Kick-off

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Most problems in physics textbooks are highly idealized to keep them analytically manageable. However, in dealing with daily phenomena, some models presented in textbooks are oversimplified. The discrepancy between what students observe and what these models predict may cause frustration or even distrust. On the other hand, it is crucial to develop intuition to discover the relevant parameters as well as appropriate optimizations—a common perspective in engineering that is not always stressed in physics. This paper addresses these topics in a concrete situation: kicking a soccer ball. The problem is formulated as follows: Given the physical constraints of the athlete, how does one achieve maximum initial velocity and range?

How To Kick a Ball

The physics of soccer has been studied by several authors.[1,2] Particular attention has been given to the ball trajectory, including the well-known Magnus effect. The prospect of kicking has also been studied, mostly using a two-body collision framework.[3-8] However, some of these models are either too complex to present to pre-college students or too simplistic to describe the richness of the phenomena.

Our model describes the ball as a mass-spring system, thus considering the elastic deformation of the ball as linear (Fig. 1). Our objective is to obtain the optimum characteristics of the ball (mass and elastic constant k) to achieve the greatest range.

The effect of the ball mass on its trajectory is easily guessed. If the ball is too light, air resistance quickly reduces its velocity and the trajectory may be disturbed by moderate winds. On the other hand, if the ball is too heavy, the athlete will not be able to impart sufficiently high velocity. The role of k is less obvious. If k is too high, e.g., a ball excessively inflated, the impact time is short and a large force is applied on the foot, which may cause muscle injuries. If k is too small, the ball does not gain enough velocity.

In a first approximation, the foot-ball interaction can be described either by: 1) fixing the external force F acting on the ball over a distance d, or 2) fixing the foot velocity. We choose the first approximation due to its mathematical simplicity. Therefore, the force applied to the spring over a distance d is considered fixed.

The foot displacement is the sum of the displacement of the mass x_b and the deformation of the spring x_d:

$$x_b + x_d = d. \quad (1)$$

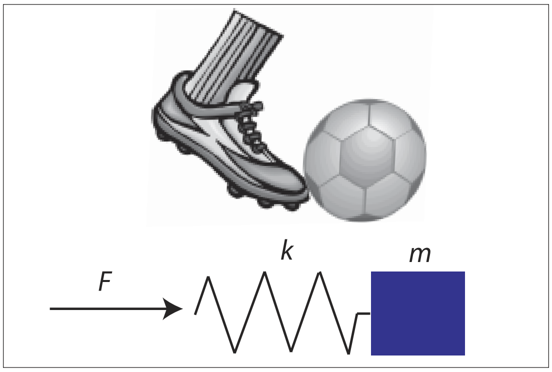

Fig. 1. Simplified model of kicking a soccer ball.

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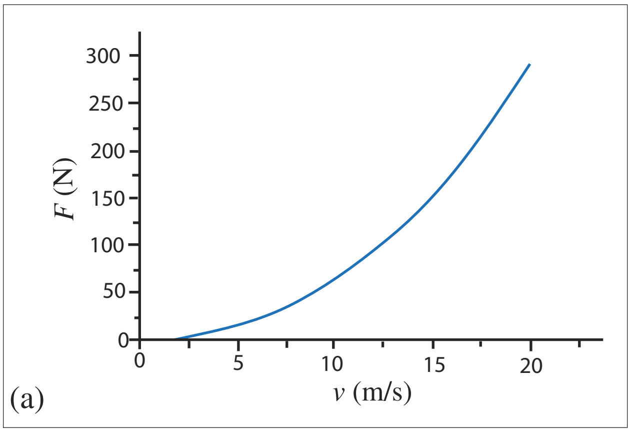

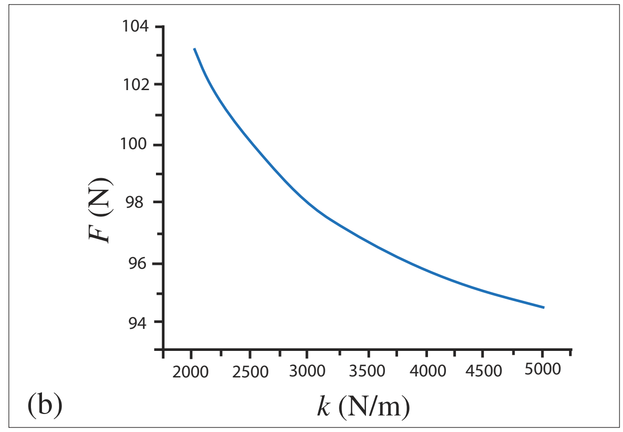

Fig. 2. (a) Force applied on the ball as a function of the velocity for *m* = 0.50 kg, *k* = 5000 N/m and *d* = 40 cm. (b) Force as a function of *k* for *m* = 0.50 kg, *d* = 40 cm, and *v* = 12 m/s.

The deformation of the spring can be calculated using Hooke's law, $x_d = F/k$. Recalling that the force applied on the spring and on the mass is the same, we have a uniform accelerated motion and the following relation for the displacement applies:

$$x_b = \frac{mv^2}{2F}. \tag{2}$$

Inserting Eq. (2) into Eq. (1) we get

$$\frac{mv^2}{2F} + \frac{F}{k} - d = 0. \tag{3}$$

This is a quadratic equation with a solution:

$$F = \frac{k}{2}\left(d - \sqrt{d^2 - \frac{2mv^2}{k}}\right). \tag{4}$$

In Fig. 2, Eq. (4) is used to represent the force necessary to achieve a certain velocity as a function of the velocity (a) and the elastic constant (b).

The force has a maximum of $F_{max} = kd/2$ when

$$v^* = \sqrt{\frac{k}{2m}}d. \tag{5}$$

For $v > v^*$, Eq. (4) has no real solution, meaning that it is impossible to achieve a velocity higher than v^* for any force acting over a distance d. Note that Eq. (5) does not depend on the mass but only on the stiffness of the ball. In addition, for a body with a high k, the force can only be applied during smaller distances to keep the force below F_{max}.

Reality, Please!

We now apply our model using typical characteristics of a professional soccer player. It has been found that during a strong kick the *instantaneous* force applied on the ball by the foot can be as high as 4000 N, although its average value rarely exceeds 1000 N.[3,5] The foot can have velocities as high as 22 m/s. The ball is accelerated during a distance of about 20 cm, corresponding to an impact time of 10 ms.

In our model we consider a force F_{max} = 500 N, a foot velocity v_{foot} = 20 m/s, and an impact distance d = 20 cm. First the constant of elasticity of the ball is obtained:

$$k = \frac{2F_{max}}{d} = \frac{1000}{0.2} = 5000 \text{ N/m}. \tag{6}$$

The deformation of the ball is $x_d = F_{max}/k$ = 500/5000 = 0.1 m = 10 cm. Note that for large distortions the deformation is no longer linear, and the player's shoe also suffers an important compression. However, a more realistic expression relating deformation of the ball with the force applied will unnecessarily complicate the analysis without altering the physics of the process.

Inserting k into Eq. (5) and solving for m, we find

$$m = \frac{kd^2}{2v_{foot}^2} = \frac{100}{v_{foot}^2}. \tag{7}$$

For $v_{foot} = v_{max}$ = 20 m/s, we find m = 250 g. If the velocity of the foot is slower, around v_{foot} = 15 m/s in a typical situation, then the optimal mass is m = 450 g. This is very close to the standard value fixed by inter-

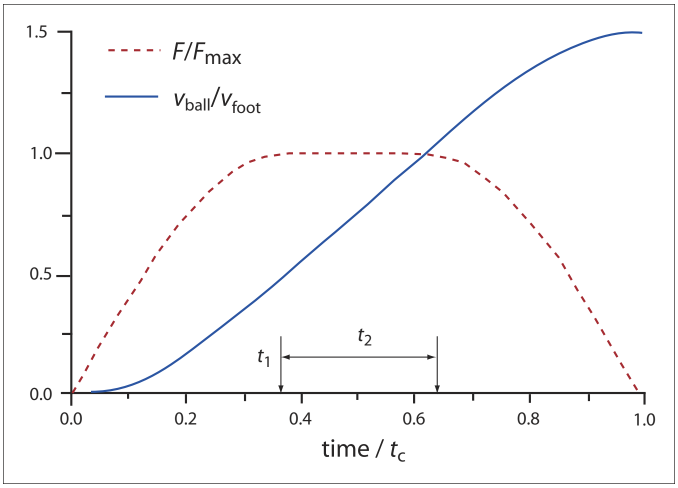

Fig. 3. Schematic representation of the force applied on the ball and the ball velocity as a function of time.

national regulations—between 400 and 440 g. Now that k and m have been obtained, Eq. (4) can be used to compute the force applied on the ball for smaller velocities.

Explaining an Intriguing Fact

It is known that the ball leaves the ground with a higher velocity than the velocity of the player's foot. In strong kicks, the ball may reach a speed of more than 30 m/s while the foot's speed is only around 20 m/s. Zernicke and Roberts[3] found that in the range from 16 to 27 m/s, the ball speed can be fitted to the following expression:

$$v_{ball} = 1.23 v_{foot} + 2.72. \tag{8}$$

This expression cannot be explained in the framework of a two-body collision since the velocity of the standing body is always equal to or smaller than the velocity of the incoming mass. In an elastic collision, the force and deformation are maximal at the same instant, which is at half the collision time t_c. At this instant the velocity of the ball is just half that of the incoming body.

However, this situation does not occur in a typical kick. During ball compression, the force applied by the foot does not begin to diminish at $t_c/2$. Due to the action of other muscles that become active under extreme tension, the player continues applying a force on the ball until its velocity is about the same as the foot. It is after this point that the elastic deformation energy of the ball is transferred to kinetic energy. This situation is explained in Fig. 3. Note that for a two-body collision, $t_2 = 0$.

Assuming energy conservation, we obtain:

$$\frac{1}{2} m v_{foot}^2 + \frac{1}{2} k x_d^2 = \frac{1}{2} m v_{ball}^2. \tag{9}$$

Using Eq. (7), the final velocity of the ball v_{ball} is

$$v_{ball} = \sqrt{1.5}\, v_{foot} = 1.22 v_{foot}, \tag{10}$$

in close agreement with Eq. (8). Note that Eq. (10) is not valid for very small values of m since Eq. (5) will require a very large v_{foot} to reach a force of 500 N.

Conclusions

In this paper we have presented a simple model to describe the kicking of a soccer ball. The results are in good agreement with observations. This approach is intended to stimulate the application of simple physical descriptions to real-life situations while not oversimplifying.

This problem can be presented to students as an open-ended one. It may be addressed in the classroom followed by some hints. Then students may work by themselves on building the adequate model and interpreting their conclusions. A simple experiment using fast video cameras and accelerometers can be easily implemented to test predictions.

Finally, the reader is invited to apply this approach to other sports such as golf or tennis making the necessary adjustments.

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