2 Introduction to Axiomatic Design Principles

2.1 Introduction

To understand the complexity theory presented in this book, one has to understand Axiomatic Design Theory, on which this complexity theory is based. Complexity theory is applicable to the design of engineered systems and to natural systems such as biological systems. The purpose of this chapter is to introduce axiomatic design for engineered systems. For a more complete treatise on axiomatic design, the readers are encouraged to refer to Suh (1990, 2001).

Complexity theory provides a broad theoretical framework for understanding and designing complicated systems. It is the 40,000-foot view that the theory provides to the designers of engineered systems and to natural scientists. The theory gives guidelines for what is possible and desirable in these systems. Furthermore, the complexity theory presented in this book augments axiomatic design theory in the design of engineered systems.

Axiomatic design theory was advanced to provide a scientific basis for the design of engineered systems. It has been used in developing software, hardware, machines and other products, manufacturing systems, materials and materials processing, organizations, and large systems such as space ships. It has provided designers with logical and rational thought processes and design tools. Axiomatic design theory has been used for the following specific purposes:

1. To provide a systematic way of designing products and large systems
2. To make human designers more creative
3. To reduce the random search process
4. To minimize the iterative trial-and-error process
5. To determine the best designs among those proposed
6. To create systems architecture that completely captures the construction of the system functions and provides ready documentation
7. To endow the computer with creative power

Although it has been applied in designing many different kinds of engineered systems, mechanical examples will be used to explain the basic concept, since it is easier to illustrate axiomatic design with visual help.

When large systems are designed and developed by traditional means -- the repetition of the "design/build/test/redesign/build/test" cycle -- the cost of development is high and the development time long. Moreover, the reliability of such a system is often less than acceptable and the cost of ownership is high. These problems -- high development cost, high cost of ownership, low reliability and safety, and high life-cycle cost -- are closely tied to the complexity of these traditionally engineered systems. They probably have large time-independent real and imaginary complexities and also have large time-dependent combinatorial complexity. These complexities should be reduced to make the system more reliable at an affordable cost.

2.2 Elements of Axiomatic Design

Axioms have played a key role in the development of modern science. Many fields of science and technology owe their advances to the development and existence of axioms. Fields as diverse as mathematics, physical sciences, biological sciences, and engineering have gone through the transition from experience-based practices to the use of scientific theories and methodologies that are based on axioms.

It is only recently that axioms were developed for the field of synthesis in the form of design axioms (see Suh, 1990, for a history of axiomatic design). Nevertheless, axiomatic design follows a historical trend in science and technology.

A brief historical perspective

Axioms, which are truths that cannot be derived but for which there are no counter-examples or exceptions, have played a major role in developing natural science, which includes fields such as physics, chemistry, and biology. These fields deal with energy, matter, living organisms, and their transformations and inter-relations. Axioms were more easily accepted in these fields because the predictions that were made based on the axioms could be objectively measured and witnessed as natural phenomena.

The scientific field of thermodynamics was born as a result of attempts to generalize how "good steam engines" work. Before the field of thermodynamics emerged, many people might have said that the steam engine was too complicated to explain and that it could be designed only by experienced, ingenious designers and through trial-and-error processes. The first law of thermodynamics, which states that energy is conserved, is believed to be true because no observations or measurements contradict either the law or predictions based on the law. It defines the universal concept of energy for all sorts of diverse situations and matter. Similarly, the second law of thermodynamics was not derived. It is a generalization of the commonly observed fact that no net mechanical work can be done by a heat engine unless it exchanges heat with two other bodies. It is also an axiom in that it is believed to be a universal truth for which there are no counter-examples or exceptions. Based on the second law of thermodynamics, the concept of entropy could be derived.

Sir Isaac Newton (1642-1727) formulated three laws or axioms of mechanics. The first law states that if there is no force acting on a body, it will remain at rest or move with constant velocity in a straight line. The second law states that the product of mass and acceleration is equal to the force acting on the body. The third law states that the force that one body exerts on another must always be equal in magnitude and opposite in direction to the force that the second body exerts on the first. These were axioms. Newton's laws established the concept of force. To prove the validity of these laws or axioms, Newton applied all three of his laws to the motion of planets around the sun. Newton predicted Kepler's three laws of planetary motion based on

> his own three laws. From this work, he could determine the gravitational force acting between two masses. Newton's three laws are universally accepted because they predict observed natural phenomena and physical measurements.

These examples show that the development of natural science has been possible because of the advent of important axioms or laws that could generalize the behavior of nature. The validity of these axioms is tested by comparing the theoretical predictions of given phenomena with experimental measurements, by testing hypotheses based on these axioms, and by analysis of observed phenomena using the axioms.

Design axioms are presumed to be valid if they lead to better designs that satisfy the functional requirements and are more reliable and robust at low cost. Also if we can take a design that violates the design axioms and, using the axioms, create better-designed products and systems, the validity of the axioms can be claimed. Furthermore, theories, such as complexity theory, that are derived from the axioms and can predict the behavior of unknown systems provide further support for the verification of the design axioms.

Axiomatic approach vs. algorithmic approach

There are two ways to deal with design and complexity: *axiomatic* and *algorithmic.* In an ideal world, the development of knowledge should proceed from axioms to algorithms to tools.

In a purely algorithmic process, we try to identify or prescribe the process, so in the end, it will lead to a solution. Generally, the algorithmic approach is founded on the notion that the best way of advancing a given field is to understand the process by following the best practice. The algorithmic approach is *ad hoc* for specific situations. It is difficult to come up with algorithms for all situations. Algorithms are generally useful at the detail level, because they are manageable.

The axiomatic approach to any subject begins with the premise that there are general principles that govern the underlying behavior of the system being investigated. Axioms generate new abstract concepts, such as force, energy, and entropy. The axiomatic approach to design is based on the *abstraction* of good design decisions and processes. The design axioms were created by identifying common elements that are present in all good designs. Once the common elements could be stated, they were reduced to two axioms through a logical reasoning process (Suh, 1990). They are general principles.

2.3 Axiomatic Design Framework

There are several key concepts that are fundamental to axiomatic design. They are the existence of domains, mapping, axioms, decomposition by zigzagging between the domains, theorems, and corollaries.

2.3.1 The Concept of Domains

The design world consists of four domains.

Design involves an interplay between "what we want to achieve" and "how we choose to satisfy the need (i.e., the what)." To systematize the thought process involved in this interplay, the concept of *domains* that create demarcation lines between four different kinds of design activities provides an important foundation of axiomatic design.

The world of design is made up of four domains: the *customer domain*, the *functional domain*, the *physical domain*, and the *process domain*. The domain structure is illustrated schematically in Figure 2.1. The domain on the left represents "what we want to achieve," relative to the domain on the right, which represents the design solution, "how we propose to satisfy the requirements specified in the left domain."

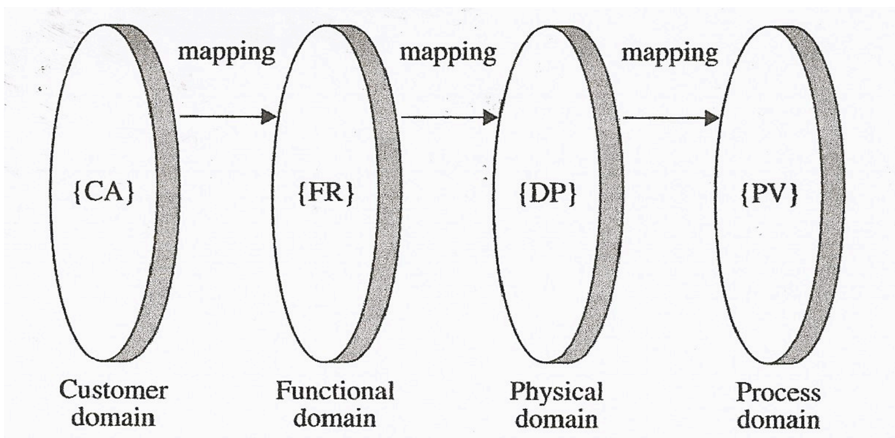

Figure 2.1 Four domains of the design world. The {x} are the characteristic vectors of each domain. During the design process we map from a left domain (i.e., what we want to know or achieve) to a domain on its right (i.e., how we hope to satisfy "what"). The process is iterative in the sense that the designer can go back to the domain on the left based on the ideas generated in the right domain.

The *customer domain* is characterized by the attributes (CAs) that the customer is looking for in a product or process or system or materials. In the *functional domain*, the customer needs are specified in terms of *functional requirements* (FRs) and *constraints* (Cs). In order to satisfy the specified FRs, we conceive *design parameters* (DPs) in the *physical domain*. Finally, to produce the product specified in terms of DPs, we develop a process that is characterized by *process variables* (PVs) in the *process domain*.

Many different fields -- software, hardware, systems, materials, organizations, and manufacturing systems -- can be described in terms of the four design domains. In the case of *product* design, the customer domain consists of the needs or attributes that the customer is looking for in a product. The functional domain consists of FRs, often defined as engineering specifications, and constraints. The physical domain is the domain in which the key DPs are chosen to satisfy the FRs. Finally, the process domain specifies the manufacturing PVs that can produce the DPs.

All design activities can be generalized in terms of the same principles. Because of this logical structure of the design world, the generalized design principles can be applied to all design applications and we can consider all design issues that arise in the four domains systematically.

Similarly, the complexity theory presented in this book should be applicable to all fields.

2.3.2 Definitions

Axioms are valid only within the bounds established by the definitions of the key terms:

> *Axiom*: Self-evident truth or fundamental truth for which there are no counter-examples or exceptions. An axiom cannot be derived from other laws or principles of nature.
>
> *Theorem*: A proposition that is not self-evident but that can be proven from accepted premises or axioms and so is established as a law or principle.
>
> *Corollary*: Inference derived from axioms or from propositions (theorems) that follow from axioms or from other propositions that have been proven.
>
> *Functional Requirement*: Functional requirements (FRs) are a minimum set of independent requirements that completely characterize the functional needs of the product (or software, organization, system, etc.) in the functional domain. By definition, each FR is independent of every other FR at the time the FRs are established.
>
> *Constraint*: Constraints (Cs) are bounds on acceptable solutions. There are two kinds of constraints: *input* constraints and *system* constraints. Input constraints are imposed as part of the design specifications. System constraints are constraints imposed by the system in which the design solution must function.
>
> *Design Parameter*: Design parameters (DPs) are the key physical variables (or other equivalent terms in the case of software design, etc.) in the physical domain that characterize the design that satisfies the specified FRs.
>
> *Process Variable*: Process variables (PVs) are the key variables (or other equivalent terms in the case of software design, etc.) in the process domain that characterize the process that can generate the specified DPs.

2.3.3 Mapping from Domain to Domain

Once we identify and define the perceived customer needs (or the attributes the customer is looking for in a product), these needs must be translated into FRs. This must be done within a "solution-neutral environment." This means that the FRs must be defined without ever thinking about something that has already been designed or what the design solution should be.

After the FRs are chosen, we map them into the physical domain to conceive a design with specific DPs that can satisfy the FRs. The mapping process is typically a one-to-many process, that is, for a given FR, there can be many possible DPs. We must choose the right DP by making sure that other FRs are not affected by the chosen DP and that the FR can be satisfied within its design range.

2.3.4 Axioms

The basic postulate of the axiomatic approach to design is that there are fundamental axioms that govern the design process. Two axioms were identified by examining the common elements that are always present in good designs.

The first axiom is called the *Independence Axiom*. It states that the independence of FRs must always be maintained. The second axiom is called the *Information Axiom*, and it states that among those designs that satisfy the Independence Axiom, the design that has the smallest information content is the best design. Because the information content is defined in terms of probability, the second axiom also states that the design with the highest probability of success is the best design. In an ideal design, the information content should be zero to satisfy the FR every time and all the time.

The axioms are formally stated as:

Axiom 1: The Independence Axiom

Maintain the independence of the functional requirements (FRs).

Axiom 2: The Information Axiom

Minimize the information content of the design.

During the mapping process, we must make the right design decisions using the Independence Axiom. When several designs that satisfy the Independence Axiom are available, the Information Axiom can be used to select the best design. When only one FR is to be satisfied by having an acceptable DP, the Independence Axiom is always satisfied and the Information Axiom is the only axiom the one-FR design must satisfy.

The case studies presented in this book will show that the performance, robustness, reliability, and functionality of products, processes, software, systems, and organizations are all significantly improved when these axioms are satisfied.

2.4 The First Axiom: The Independence Axiom

A set of FRs is a description of the design goals. The Independence Axiom states that when there are two or more FRs, the design solution must be such that each of the FRs can be satisfied without affecting any of the other FRs. This means that we have to choose a correct set of DPs to be able to satisfy the FRs and maintain their independence.

After the FRs are established, the next step in the design process is the conceptualization process, which occurs during the mapping process going from the functional domain to the physical domain.

To design, we have to go from "what" in the functional domain to "how" in the physical domain, which requires *mapping*. After the overall design concept is generated by mapping, we must identify the DPs and complete the mapping process. During this process, we must think of the different ways of fulfilling each of the FRs by identifying plausible DPs. Sometimes it is convenient to think about a specific DP to satisfy a specific FR, repeating the process until the design is completed. Identifying a DP for a given FR is somewhat straightforward, but when there are many FRs that we must satisfy, the design task becomes difficult since the Independence Axiom cannot be violated.

The mapping process between the domains can be expressed mathematically in terms of the characteristic vectors that define the design goals and design solutions. At a given level of the design hierarchy, the set of functional requirements that define the specific design goals constitutes the {FR} vector in the functional domain. Similarly, the set of design parameters in the physical domain that has been chosen to satisfy the FRs constitutes the {DP} vector. The relationship between these two vectors can be written as

$$\{FR\} = [A]\{DP\} \tag{2.1}$$

where [A] is called the *Design Matrix* that relates FRs to DPs and characterizes the product design. Equation (2.1) is a design equation for the design of a product. The design matrix is of the following form for a design which has three FRs and three DPs:

$$[A] = \begin{bmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{bmatrix} \tag{2.2}$$

When Equation (2.1) is written in a differential form as

$$\{dFR\} = [A]\{dDP\}$$

the elements of the design matrix are given by

$$Aij = \partial FRi / \partial DPj$$

With three FRs and three DPs, Equation (2.1) may be written in terms of its elements as
or

$$\begin{aligned} FR1 &= A11\ DP1 + A12\ DP2 + A13\ DP3 \\ FR2 &= A21\ DP1 + A22\ DP2 + A23\ DP3 \\ FR3 &= A31\ DP1 + A32\ DP2 + A33\ DP3 \end{aligned} \qquad (2.3)$$

In general,

$$FRi = \sum_{i=1}^{n} Aij\,DPj$$

where n = the number of DPs.

For a linear design, Aij are constants; for a nonlinear design, Aij are functions of the DPs. There are two special cases of the design matrix:

(1) The diagonal matrix, where all Aij = 0 except those where i=j.

$$[A] = \begin{bmatrix} A11 & 0 & 0 \\ 0 & A22 & 0 \\ 0 & 0 & A33 \end{bmatrix} \qquad (2.4)$$

(2) The triangular matrix; in a lower triangular (LT) matrix, all upper triangular elements are equal to zero, as shown below.

$$[A] = \begin{bmatrix} A11 & 0 & 0 \\ A21 & A22 & 0 \\ A31 & A32 & A33 \end{bmatrix} \qquad (2.5)$$

In an upper triangular (UT) matrix, all lower triangular elements are equal to zero. A UT matrix can always be changed to a LT matrix.

For the design of processes involving mapping from the {DP} vector in the physical domain to the {PV} vector in the process domain, the design equation may be written as

$$\{DP\} = [B]\{PV\} \qquad (2.6)$$

where [B] is the design matrix that defines the characteristics of the process design and is similar in form to [A].

To satisfy the Independence Axiom, the design matrix must be either diagonal or triangular. When the design matrix [A] is diagonal, each of the FRs can be satisfied independently by means of its respective DP. Such a design is called an *uncoupled* design. When the matrix is triangular, the independence of FRs can be guaranteed if and only if the DPs are determined in a proper sequence. Such a design is called a *decoupled* design. Any other form of the design matrix is called a full matrix and results in a *coupled* design.

When the matrix is a full matrix producing a coupled design, we may get a unique solution that gives the right values for FRs, but such a design has many problems. Coupled designs are not robust and cannot survive random variations of DPs and the environment surrounding the design. For example, when one of the FRs is changed, all DPs must be changed to balance the system out again. Also whenever the DPs are not exact and deviate from the desired (or set) values, the FRs may not be satisfied. Therefore, when several FRs must be satisfied, we must develop designs that will enable us to create either a diagonal or a triangular design matrix.

What are constraints?

The design goals are often subject to constraints (Cs). Constraints provide bounds on acceptable design solutions and differ from FRs in that they do not have to be independent.

There are two kinds of constraints: *input* constraints and *system* constraints. Input constraints are specific to the overall design goals (i.e., all designs that are proposed must satisfy these). System constraints are specific to a given design; they are the result of design decisions made.

The designer often has to specify input constraints at the beginning of the design process because the designed product (or process or system or software or organization) must satisfy external boundary conditions, such as the voltage and the maximum current of the power supply. The environment within which the design must function may also impose many constraints. All of these constraints must be satisfied by all proposed design embodiments regardless of the specific details of the design.

Some constraints are generated because of design decisions made as the design proceeds. All higher-level decisions act as constraints at lower levels. For example, if we have chosen to use a diesel engine in a car, all subsequent decisions related to the vehicle must be compatible with this decision. These are system constraints.

2.4.1 Ideal Design, Redundant Design, and Coupled Design -- A Matter of Relative Numbers of DPs and FRs

Depending on the relative numbers of DPs and FRs, the design can be classified as coupled, redundant, or ideal.

Case 1. Number of DPs < Number of FRs: Coupled Design

When the number of DPs is less than the number of FRs, we always have a coupled design. This is stated as Theorem 1, which is given below:

<u>Theorem 1</u> (Coupling Due to Insufficient Number of DPs)

When the number of DPs is less than the number of FRs, either a coupled design results or the FRs cannot be satisfied.

Case 2. Number of DPs > Number of FRs: Redundant Design

When there are more DPs than there are FRs, the design is called a redundant design. A redundant design may or may not violate the Independence Axiom as illustrated below.

Consider the following two-dimensional case:

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & A13 & A14 & A15 \\ A21 & A22 & 0 & A24 & 0 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \\ DP4 \\ DP5 \end{Bmatrix}$$

This design takes on various characteristics, depending on which design parameters are varied and which ones are fixed. If DP1 and DP4 are varied after DP2, DP3, and DP5 are fixed to control the values of FRs, the design is a coupled design. On the other hand, if we fix the values of DP1, DP4, and DP5, the design is an uncoupled design. If DP3, DP4, and DP5 are fixed, then the design is a decoupled design. If DP1 and DP4 are set first, then the design behaves as an uncoupled redundant design. Theorem 3 states this fact:

<u>Theorem 3</u> (Redundant Design)

When there are more DPs than FRs, the design is either a redundant design or a coupled design.

Case 3. Number of DPs = Number of FRs: Ideal Design

When the number of FRs is equal to the number of DPs, the design is an ideal design, provided that the Independence Axiom is satisfied. This is stated as Theorem 4.

<u>Theorem 4</u> (Ideal Design)

In an ideal design, the number of DPs is equal to the number of FRs, and the FRs are always kept independent from each other.

Many other theorems and corollaries are presented in Appendix 2-A. They may be used as design rules for specific cases.

2.4.2 Decomposition, Zigzagging, and Hierarchy

When the design details are missing at the highest level of design, the design equation represents the design *intent*. We must decompose the highest-level design to develop design details that can be implemented. As we decompose the highest-level design, the lower-level design decisions must be consistent with the highest-level design intent.

When the Independence Axiom is violated by design decisions made, we should go back and re-design rather than proceed with a flawed design.

How do we decompose FRs and DPs?

To decompose FR and DP characteristic vectors, we must zigzag between the domains. That is, we start out in the "what" domain and go to the "how" domain. This is illustrated in Figure 2.2. From an FR in the functional domain, we go to the physical domain to conceptualize a design and determine its corresponding DP. Then we come back to the functional domain to create FR1 and FR2 at the next level that collectively satisfy the highest-level FR. FR1 and FR2 are the FRs for the highest-level DP. Then we go to the physical domain to find DP1 and DP2, which satisfy FR1 and FR2, respectively. This process of decomposition is continued until the highest-level FR can be satisfied without further decomposition, that is, when all of the branches reach the final state. The final state is indicated by thick boxes in Figure 2.2, which are called “leaves”.

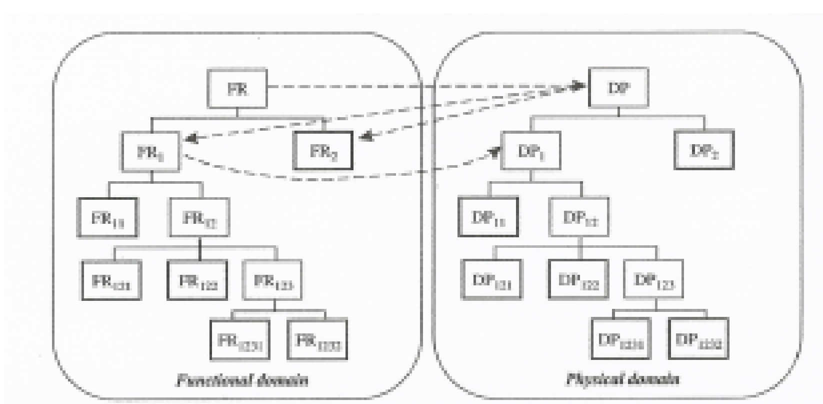

Figure 2.2 Zigzagging to decompose FRs and DPs in the functional and the physical domains to create the FR and DP hierarchies. Boxes with thick lines represent “leaves” that do not require further decomposition.

To be sure that we have made the right design decision, we must write down the design equation -- {FR} = [A]{DP} -- at each level of decomposition. For example, in the case shown in Figure 2.2, after FR and DP are decomposed into FR1, FR2 and DP1, DP2, we must write down the design equation to indicate our design *intent* at this level. At this high level of the design process, we can only state our design intent, since we have not yet developed the lower-level detailed designs. We know that the design must be either uncoupled or decoupled, and therefore, the intended design must have either a diagonal or a triangular matrix.

Suppose that the designer wanted to have a decoupled design represented by the design equation

$$\begin{Bmatrix} FR1 \\ FR2 \end{Bmatrix} = \begin{bmatrix} X & 0 \\ X & X \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \end{Bmatrix}$$

Since design details are unknown at this stage of the design process, the triangular matrix represents the design *intent*. All subsequent lower-level design decisions must be consistent with this high-level design decision. The consistency of all lower-level design decisions can be checked by constructing the master design matrix.

Through the design decomposition process, the designer is transforming design intent into realizable design details.

At the highest level of the design process, the designer develops the design concept based on the available data; that is, the designer develops design *intent*. To complete the detailed design, the FR and DP vectors must be decomposed to the lowest level of FRs and DPs, i.e., to leaf-level FRs and DPs. Throughout the decomposition process, the designer is transforming the design *intent* expressed by the higher-level design matrices into realizable detailed designs given by the lowest-level design matrices.

At each level of decomposition, the design decisions made must be consistent with all higher-level design decisions that were previously made. That is, if the highest-level design matrix is a diagonal matrix, all lower-level decisions must not make the off-diagonal elements of the highest-level design matrix non-zeroes -- either intentionally or inadvertently. To check this fidelity and consistency of design decisions, the master design matrix must be constructed by combining all lower-level design matrices into a single master matrix.

How does this design process affect inventions and innovations?

As a designer tries to develop detailed designs that do not violate the original design intent, the designer may find that existing technologies cannot be used. Then the designer may develop a new technology that can achieve the original design goals. This process of recognizing the shortcomings of existing technologies and/or designs often leads to inventions and innovations. When a coupled design is replaced by an uncoupled or a decoupled design, major improvements can be made. These novel solutions often constitute inventions or innovations.

What is the current state of design practice as far as the decomposition process is concerned?

In some large organizations, there exists a "division for engineering specification" that is charged with creating FRs at all levels. The major task of the division is to develop functional requirements or specifications for their products. These divisions are typically organized so that they have to create FRs at all levels without zigzagging, i.e., by remaining only in the functional or physical domain. As should be quite obvious by now, these divisions cannot do their job right, since FRs cannot be decomposed by remaining in one domain, i.e., without zigzagging. Thus, when designers/engineers are forced to work in such an organization, they often develop FRs or specifications by thinking of an already existing design, which results in re-specifying that which already exists.

To decompose FRs and DPs, the designer must zigzag. For example, suppose you want to design a vehicle that satisfies the following four FRs: go forward, go backward, stop, and turn. We cannot decompose these FRs unless we first conceptualize DPs that can satisfy these highest-level FRs. If we decide to use an electric motor as a DP to satisfy the FR of moving forward, the decomposed FRs at the next level would be quite different from those that would have resulted had we chosen gas turbines as the DP. Therefore, when we define the FRs in a solution-neutral environment, we have to "zig" to the physical domain, and after proper DPs are chosen, we have to "zag" to the functional domain for further decomposition. Organizations that have created a division for the specific task of specifying FRs at all levels without zigzagging between the domains will not get the results they are looking for and will miss important opportunities for innovation.

When does analysis come into the picture during the design process?

To refine the design, we must model and analyze the proposed design whenever possible. In the preceding examples, the design matrix was formulated in terms of X and 0. In some cases, it may be sufficient to complete the design using simply X and 0. In many cases, we may take further steps to determine the precise values of design parameters. After the conceptual design is done in terms of X and 0, we need to model the design more precisely to replace the Xs with equations or numbers. Through modeling, we can replace each X with either a constant or a function that involves the DP. We then have a set of equations that relates the FRs to the DPs. This set of equations can be solved separately for uncoupled designs or by following the sequence given by the design matrix for decoupled designs.

2.5 The Second Axiom: The Information Axiom

In the preceding section, the Independence Axiom was discussed and its implications were presented. The design effort may produce several designs, all of which may be acceptable in terms of the Independence Axiom. Even for the same task defined by a given set of FRs, it is likely that different designers will come up with different designs because there can be many designs that satisfy a given set of FRs. However, one of these designs is likely to be superior to the others. The Information Axiom provides a quantitative measure of the merits of a given design, and thus is useful in selecting the best among those designs that are acceptable. In addition, the Information Axiom provides the theoretical basis for design optimization and robust design.

Among the designs that are equally acceptable from the functional point of view, one may be superior to others in terms of the probability of achieving the design goals as expressed by the functional requirements. The Information Axiom states that the design with the highest probability of success is the best design.

Information content I_i for a given FR_i is defined in terms of the probability P_i of satisfying FR_i.

$$I_i = \log_2 \frac{1}{P_i} = -\log_2 P_i \tag{2.7}$$

The information is given in units of bits[1]. The logarithmic function is chosen so that the information content will be additive when there are many FRs that must be satisfied simultaneously. Either the logarithm based on 2 (with the unit of bits) or the natural logarithm (with the unit of nats) may be used.

In the general case of m FRs, the information content for the entire system I_{sys} is

$$I_{sys} = -\log_2 P_{\{m\}} \tag{2.8}$$

where $P_{\{m\}}$ is the joint probability that all m FRs are satisfied.

When all FRs are statistically independent, as is the case for an uncoupled design,

$$P_{\{m\}} = \prod_{i=1}^{m} P_i$$

Then I_{sys} may be expressed as

$$I_{sys} = \sum_{i=1}^{m} I_i = -\sum_{i=1}^{m} \log_2 P_i \tag{2.9}$$

When all FRs are not statistically independent, as is the case for a decoupled design,

[1] Although the mathematical formula for information is the same as that used in information theory, the information content in axiomatic design and that in information theory have different significance. (See Appendix 2A)

$$P_{\{m\}} = \prod_{i=1}^{m} P_{i|\{j\}} \qquad \text{for } \{j\} = \{1, \ldots, i\text{-}1\}$$

where $P_{i\,|\,\{j\}}$ is the conditional probability of satisfying FRi given that all other relevant (correlated) $\{FRj\}_{j=1,\ldots,i-1}$ are also satisfied. In this case, I_{sys} may be expressed as

$$I_{sys} = -\sum_{i=1}^{m} \log_2 P_{i|\{j\}} \qquad \text{for } \{j\} = \{1, \ldots, i\text{-}1\} \tag{2.10}$$

The Information Axiom states that the design with the smallest I is the best design, since it requires the least amount of information to achieve the design goals. When all probabilities are equal to 1.0, the information content is zero, and conversely, the information required is infinite when one or more probabilities are equal to zero. That is, if the probability is small, we must supply more information to satisfy the FRs.

A design is called *complex* when its probability of success is low, that is, when the information content required to satisfy the FRs is high. This occurs when the tolerances of FRs for a product (or DPs for a process) are small, requiring high accuracy. This situation also arises when there are many parts because as the number of parts increases, the likelihood that some of the components do not meet the specified requirements also increases. In this sense, the quantitative measure for complexity is the information content because complex systems may require more information to make the system function. A physically large system is not necessarily complex if the information content is low. Conversely, even a small system can be complex if the information content is high. Therefore, the notion of complexity is tied to the design range for the FRs -- the tighter the design range, the more difficult it becomes to satisfy the FRs.

The probability of success is governed by the intersection of the design range defined by the designer to satisfy the FRs and the ability of the system to produce the part within the specified range. For example, if the design specification for cutting a rod is 1 meter plus or minus one micron and the available tool (i.e., system) for cutting the rod consists of only a hacksaw, the probability of success will be extremely low. In this case, the information required to achieve the goal would approach infinity. Therefore, this may be called a complex design. On the other hand, if the rod needs to be cut within an accuracy of 10 cm, the hacksaw may be more than adequate, and therefore the information required is close to zero. In this case, the design is simple.

The probability of success can be computed by specifying the *design range* (*dr*) for the FR and by determining the *system range* (*sr*) that the proposed design can provide to satisfy the FR. Figure 2.3 illustrates these two ranges graphically.

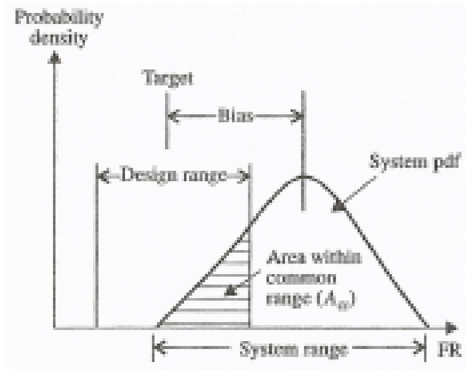

Figure 2.3 Design range, system range, common range, and system pdf for an FR.

The vertical axis (the ordinate) represents the probability density and the horizontal axis (the abscissa) represents either the FR or DP, depending on the mapping domains involved. When the mapping is between the functional domain and the physical domain as in product design, the abscissa is for the FR. When the mapping is between the physical domain and the process domain as in process design, the abscissa is for the DP.

In Figure 2.3, the system probability density function (pdf) is plotted over the system range for the specified FR. The overlap between the design range and system range is called the *common range* (*cr*), and this is the only region where the FR is satisfied. Consequently, the area under the system pdf within the common range, A_{cr}, is the design's probability of achieving the specified goal. Then the information content may be expressed as [Suh, 1990]:

$$I = \log_2 \frac{1}{A_{cr}} \tag{2.11}$$

In terms of the system pdf $p_s(FR_i)$, the probability p_i of satisfying FR_i is the integral of $p_s(FR_i)$ over the FR_i design range, which may be expressed as

$$p_i = \int_{design\ range} p_s(FR_i)dFR_i \tag{2.12}$$

When there are many FRs, the probability $p_{1,2,\ldots,n}$ of satisfying all the FRs, is given by integrating the joint density function $p_s(FR_1, FR_2, \ldots, FR_n)$ over the design space, which may be expressed as

$$p_{1,2,..,n} = \int_{design\ space} p_s(FR_1, FR_2, .., FR_n)dFR_1, dFR_2, ..., dFR_n \tag{2.13}$$

When the outcome for FR is binary, i.e., either 0 or 1, such as in software (see Suh, 2001), the probability p_i is estimated by the ratio of the number of positive outcomes divided by the total number of trials. The larger the number of trials, the more accurate will be the probability estimate.

The Information Axiom is a powerful tool for selecting the best set of DPs when there are many FRs to be satisfied, but should we also use weighting factors?

Often design decisions must be made when there are many FRs that must be satisfied at the same time. The Information Axiom provides a powerful criterion for making such decisions without the arbitrary weighting factors used in other decision-making theories. In Equation (2.9), the information content for each FR is simply summed with all other information terms without a weighting factor for two reasons. First, if we sum the information terms, each of which has been modified by multiplying it by a weighting factor, the total information content no longer represents the total probability (Homework 1.1). Second, the intention of the designer and the importance assigned to each FR by the designer are represented by the design range. If the design ranges for all of the FRs are precisely specified and if every specified FR is satisfied within its design range, the goal of the design is fully satisfied. Then there is no need for rank ordering or giving weighting factors to FRs, since the design range specifies their relative importance.

When there is only one FR, the Independence Axiom is always satisfied if there is an appropriate DP that satisfies the FR. In the one-FR case, the only task left is the selection of the right values for the design matrix element and the DP to come up with a robust design based on the Information Axiom. In the case of one-FR nonlinear design, various optimization techniques have been developed to deal with the task of finding a maximum or minimum of an objective function. However, when there are more than two FRs, some of these optimization techniques do not work.

To develop a design with more than one FR, we must first develop a design that is either uncoupled or decoupled. If the design is uncoupled, each FR can be satisfied and the optimum points for all FRs can be found because each FR is controlled only by its corresponding DP. If the design is decoupled, the FRs must be satisfied following a set sequence, which is further discussed in Chapter 3. The Information Axiom provides a measurement that enables us to measure the information content and thus be able to judge a superior design.

2.5.1 Reduction of the Information Content – Robust Design

The ultimate goal of design is to reduce the additional information required to make the system function as designed, i.e., minimize the information content, as stated by the Information Axiom. To achieve this goal, the design must be able to accommodate large variations in design parameters and process variables and yet still satisfy the functional requirements. Such a design is called a *robust* design.

To achieve a robust design, the variance of the system must be small and the bias must be eliminated to make the system range lie inside the design range, thus reducing the information content to zero (see Figure 2.3). The bias can be eliminated if the design satisfies the Independence Axiom. There are four different ways of reducing the variance of a design if the design satisfies the Independence Axiom.

2.5.1.1 Elimination of Bias

In Figure 2.3, the target value of the FR is shown at the middle of the design range. The distance between the target value and the mean of the system pdf is called the *bias*. In order to have an acceptable design, the bias associated with each FR should be very small or zero. That is, the mean of the system pdf should be equal to the target value inside the design range.

How can we eliminate bias? What are the pre-requisites for eliminating bias?

In a one-FR design, the bias can be reduced or eliminated by changing the appropriate DP, because the DP controls only this FR so we do not have to worry about its effect on other FRs. Therefore, it is easy to eliminate the bias when there is only one FR.

When there is more than one FR to be satisfied, we may not be able to eliminate the bias unless the design satisfies the Independence Axiom. If the design is coupled, each time a DP is changed to eliminate the bias for a given FR, the bias for the other FRs changes also, making the design uncontrollable. If the design is uncoupled, the design matrix is diagonal and the bias associated with each FR can be changed independently as if the design were a one-FR design. When the design is decoupled, the bias for all FRs can be eliminated by following the sequence dictated by the triangular matrix.

2.5.1.2 Reduction of Variance

What is variance? What causes variance? How do we control it? How is it related to redundant design?

Variance is a statistical measure of the variability of a pdf. Variability is caused by a number of factors, such as noise, coupling, environment, and random variations in design parameters. In a multi-FR design, the pre-requisite for variance reduction is the satisfaction of the Independence Axiom. In all situations, the variance must be minimized. The variation can be reduced in a few specific situations discussed below.

a. Reduction of the Information Content through Reduction of Stiffness

Suppose there is only one FR that is related to its DP as

$$FR1 = (A11)\ DP1 \qquad (2.14)$$

In a linear design, the allowable tolerance for DP1, given the specified design range for FR1, depends on the magnitude of A11, i.e., the stiffness. As shown in Figure 2.4, the smaller the stiffness, A11, the larger is the allowable tolerance of DP1.

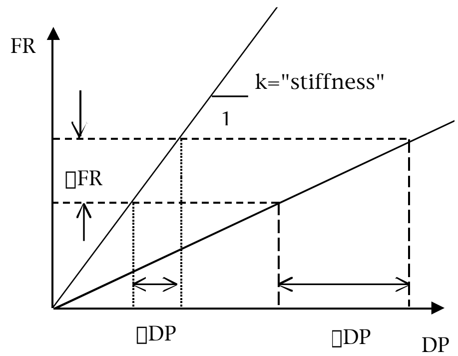

Figure 2.4 Allowable variation of DP as a function of stiffness. For a specified FR, the allowable variation of DP increases with a decrease in the stiffness, A11.

b. Reduction of the Information Content through the Design of a System That Is Immune to Variation

When the stiffness, as shown in Figure 2.4, is zero, the system will be completely insensitive to variation in DP. However, if the goal is to vary the FR by changing the DP, the stiffness must be large enough to allow control of the FR, although from the robustness point of view, low stiffness is desired. When there are many DPs that affect a given FR, design should be done so that the FR will be "immune" to variation of all these other DPs except the one specific DP chosen to control the FR. In the case of non-linear design, we should search for such a design window where this condition is satisfied.

The variance is the statistical measure of the spread of the distribution of the output. If a number of DPs are affecting an FR, the total variance of the FR is equal to the sum of the separate variances of the DPs when these DPs are statistically independent.

Often the variation in the system range may be due to many factors that affect the FR. Consider the one-FR design problem. The designer might have created a redundant design as follows:

$$FR = f(DPa, DPb, DPc)$$

or

$$FR = Aa.\ DPa + Ab.\ DPb + Ac.\ DPc \qquad (2.15)$$

where Aa, Ab, and Ac are coefficients and the DPs are design parameters that affect the FR. In this case, variation in FR can be introduced by any uncontrolled variation in all coefficients and DPs. The variance can be reduced by making the design so that the FR is not sensitive to (is immune to) changes in DPb and DPc, which can be done either if Ab and Ac are small or if DPb and DPc are fixed so that they remain constant. In this case, since the FR would be a function of only DPa, the FR can be controlled by changing DPa. In this case, the only source of variation is the random variation of Aa.

Now consider the case of the multi-FR design given by

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ 0 & A22 & 0 \\ 0 & 0 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \tag{2.16}$$

In this ideal design with a diagonal design matrix, the variance will be minimized if the random variation in A11, A22, and A33 can be eliminated. Therefore, the coefficients A11, A22, and A33 should be small, but large enough to exceed the required signal-to-noise ratio. It should be noted that any variation in DPs would also contribute to the variance and the bias.

c. Reduction of the Information Content by Fixing the Values of Extra DPs

When the design is a redundant design, the variance of the FRs can be reduced by identifying the key DPs and preventing the extra DPs from varying by fixing their values.

Consider a multi-FR design given by

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 & A14 & A15 & 0 \\ 0 & A22 & 0 & 0 & A25 & A26 \\ 0 & 0 & A33 & A34 & 0 & A36 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \\ DP4 \\ DP5 \\ DP6 \end{Bmatrix} \tag{2.17}$$

Equation (2.17) represents a redundant design because there are more DPs than FRs. The task now is to reduce the information content of this redundant design. The first thing we have to do is to find a way to make the design represented by Equation (2.17) an ideal, uncoupled design as shown by Equation (2.16). This can be done either by fixing DP4, DP5, and DP6 so that they do not act as design parameters or by making the coefficients associated with these DPs equal to zero. Fixing DP4, DP5, and DP6 also will minimize the variance in the FRs due to any variation of these three DPs. The variation can also be reduced by setting A14, A15, A25, A26, A34, and A36 to zero so that the FRs will be immune to changes in DP4, DP5, and DP6. If the design matrix were different from the one shown above, other appropriate design elements should be made zero or other appropriate DPs could be fixed to reduce the variance of the FRs.

d. Reduction of Information Content by Minimizing the Random Variation of DPs and PVs

One way of reducing the variance of the FRs is to reduce the random variation of input parameters since they contribute to the total random variation of the FRs.

The variance of FR may be expressed as

$$\sigma^2_{\mathrm{FRi}} = \sum_{j=1}^{n} \mathrm{Aij}^2 \sigma^2_{\mathrm{DPj}} + 2\sum_{j=1}^{n}\sum_{k=1}^{j-1} \mathrm{Aij\ Aik\ Cov(DPj,DPk)} \tag{2.18}$$

By reducing the variance of any of the DPj, we can reduce the contributions to the variance of FRi. Moreover, if some of the DPs are independent of one another, the relevant covariance terms disappear from Equation (2.18), further reducing the contributions to the variance of FRi.

It is clear from Equation (2.18) why it is easier to reduce the information content for uncoupled designs because only one DP contributes to the variance of FRi and there are no covariance terms.

e. Reduction of the Information Content by Compensation

The one-FR design given by Equation (2.15) was a redundant design, having three DPs rather than one DP. For the design given by Equation (2.15), we could satisfy the FR with only one DP (Theorem 4 (Ideal Design)). Therefore, the best solution for dealing with random variation (noise) for one-FR design is to eliminate the unnecessary DPs and lower the stiffness of the one DP that has been selected to satisfy the FR. However, there may be situations where a given redundant design must be made to work.

Suppose that we have to work with the less-than-ideal design represented by Equation (2.15), and that the design cannot be made to be "immune" to random variations by having low stiffness, because the coefficients associated with the redundant DPs cannot be made sufficiently small. In this case, the effect of random variation of the extra DPs on FR can be eliminated by "compensating" for the effects through the adjustment of the selected DP.

In Equation (2.15), suppose that the following is true:

Aa >> Ab and Aa >> Ac

Then we should choose DPa as the chosen DP and try to minimize the effect of the random variation of DPb and DPc on FR. The random variation will be represented as dDPb and dDPc. If we want to change FR from one state to another state, which is represented by OFR, it can be done by changing DPa by ODPa. For this change of state of FR, Equation (2.15) may be written as

$$\Delta FR = A_a\, \Delta DP_a + \sum_{i=\text{noise terms}} A_i\, \Delta DP_i \tag{2.19}$$

If the allowable random variation of FR, i.e., the design range of FR, is represented as ΔFR, the random noise term represented by the second term of RHS of Equation (2.19) can be compensated by adjusting DPa. The necessary adjustment ΔDPa to compensate for the random variation is given by

$$\Delta DP_a = \frac{\Delta FR - \sum_{i=noise\ terms} A_i \Delta DP_i}{A_a} \tag{2.20}$$

In Equation (2.20), if the noise term is larger than the allowable tolerance of FR, we have to look for a new design by choosing new DPs.

This means of compensating for the random error can be done with multi-FR designs as well as with one-FR designs if the Independence Axiom is satisfied by the multi-FR design. This kind of compensation scheme can be used to eliminate the effect of the random variation introduced during manufacturing. Such an example is given later in Example 2.6 (Van seat assembly).

f. Reduction of the Information Content by Increasing the Design Range

In some special cases, the design range can be increased without jeopardizing the design goals. The system range may then be inside the design range.

2.5.2 Reduction of the Information Content through Integration of DPs

The preceding section presented a means of reducing the information content of a design by making the system range fit inside the design range. This technique is normally called "robust design." Another equally significant means of reducing the information content is through integration of DPs in a single physical part without compromising the independence of FRs. In this way, the information content can be made small by reducing the likelihood of introducing errors when many physical parts are assembled or by making the manufacturing operation simple.

A good example of DP integration is the beverage can, which has twelve DPs, but only three physical pieces. Another example is a can and bottle opener that must open bottles and cans, but not at the same time. In this case, the DP that opens the bottle and the DP that opens the can (by punching a triangular opening in the lid of the can) may be integrated in the same steel sheet stock -- the can opener at one end and the bottle opener at the other end (Figure 3.3 in *The Principles of Design* (Suh, 1990)).

When the design has been achieved by decomposing FRs and DPs to many levels, the integration of DPs can be done in the physical domain. In this case, only the leaf-level DPs of each branch need to be integrated, since higher-level DPs are made up of the leaf-level DPs.

To create a system, all physical parts that contain the leaf-level DPs must be integrated into a physically functioning system. This system-level integration must be done from the viewpoint of minimizing information content. As of now, there is no automatic means of assembling a system without human intervention.

2.5.3 Designing with Incomplete Information

During design, we encounter situations where the necessary knowledge about the proposed design is insufficient and thus design must be executed in the absence of complete information. The basic questions are:

- Under what circumstances can design decisions be made in the absence of sufficient information?
- What are the most essential kinds of information for making design decisions?

These questions will be explored in this section.

Throughout the design process, the designer collects, manipulates, creates, classifies, transforms, and transmits information. Information in design assumes a variety of forms -- knowledge, databases, causality, paradigms, etc. The information necessary to design must be distinguished from the *information content* we need to minimize as required by the Information Axiom. Information is not as specific as the *information content*, which was specifically defined as a function of the probability of satisfying the FR in terms of design range and system range (see Equations (2.7) and (2.8)).

For example, in mapping from the CAs of the customer domain to the FRs of the functional domain, the information needed is in the form of customer preference, potential FRs, and the relationship between the CAs and the FRs. Similarly, information is needed when FRs are mapped into the physical domain and when the DPs are mapped into the process domain.

The information we need is indicated by the design equations. First, we need information on the characteristic vectors (i.e., what they are, etc.). Given an FR, the most appropriate DP must be chosen, the likelihood of which increases with the size of the library of DPs that satisfy the FR. Similarly, given a DP, the more PVs we have, the more options we will have. Once DPs and PVs are chosen, information must be available on the elements of the design matrix, which define the relationship between "what we want to achieve" and "how we want to achieve it."

One of the central issues in the design process is: "What is the minimum information that is necessary and sufficient for making design decisions given a set of DPs for a given set of FRs. The necessary information depends on whether or not the proposed design satisfies the Independence Axiom. In the case of a coupled design, which violates the Independence Axiom, all of the information associated with all elements of the design matrix is required. That is, in the case of coupled designs, design cannot be done rationally without complete information.

a. Information Required for an Uncoupled Design

Consider an ideal uncoupled design that satisfies the Independence Axiom and consists of three FRs. For this uncoupled design, which is the simplest case, the design equation may be written as

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ 0 & A22 & 0 \\ 0 & 0 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \tag{2.21}$$

A11, A22, and A33 relate FRs to DPs. They are constants in the case of linear design, whereas in the case of nonlinear design, A11 is a function of DP1, etc. To proceed with this design, we must know the diagonal elements. Therefore, the minimum information required is the information associated with the diagonal elements. The information required for the uncoupled case is less than that for the coupled case because the off-diagonal elements are zeros.

b. Information Required for a Decoupled Design

Again consider the three-FR case, but this time the design is a decoupled design given by the following design equation:

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ A21 & A22 & 0 \\ A31 & A32 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \tag{2.22}$$

As in the case of the uncoupled design given by Equation (2.21), we need to know the diagonal elements Aii. It is also desirable to know the off-diagonal elements Aij. However, information on the off-diagonal elements may not be required to satisfy the given set of FRs with a given set of DPs. We can proceed with the design if the diagonal elements are known and if the magnitudes of the off-diagonal elements are smaller than those of the diagonal elements, i.e., if Aii>Aij. This can be done because the value of FR1 can be set first, and then the value of FR2 can be set by varying the value of DP2, regardless of the value of A21. When DP2 is chosen, we must be certain that it does not affect FR1, but it is not necessary that any information for A21 be available if DP2 has the dominant effect on FR2, i.e., if A22>A21. Similarly, as long as DP3 does not affect FR1 or FR2, the design can be completed even if we do not have any information on A31 and A32. This is the only case when design can proceed in the absence of complete information. This is stated as Theorem 17.

Suppose that the upper triangular elements are not quite equal to zero but have very small values, as shown in Equation (2.23):

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & a12 & a13 \\ A21 & A22 & a23 \\ A31 & A32 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \tag{2.23}$$

The absolute magnitudes of the elements a_{ij} are much smaller than those of A_{ij}, i.e., $|a_{ij}| << |A_{ij}|$. In this case, FR1 will still be affected by large state changes of DP2 and DP3 and this effect may not be negligible since

$$\Delta FR1 = A11\ \Delta DP1 + a12\ \Delta DP2\ + a13\ \Delta DP3 \qquad (2.24)$$

where O signifies a large change in the value of FRi due to large state changes in the DPs. In this case, we must compensate for the effect of the DP state changes if the design range of FRi is smaller than the variability caused by these state changes.

2.6 Allowable Tolerances of Uncoupled, Decoupled, and Coupled Designs

As stated in the preceding sections, an ideal design is an uncoupled design with a diagonal design matrix with zero information content. In this case, a multi-FR design is almost identical to a one-FR design problem. For each FR, we can write a design equation relating the FR to a single DP. If there are *m* FRs, there are *m* design equations, each of which can be solved independently. Modeling of the design also becomes simple because the modeling can be limited to relating one FR to one DP. The element of the design matrix can be expressed quantitatively or analytically. Furthermore, the design can be made robust using the techniques discussed in Section 2.5.

Decoupled designs can also be modeled similarly, although this involves additional consideration of the off-diagonal elements and the sequence of the operation. However, there is a substantial difference between the uncoupled design and the decoupled design in the allowable DP and PV tolerances.

How does the tolerance propagate from domain to domain in the case of an uncoupled design?

Tolerance specification is simple in the case of an uncoupled design. If the specified design range for FRi is ΔFRi, then the tolerance for DPi is simply

$$\Delta DPi = \frac{\Delta FRi}{Aii} \qquad (2.25)$$

Because the goal of a robust design is to make ΔDPi as large as possible, Aii should be made small. Similarly, the tolerance for PVi is

$$\Delta PVi = \frac{\Delta DPi}{Bii} \qquad (2.26)$$

The *design range* is defined by ΔFR. The actual variation of FR, which is determined by the variation of DPs and PVs as well as by the magnitude of the design matrix elements, defines the system range. If the system range determined by the random variation of FR is completely contained within the specified design range ΔFRi, then the information content is equal to zero.

How does the tolerance propagate from domain to domain in the case of a decoupled design?

Is the propagation of tolerance different for a decoupled design from the case of an uncoupled design discussed so far? Can a decoupled design be as robust as an uncoupled design? Why? How are they different?

Consider the decoupled design shown below.

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & 0 & 0 \\ A21 & A22 & 0 \\ A31 & A32 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \tag{2.27}$$

The Independence Axiom can be satisfied if we change the DPs in the order shown. However, to have a robust design, we must be sure that the off-diagonal elements are much smaller than the diagonal elements, i.e., Aii>>Aij.

If the specified design ranges for the FRs are ΔFR1, ΔFR2, and ΔFR3, the maximum allowable tolerances for the DPs may be expressed as

$$\begin{aligned} \Delta DP1 &= \frac{\Delta FR1}{A11} \\ \Delta DP2 &= \frac{\Delta FR2 - \left|A21\ \Delta DP1\right|}{A22} \\ \Delta DP3 &= \frac{\Delta FR3 - \left|A31\ \Delta DP1\right| - \left|A32\ \Delta DP2\right|}{A33} \end{aligned} \tag{2.28}$$

The fluctuation of ΔDP2 due to the term A21 ΔDP1 can make ΔDP2 larger or smaller depending on its sign. However, the maximum allowable ΔDP2 corresponds to the worst possible case, i.e., when ΔDP2 is made smaller by the term (A21 ΔDP1). A similar argument holds for ΔDP3. Therefore, the absolute value represented by |x| is used to represent the worst possible case.

According to Equation (2.28), the maximum tolerances for DPs of a decoupled design are less than the corresponding tolerances for DPs of an uncoupled design. This means that the decoupled design is inherently less robust than the uncoupled design. This may be stated as Theorem 22[2].

<u>Theorem 22</u> (Comparative Robustness of a Decoupled Design)

Given the maximum tolerances for a given set of FRs, decoupled designs cannot be as robust as uncoupled designs in that the allowable tolerances for DPs of a

[2] These theorem numbers correspond to those given in Suh, 2001. See Appendix 2-A.

decoupled design are less than those of an uncoupled design.

Equation (2.28) was for a decoupled design with three FRs and three DPs. Extending the argument given above to the case of *m* FRs and *m* DPs, it becomes obvious that as *m* increases, the allowable tolerance for the last DP of the triangular matrix becomes increasingly smaller. This means that the robustness of a decoupled design diminishes as the number of FRs increases.

Theorem 23 (Decreasing Robustness of a Decoupled Design)

The allowable tolerance and thus the robustness of a decoupled design with a full triangular matrix diminish with an increase in the number of functional requirements.

How does the tolerance propagate in the case of coupled designs? Can a coupled design be robust?

In the case of a coupled design, the maximum allowable tolerance is even smaller than was the case for a decoupled design. Consider the following coupled design

$$\begin{Bmatrix} FR1 \\ FR2 \\ FR3 \end{Bmatrix} = \begin{bmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{bmatrix} \begin{Bmatrix} DP1 \\ DP2 \\ DP3 \end{Bmatrix} \qquad (2.29)$$

The above equation may be solved for DPs if the determinant of the design matrix |A| is not equal to zero, which is likely to be the case. The solution for DP1 is

$$DP1 = \frac{1}{|A|}\{\alpha\, FR1 - \beta\, FR2 - \gamma\, FR3\} \qquad (2.30)$$

where

$$\alpha = \text{A22 A33} - \text{A23 A32}$$
$$\beta = \text{A12 A33} - \text{A32 A13}$$
$$\gamma = \text{A22 A13} - \text{A12 A23}$$

The expressions for DP2 and DP3 are of a similar form.

For a given set of design ranges of FRs, the maximum allowable tolerances for DPs may be expressed as

$$\Delta DP1 = \frac{1}{|A|}\left\{\alpha\, \Delta FR1 + |\beta\, \Delta FR2| + |\gamma \Delta FR3|\right\} \qquad (2.31)$$

As argued before, although the magnitudes of $\beta\Delta$FR2 and $\gamma\Delta$FR3 can be either positive or negative, the maximum allowable ΔDP1 is given by Equation (2.31). Therefore, the allowable tolerances of DPs for coupled designs are smaller than those for uncoupled or decoupled designs.

The main point of this section is to show that because coupling terms are present in the design matrix, the time-independent real complexity is likely to increase since the allowable tolerances of DPs and PVs decrease.

2.7 Summary

Since the complexity theory presented in this book is based on axiomatic design theory, the basic elements of axiomatic design theory are reviewed in this chapter. The basic concepts and methodologies of Axiomatic Design include domains, mapping, the two design axioms (the Independence Axiom and the Information Axiom), decomposition, hierarchy, and zigzagging.

Several key terms, such as functional requirement (FR), design parameter (DP), and process variable (PV), are carefully defined, because strict adherence to definitions is required in an axiomatic treatise of the subject matter for internal consistency, logical deduction, and mathematical derivation of the resulting relationships. The acceptance of these definitions is a pre-requisite in applying the axiomatic principles for design.

Mapping between the domains generates design equations and design matrices. The design equation models the relationship between the design objectives (*what* the design is trying to achieve) and the design features (*how* the design goals are to be satisfied). The design matrix describes the relationship between the characteristic vectors of the domains and forms the basis for functional analysis of the design in order to identify acceptable designs. Uncoupled and decoupled designs are shown to satisfy the Independence Axiom and thus are acceptable. Coupled designs do not satisfy the Independence Axiom and thus are unacceptable.

The Independence Axiom states that the FRs must always be maintained independent of one another by choosing appropriate DPs. To be able to satisfy the FRs, the designer must always think in terms of FRs before any solution is sought. Robust design is a design that satisfies the FRs easily, although large tolerances are given to DPs and PVs. Decomposition of FRs and DPs can be done by zigzagging between the functional and the physical domains to deal with complex designs and complex systems.

The Information Axiom deals with information content, the probability of satisfying the FRs, and complexity. Information content is defined in terms of the probability of success and is the additional information required to satisfy the FR. Complexity is related to information content, since it is more difficult to meet the design objectives when the probability of success is low. Computing the information content in a design is facilitated by the notion of the design range and the system range. The design range is specified for each FR by the designer, whereas the system range is the resulting actual performance of the design embodiment.

References

Suh, N. P. *The Principles of Design*, Oxford University Press, New York, 1990.

Suh, N. P. "Design and Operation of Large Systems," *Journal of Manufacturing Systems*, Vol. 14, No. 3, pp 203-213, 1995.

Suh, N. P. *Axiomatic Design: Advances and Applications*, Oxford University Press, New York, 2001.

Appendix 2-A Corollaries and Theorems

Some of these theorems are derived in this book as well as in the references given. For those theorems not derived in this book, the readers may consult the original references.

1. Corollaries

Corollary 1 (Decoupling of Coupled Designs)
Decouple or separate parts or aspects of a solution if FRs are coupled or become interdependent in the designs proposed.

Corollary 2 (Minimization of FRs)
Minimize the number of FRs and constraints.

Corollary 3 (Integration of Physical Parts)
Integrate design features into a single physical part if the FRs can be independently satisfied in the proposed solution.

Corollary 4 (Use of Standardization)
Use standardized or interchangeable parts if the use of these parts is consistent with the FRs and constraints.

Corollary 5 (Use of Symmetry)
Use symmetrical shapes and/or components if they are consistent with the FRs and constraints.

Corollary 6 (Largest Design Ranges)
Specify the largest allowable design range in stating FRs.

Corollary 7 (Uncoupled Design with Less Information)
Seek an uncoupled design that requires less information than coupled designs in satisfying a set of FRs.

Corollary 8 (Effective Reangularity of a Scalar)
The effective reangularity R for a scalar coupling "matrix" or element is

unity. [Note: Reangularity is defined in Suh (1990).]

2. Theorems of General Design

Theorem 1 (Coupling Due to Insufficient Number of DPs)
When the number of DPs is less than the number of FRs, either a coupled design results or the FRs cannot be satisfied.

Theorem 2 (Decoupling of Coupled Design)
When a design is coupled because of a larger number of FRs than DPs (i.e., $m > n$), it may be decoupled by the addition of new DPs so as to make the number of FRs and DPs equal to each other if a subset of the design matrix containing $n \times n$ elements constitutes a triangular matrix.

Theorem 3 (Redundant Design)
When there are more DPs than FRs, the design is either a redundant design or a coupled design.

Theorem 4 (Ideal Design)
In an ideal design, the number of DPs is equal to the number of FRs and the FRs are always maintained independent of each other.

Theorem 5 (Need for New Design)
When a given set of FRs is changed by the addition of a new FR, by substitution of one of the FRs with a new one, or by selection of a completely different set of FRs, the design solution given by the original DPs cannot satisfy the new set of FRs. Consequently, a new design solution must be sought.

Theorem 6 (Path Independence of Uncoupled Design)
The information content of an uncoupled design is independent of the sequence by which the DPs are changed to satisfy the given set of FRs.

Theorem 7 (Path Dependency of Coupled and Decoupled Design)
The information contents of coupled and decoupled designs depend on the sequence by which the DPs are changed to satisfy the given set of FRs.

Theorem 8 (Independence and Design Range)
A design is an uncoupled design when the designer-specified range is greater than

$$\left(\sum_{\substack{i\neq j\\ j=1}}^{n} \frac{\partial \mathrm{FRi}}{\partial \mathrm{DPj}} \Delta \mathrm{DPj}\right)$$

in which case, the non-diagonal elements of the design matrix can be neglected from design consideration.

Theorem 9 (Design for Manufacturability)
For a product to be manufacturable with reliability and robustness, the design matrix for the product, [A] (which relates the FR vector for the product to the DP vector of the product), times the design matrix for

the manufacturing process, [B] (which relates the DP vector to the PV vector of the manufacturing process), must yield either a diagonal or a triangular matrix. Consequently, when either [A] or [B] represents a coupled design, the independence of FRs and robust design cannot be achieved. When they are full triangular matrices, either both of them must be upper triangular or both, lower triangular for the manufacturing process to satisfy independence of functional requirements.

Theorem 10 (Modularity of Independence Measures)
Suppose that a design matrix [DM] can be partitioned into square submatrices that are nonzero only along the main diagonal. Then the reangularity and semangularity for [DM] are equal to the product of their corresponding measures for each of the non-zero submatrices. [Note: See Suh (1990).]

Theorem 11 (Invariance)
Reangularity and semangularity for a design matrix [DM] are invariant under alternative orderings of the FR and DP variables, as long as the orderings preserve the association of each FR with its corresponding DP.

Theorem 12 (Sum of Information)
The sum of information for a set of events is also information, provided that proper conditional probabilities are used when the events are not statistically independent.

Theorem 13 (Information Content of the Total System)
If each DP is probabilistically independent of other DPs, the information content of the total system is the sum of the information of all individual events associated with the set of FRs that must be satisfied.

Theorem14 (Information Content of Coupled versus Uncoupled Designs)
When FRs are changed from one state to another in the functional domain, the information required for the change is greater for a coupled design than for an uncoupled design.

Theorem 15 (Design-Manufacturing Interface)
When the manufacturing system compromises the independence of the FRs of the product, either the design of the product must be modified or a new manufacturing process must be designed and/or used to maintain the independence of the FRs of the products.

Theorem 16 (Equality of Information Content)
All information contents that are relevant to the design task is equally important regardless of its physical origin, and no weighting factor should be applied to them.

Theorem 17 (Design in the Absence of Complete Information)
Design can proceed even in the absence of complete information only in the case of a decoupled design if the missing information is related to the off-diagonal elements.

Theorem 18 (Existence of an Uncoupled or Decoupled Design)
There always exists an uncoupled or a decoupled design that has less information than a coupled design.

Theorem 19 (Robustness of Design)
An uncoupled design and a decoupled design are more robust than a coupled design in the sense that it is easier to reduce the information content of designs than to satisfy the Independence Axiom.

Theorem 20 (Design Range and Coupling)
If the design ranges of uncoupled or decoupled designs are tightened, they may become coupled designs. Conversely, if the design ranges of some coupled designs are relaxed, the designs may become either uncoupled or decoupled.

Theorem 21 (Robust Design When the Design Range has a Non-Uniform pdf)
If the probability distribution function (pdf) of the FR in the design range is non-uniform, the probability of success is equal to 1 when the system range is inside the design range.

Theorem 22 (Comparative Robustness of a Decoupled Design)
Given the maximum design ranges for a given set of FRs, decoupled designs cannot be as robust as uncoupled designs in that the allowable tolerances for the DPs of a decoupled design are less than those of an uncoupled design.

Theorem 23 (Decreasing Robustness of a Decoupled Design)
The allowable tolerance and thus the robustness of a decoupled design with a full triangular matrix diminish with an increase in the number of functional requirements.

Theorem 24 (Optimum Scheduling)
Before a schedule for robot motion or factory scheduling can be optimized, the design of the tasks must be made to satisfy the Independence Axiom by adding decouplers to eliminate coupling. The decouplers may be in the form of a queue or of separate hardware or buffer.

Theorem 25 ("Push" System vs "Pull" System)
When identical parts are processed through a system, a "push" system can be designed with the use of decouplers to maximize productivity, whereas when irregular parts requiring different operations are processed, a "pull" system is the most effective system.

Theorem 26 (Conversion of a System with Infinite Time-Dependent Combinatorial Complexity to a System with Periodic Complexity)
Uncertainty associated with a design (or a system) can be reduced significantly by changing the design from one of serial combinatorial complexity to one of periodic complexity.

3. Theorems Related to Design and Decomposition of Large Systems

Theorem S1 (Decomposition and System Performance)
The decomposition process does not affect the overall performance of the design if the highest-level FRs and Cs are satisfied and if the information content is zero, irrespective of the specific decomposition process.

Theorem S2 (Cost of Equivalent Systems)
Two "equivalent" designs can have substantially different cost structures, although they perform the same set of functions and they may even have the same information content.

Theorem S3 (Importance of High-Level Decisions)
The quality of design depends on the selection of FRs and the mapping from domain to domain. Wrong selection of FRs made at the highest levels of design hierarchy cannot be rectified through the lower-level design decisions.

Theorem S4 (The Best Design for Large Systems)
The best design for a large flexible system that satisfies *m* FRs can be chosen among the proposed designs that satisfy the Independence Axiom if the complete set of the subsets of FRs that the large flexible system must satisfy over its life is known *a priori.*

Theorem S5 (The Need for a Better Design)
When the complete set of the subsets of FRs that a given large flexible system must satisfy over its life is not known *a priori,* there is no guarantee that a specific design will always have the minimum information content for all possible subsets and thus there is no guarantee that the same design is the best at *all times*.

Theorem S6 (Improving the Probability of Success)
The probability of choosing the best design for a large flexible system increases as the known subsets of FRs that the system must satisfy approach the complete set that the system is likely to encounter during its life.

Theorem S7 (Infinite Adaptability versus Completeness)
A large flexible system with infinite adaptability (or flexibility) may not represent the best design when the large system is used in a situation where the complete set of the subsets of FRs that the system must satisfy is known *a priori*.

Theorem S8 (Complexity of a Large Flexible System)
A large system is not necessarily complex if it has a high probability of satisfying the FRs specified for the system.

Theorem S9 (Quality of Design)
The quality of design of a large flexible system is determined by the quality of the database, the proper selection of FRs, and the mapping process.

4. Theorems for Design and Operation of Large Organizations (Suh, 1995)

Theorem M1 (Efficient Business Organization)
In designing large organizations with finite resources, the most efficient organizational design is the one that specifically allows reconfiguration by changing the organizational structure and by having flexible personnel policy when a new set of FRs must be satisfied.

Theorem M2 (Large System with Several Sub-Units)
When a large system (e.g., organization) consists of several sub-units, each unit must satisfy independent subsets of FRs so as to eliminate the possibility of creating a resource-intensive system or a coupled design for the entire system.

Theorem M3 (Homogeneity of Organizational Structure)
The organizational structure at a given level of the hierarchy must be either all functional or product-oriented to prevent duplication of effort and coupling.

5. Theorems Related to Software Design

Theorem Soft 1 (Knowledge Required to Operate an Uncoupled System)
Uncoupled software or hardware systems can be operated without precise knowledge of the design elements (i.e., modules) if the design is truly an uncoupled design and if the FR outputs can be monitored to allow closed-loop control of FRs.

Theorem Soft 2 (Making Correct Decisions in the Absence of Complete Knowledge for a Decoupled Design with Closed-Loop Control)
When the software system is a decoupled design, the FRs can be satisfied by changing the DPs if the design matrix is known to the extent that knowledge about the proper sequence of change is given, even if precise knowledge about the elements of the design matrix may not be known.

6. Theorems Related to Complexity

Theorem C1 (Complexity of an Uncoupled System with Many Interconnected Parts)
Complexity of an uncoupled system with many interconnected parts is not necessarily greater than that of a system with fewer interconnected parts unless the interfaces between the interconnected parts of the uncoupled system increase uncertainty by reducing the overlap between the system range and the design range.

Theorem C2 (Complexity of a Decoupled System with Many Interconnected Parts)
Complexity of a decoupled system with many interconnected parts is not necessarily greater than that of a system with fewer interconnected parts unless the interfaces between the interconnected parts of the decoupled system increase uncertainty by reducing the overlap between the system range

and the design range.

Theorem C3 (Complexity of a Coupled System with Many Interconnected Parts)
Complexity of a coupled system with many interconnected parts is greater than that of a system with fewer interconnected parts since any variation at the interfaces between the interconnected parts of the coupled system increases uncertainty by reducing the overlap between the system range and the design range.

Theorem C4 (Complexity of an Uncoupled System with Complicated Arrangement of Parts)
Complexity of an uncoupled system with complicated arrangement of parts is not necessarily greater than that of a system with less complicated arrangement of parts unless the interfaces between the parts of the uncoupled system increase uncertainty by reducing the overlap between the system range and the design range.

Theorem C5 (Complexity of a Decoupled System with Complicated Arrangement of Parts)
Complexity of a decoupled system with complicated arrangement of parts is not necessarily greater than that of a system with less complicated arrangement of parts unless the interfaces between the parts of the decoupled system increase uncertainty by reducing the overlap between the system range and the design range.

Theorem C6 (Complexity of a Coupled System with Complicated Arrangement of Parts)
Complexity of a coupled system with complicated arrangement of parts is greater than that of a system with less complicated arrangement of parts since any variation at the interfaces between the parts of the coupled system increases uncertainty by reducing the overlap between the system range and the design range.

Theorem C7 (Imaginary Complexity of a Decoupled System with Complicated Arrangement of Parts)
The time-independent imaginary complexity of a decoupled system with complicated arrangement of parts can be large if the design parameters (DPs) are not changed in the sequence given by the design matrix.

Homework

2.1 Prove that if each information content term of the right-hand side of Equation (2.9) is multiplied by a weighting factor k_i, the total information content will not be equal to information.

2.2 With the result of question 2.1 in mind, prove Theorem 16 (Equality of Information Content): All information contents that are relevant to the design task are equally important regardless of their physical origin, and no weighting factor should be applied to them.

2.3 Consider the design of a hot and cold water tap. The functional requirements are the flow rate and the temperature of the water. If we have a faucet that has one valve for hot water and another valve for cold water, the design is coupled since the temperature and flow rate cannot be controlled independently. We can design an uncoupled faucet that has one knob only for temperature control and another knob only for the flow-rate control. Design such an uncoupled faucet by decomposing the FRs and DPs. Integrate the DPs to reduce the number of parts.

2.4 Professor Smith of the University of Edmonton raised the following question about the water-faucet design. If we take the coupled design (i.e., the design with two valves, one for cold water and the other for hot water) and then add a servo-control mechanism, we may be able to control the flow rate and the temperature independently. Therefore, Professor Smith says that a coupled design is as good as the uncoupled design.

How would you answer Professor Smith's question? Analyze the design proposed by Professor Smith by establishing FRs and DPs, by creating a design hierarchy through zigzagging, and by constructing the design matrices at each level. Is Professor Smith's design coupled, uncoupled, or decoupled?

2.5 In some design situations, we may find that we have to make design decisions in the absence of sufficient information. In terms of the Independence Axiom and the Information Axiom, explain when and how we can make design decisions even when we do not have sufficient information. What kinds of information can we do without and what kinds of information must we have in design? Illustrate your argument using a design task with three FRs as an example.

2.6 Prove Theorem 18, which states that there is always an uncoupled design that has a lower information content than coupled designs.

2.7 Prove Theorems 2, 6, 15, and 16.

2.8 A surgical operating table for hospitals is to be designed. The position of the table must be adjustable along the horizontal and the vertical directions as well as the inclination of the table. Design a mechanism that can satisfy these functional requirements.

If the functional requirements of the table are modified so that the table has to change from one fixed position (i.e., fixed horizontal, vertical, and inclination) to another fixed position, how would you design the mechanism?

2.9 One of the major problems in the automobile business is the warranty cost associated with the weather-strip. It is typically made of extruded rubber to prevent dust, water, and noise from coming into the vehicle. The weather-strip also affects the force required to close the door. One of the problems identified is that the gap between the door and the body can vary from about 10 to 20 mm. Design the weather-strip.

2.10 Compare elements of axiomatic design theory to those of other design methodologies, specifically Quality Function Deployment (QFD), Robust Design (Taguchi methods), and Pugh Concept Selection. Where do they agree and where do they differ?

2.11 The two linear equation sets below describe two designs. Each of the design matrices can be made uncoupled or decoupled depending on whether the variable x is set to 0 or 1, respectively. Analytically compute the probability of success of each of the designs for each value of x (four cases in total). All distributions are uniform. What can you conclude about the relationship between information content and coupling?

$$\begin{Bmatrix} -1 < FR_1 < 1 \\ -1 < FR_2 < 1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix} \begin{Bmatrix} 0 < DP_1 < 2 \\ 0 < DP_2 < 2 \end{Bmatrix}$$

$$\begin{Bmatrix} -1 < FR_1 < 1 \\ 1.5 < FR_2 < 3.5 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix} \begin{Bmatrix} 0 < DP_1 < 2 \\ 0 < DP_2 < 2 \end{Bmatrix}$$

2.12 The equation below describes a design with two DPs and two FRs. The first DP has a uniform distribution and the second DP has a normal (2, 0.8) distribution. Write a short program (in MATLAB, for example) that numerically computes the probability of success of this design, and plot FR(DP1, DP2).

$$\begin{Bmatrix} 2 < FR_1 < 5 \\ 1.5 < FR_2 < 3.5 \end{Bmatrix} = \begin{bmatrix} 0.7 & 1.6 \\ 1.1 & 0.5 \end{bmatrix} \begin{Bmatrix} 1 < DP_1 < 3 \\ DP_2 = 2, \quad \sigma = 0.8 \end{Bmatrix}$$

2.13 Given a system with m independent events with probability of success Pi, prove that the total information content is the sum of individual information content of these events.