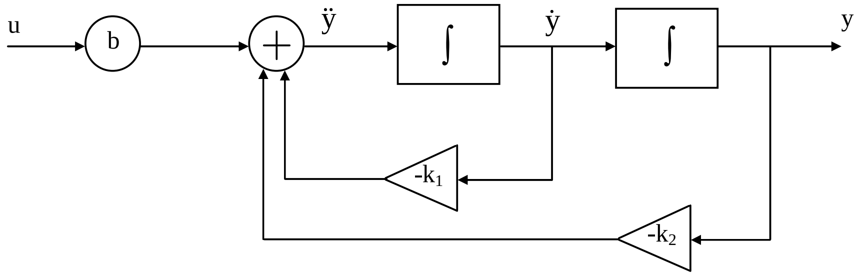
PTC2307 - Sistemas e Sinais I – Junho de 2010 – 6ª Lista de Exercícios – Respostas

1)

a)



b) $G(s) = \dfrac{b}{s^2 + k_1 s + k_2}$

c) $h(t) = 5te^{-t} \cdot 1(t)$

2)

Descrição de estados

Equação de estado: $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -2x_1 - 5x_2 + u \end{cases}$

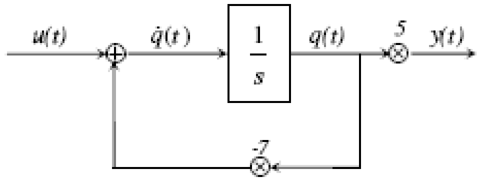
Equação de saída: $y = 2x_1 + 5x_2$

Descrição entrada-saída

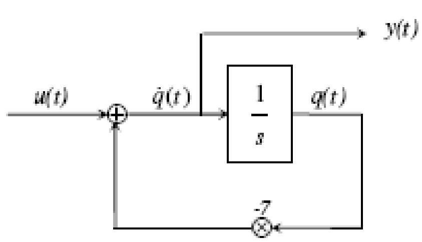
$\ddot{y} + 5\dot{y} + 2y = 5\dot{u} + 2u$

3)

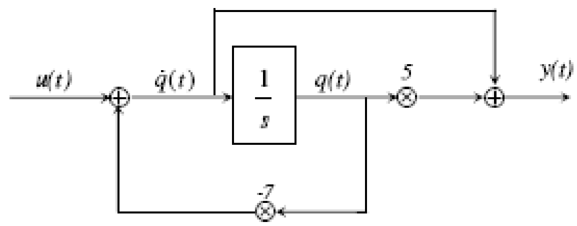
a)



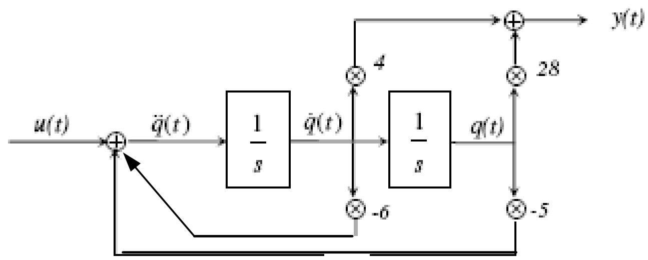
b)



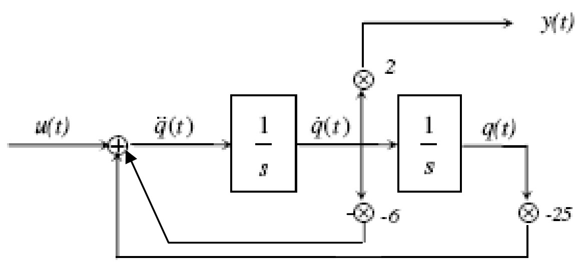
c)



d)

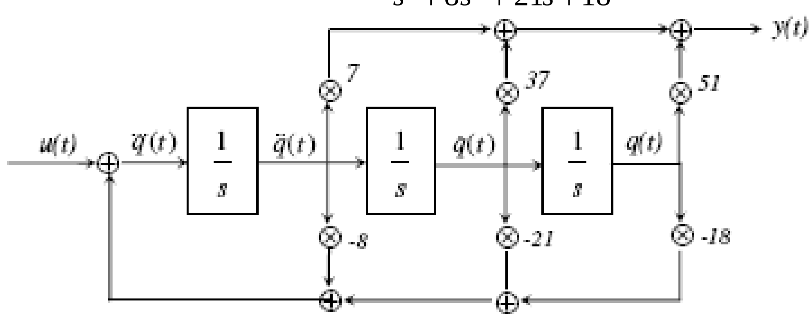


e)

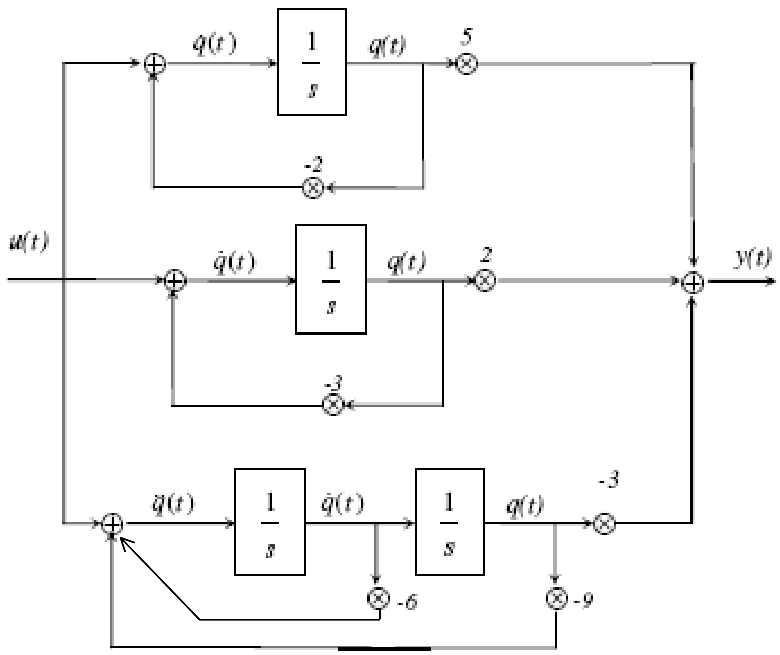


4)

Forma canônica controlável: $H(s) = \dfrac{7s^2 + 37s + 51}{s^3 + 8s^2 + 21s + 18}$

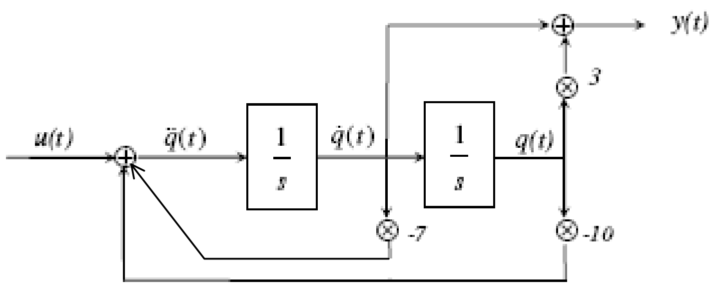


Realização paralela: $H(s)=\dfrac{7s^2+37s+51}{(s+2)(s+3)^2}=\dfrac{5}{(s+2)}+\dfrac{2}{(s+3)}+\dfrac{-3}{(s+3)^2}$

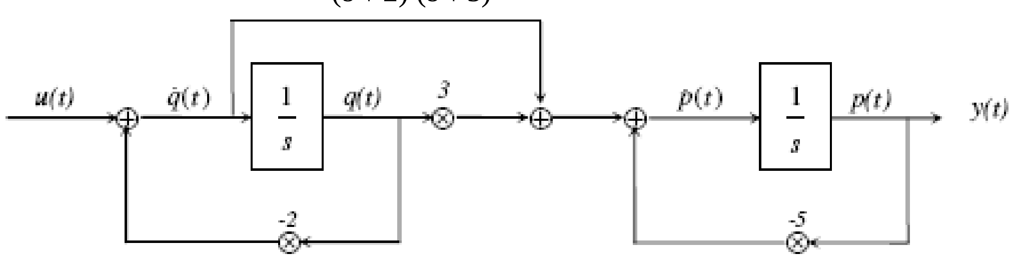


5)

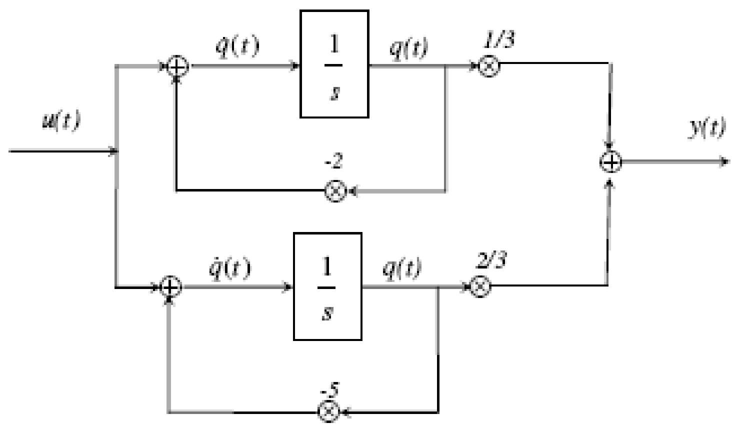
Realização canônica controlável $H(s)=\dfrac{s+3}{s^2+7s+10}$



Realização em cascata $H(s)=\dfrac{(s+3)}{(s+2)}\dfrac{1}{(s+5)}$



Realização em paralelo $H(s)=\dfrac{(s+3)}{(s+2)(s+5)}=\dfrac{1/3}{(s+2)}+\dfrac{2/3}{(s+5)}$



6)

$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mathbf{u}(\mathbf{t})$$

$$y(t) = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix} \mathbf{x}(\mathbf{t})$$

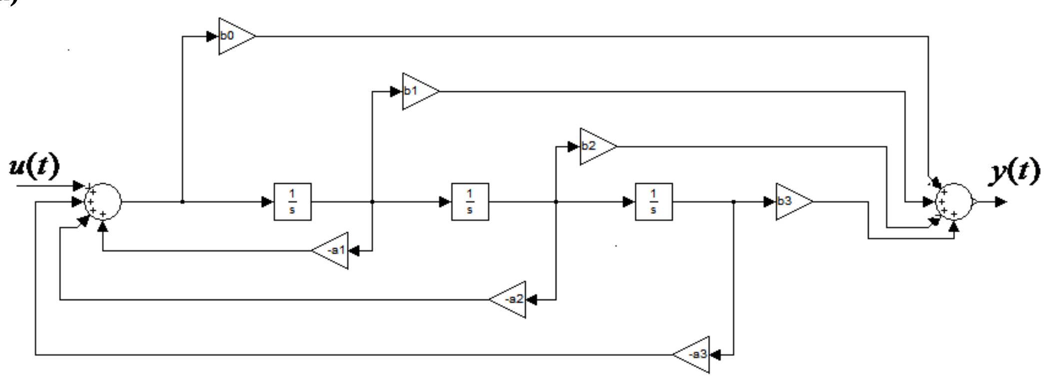
7)

$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}(\mathbf{t})$$

$$y(t) = \begin{bmatrix} 4 \end{bmatrix} \mathbf{x}(\mathbf{t})$$

8)

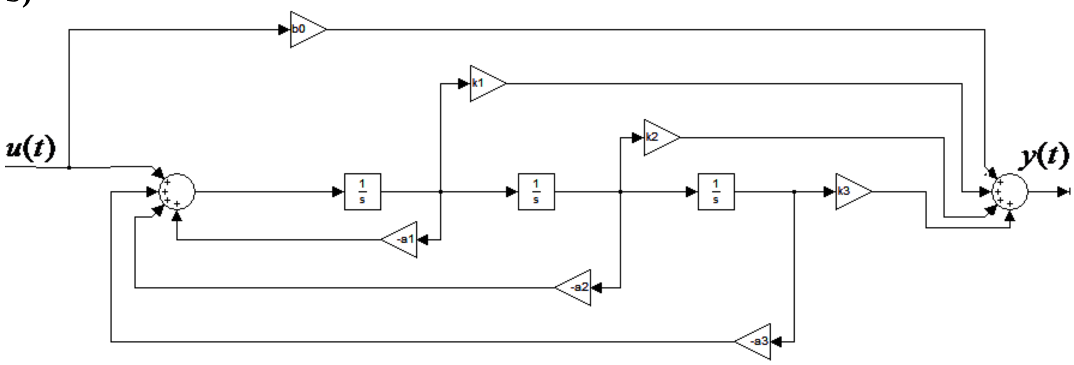
a)



$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} (b_3 - a_3 b_0) & (b_2 - a_2 b_0) & (b_1 - a_1 b_0) \end{bmatrix} \mathbf{x}(\mathbf{t}) + b_0 u(t)$$

b)



Onde: $k_1 = b_1 - a_1 b_0$, $k_2 = b_2 - a_2 b_0$ e $k_3 = b_3 - a_3 b_0$

9)

$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -10 & -17 & -8 \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{u}(\mathbf{t})$$

$$y(t) = \begin{bmatrix} 7 & 3 & 0 \end{bmatrix} \mathbf{x}(\mathbf{t})$$

10)

$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} 0 & 1 \\ \frac{-k_1}{m} & \frac{-k_2}{m} \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \mathbf{u}(\mathbf{t})$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(\mathbf{t})$$

11)

$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-R}{L} \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}(\mathbf{t})$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x}(\mathbf{t})$$

12)

$$G(s) = \frac{1}{(s+1)^2 (s+2)}$$

13)

$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} \frac{-R_1}{L} & \frac{-1}{L} \\ \frac{1}{C} & \frac{-1}{R_2 C} \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{1}{R_2 C} \end{bmatrix} \mathbf{u}(\mathbf{t})$$

$$\mathbf{y}(\mathbf{t}) = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{R_2} \end{bmatrix} \mathbf{u}(\mathbf{t})$$

14)

a)

$$\dot{\mathbf{x}}(\mathbf{t}) = \begin{bmatrix} K_2 & K_4 \\ K_5 & K_3 \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 1 & K_1 \\ 0 & 1 \end{bmatrix} \mathbf{u}(\mathbf{t})$$

$$\mathbf{y}(\mathbf{t}) = \begin{bmatrix} K_6 & 1 \\ 0 & 1 \end{bmatrix} \mathbf{x}(\mathbf{t}) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}(\mathbf{t})$$

b)

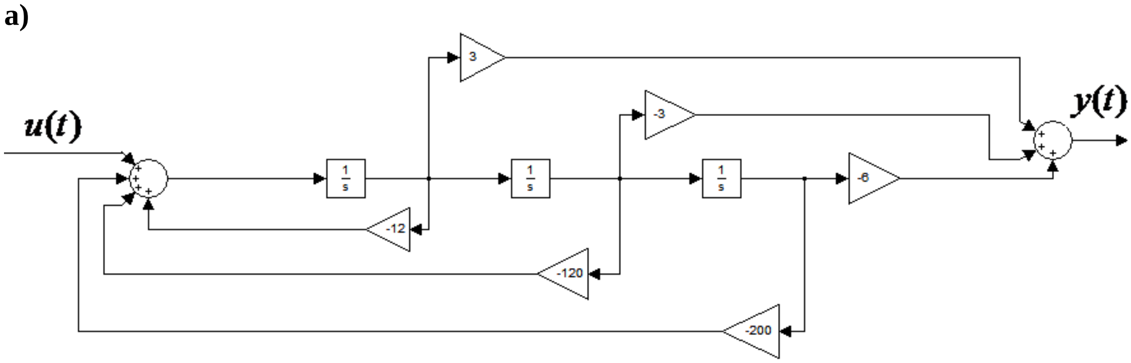
$$\frac{Y_2(s)}{U_1(s)} = \frac{K_5}{s^2 - (K_2 + K_3)s - K_4 K_5 + K_2 K_3}$$

c)

$$K_5 = 0$$

15)

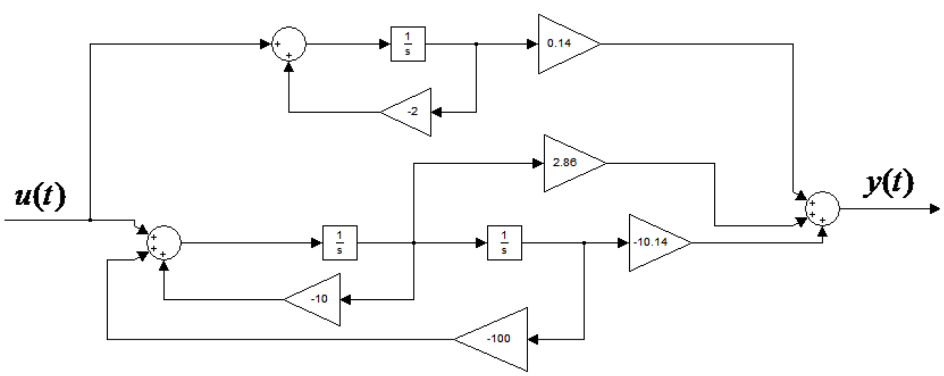
a)



$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -200 & -120 & -12 \end{bmatrix} \cdot \mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u(t)$$

$$y(t) = \begin{bmatrix} -6 & -3 & 3 \end{bmatrix} \cdot \mathbf{x}(t)$$

b)



16) (Primeira Prova de 2008)

$$h(t) = 3 \cdot e^{-4 \cdot t} \cdot \mathbf{1}(t) + \delta(t)$$

17) (Primeira Prova de 2009)

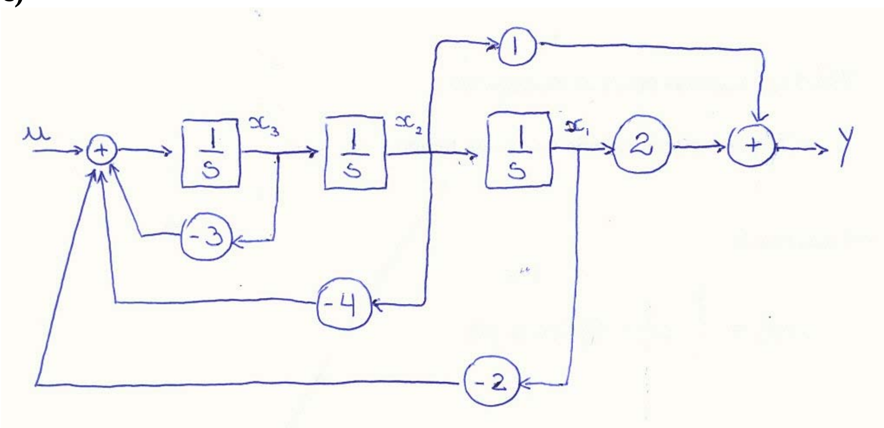
a)

$$h_1(t) = e^{-t} \cdot \mathbf{1}(t)$$

b)

$$\dddot{y}(t) + 3\ddot{y}(t) + 4\dot{y}(t) + 2y(t) = \dot{u}(t) + 2u(t)$$

c)



d)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

18) (Primeira Prova de 2009)

a)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -13 & 1 & 0 \\ -39 & 0 & 1 \\ -27 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 10 \\ 24 \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b)

Corresponde