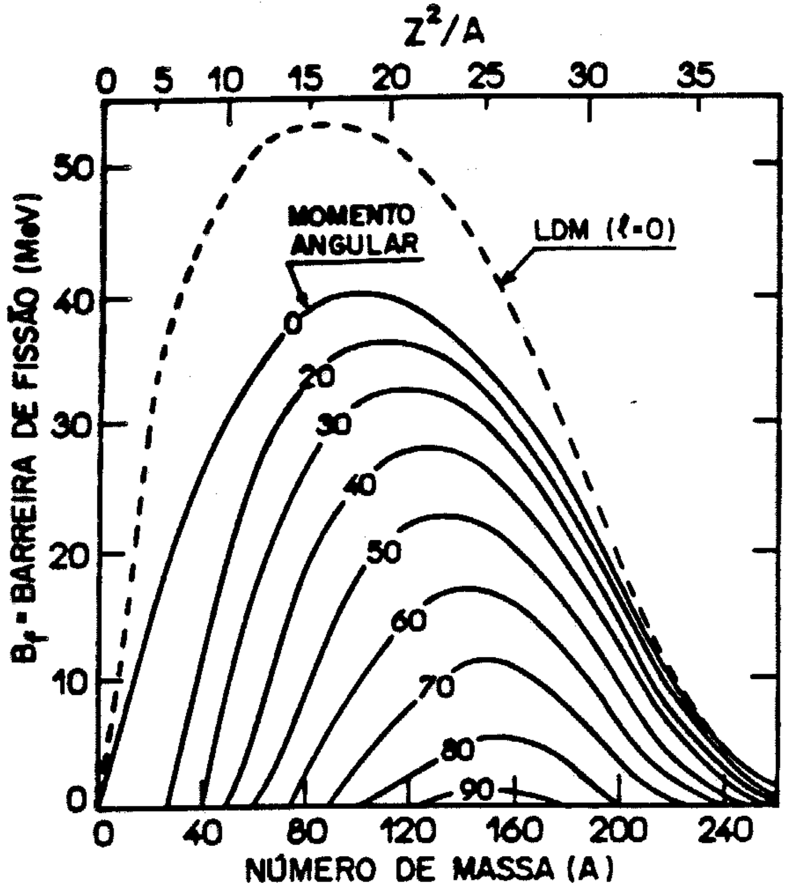
$$B = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_{sim}(A-2Z)^2/A + \delta(Z,A)$$

$$\delta(Z,A) = \begin{cases} -34A^{-3/4}\ \text{MeV} & \text{impar-impar} \\ 0 & \text{impar-par ou par-impar} \\ +34A^{-3/4}\ \text{MeV} & \text{par-par} \end{cases}$$

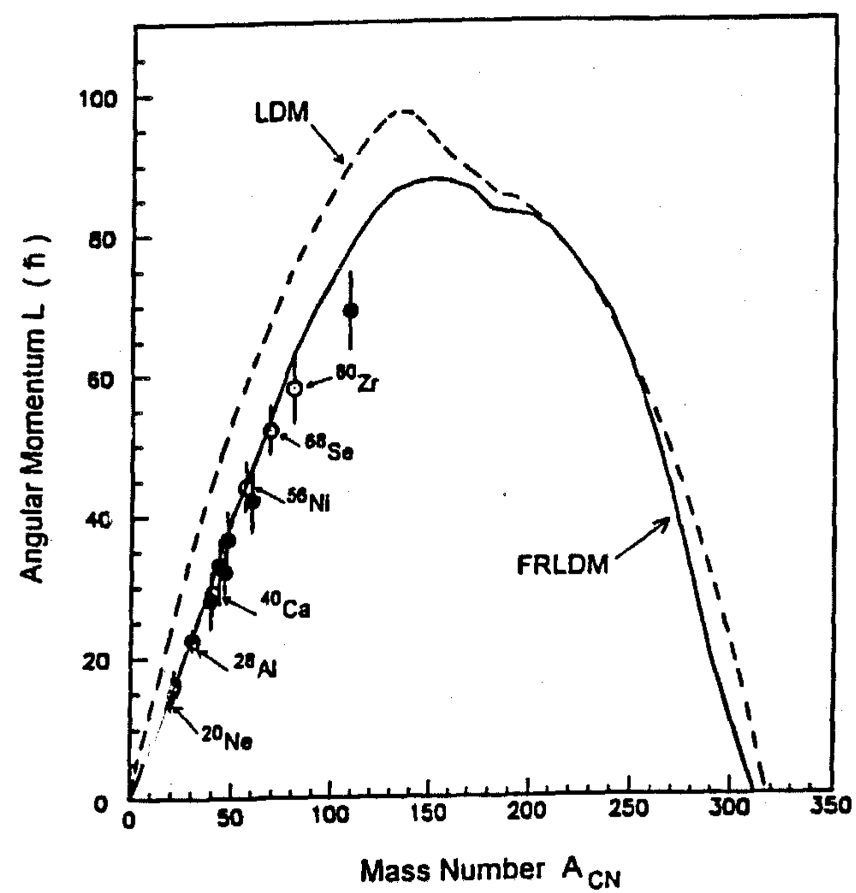


$$\eta = \frac{A_1 - A_2}{A_1 - A_2}$$





Z²/A
0 5 10 15 20 25 30 35
MOMENTO
ANGULAR
LDM (ℓ=0)
0
20
30
40
50
60
70
80
90
Bℓ = BARREIRA DE FISSÃO (MeV)
50
40
30
20
10
0
0 40 80 120 160 200 240
NÚMERO DE MASSA (A)



LDM
FRLDM
80Zr
68Se
56Ni
40Ca
28Al
20Ne
Angular Momentum L (ħ)
100
80
60
40
20
0
0 50 100 150 200 250 300 350
Mass Number A CN

momentos eletromagneticos nucleares

Para um eletron orbitando em uma orbita de raio r e area A

$$|\mu| = iA$$

$$|\mu| = \frac{e}{(2\pi r)/v}\pi r^2 = \frac{evr}{2}$$

quanticamente

Como $|\ell| = r.p = r.m.v$
$r = |\ell|/mv$

$$|\mu| = \frac{e}{2m}|\ell|$$

ou

$$|\mu| = \frac{e\hbar}{2m}|\ell|$$

$$\frac{e\hbar}{2m} \equiv \textbf{magneton}$$

$\mu_B = 5.7884 \times 10^{-5}$ eV/T

$\mu_N = 3.1525 \times 10^{-8}$ eV/T

reescrevendo $\mu = g_\ell \, \ell \, \mu_N$

$g_\ell = 1$ *para protons*
$g_\ell = 0$ *para neutrons*