Reconstrução tomográfica a partir de projeções

EPUSP/PTC-5892
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Tomografia: secções de objetos

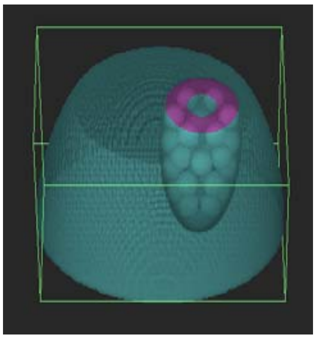
- Reflexão
  - Acústica: Ultra-som, Radar, ..
  - Ótica: microscópio confocal
- A partir das projeções
  - No domínio do espaço
    - transmissão : CT
    - emissão: SPECT, PET
  - No domínio da frequência
    - Ressonância Magnética (geometric projection, Fourier projection)

Tomografia a partir de projeções no espaço

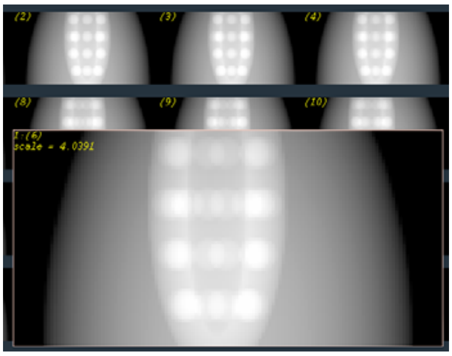
- Conceito
  - Matemática da reconstruçao: Radon, 1917
- Aplicação
  - Astronomia: Bracewell, 1956
  - Medicina (revolução após Roentgen, 1895)
    - Primeiras publicações: Oldendorf, 1961
    - Primeiros experimentos: Kuhl (UPENN, 1963)
    - Equipamento médico: G Hounsfield (EMI, UK, 1971) e A Cormack (Tufts Univ) => Nobel, 1979

Estudo de caso: Phantom 3D

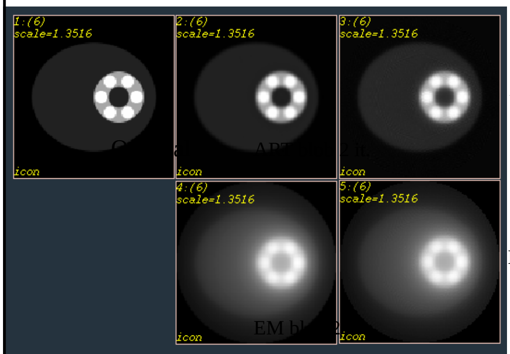
- background
- myocard.
- spots



Dados disponíveis: Projeções



Processamento: Reconstrução

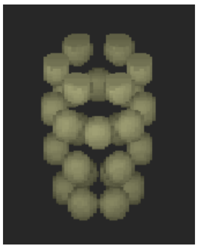


ART vox 2 it

EM vox 2 it.

- Noiseless proj.

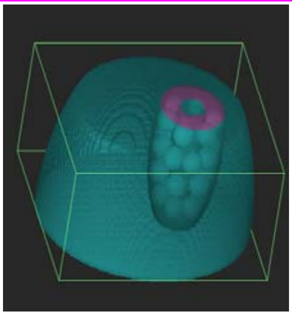
Motivação: 3D Reconstruction



- ART Blob
- Noisyless data
- 2 iterations

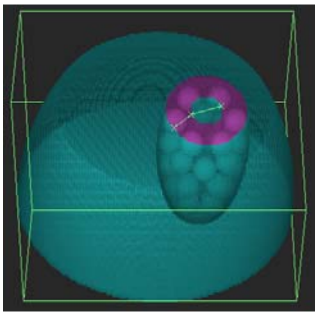
Motivação: 3D rendering (surface)

- phantom
- segmentation
- surface rendering



Motivation: Measuring in 3D

- distance
- area
- volume
- ejection fraction
- velocity
- ...



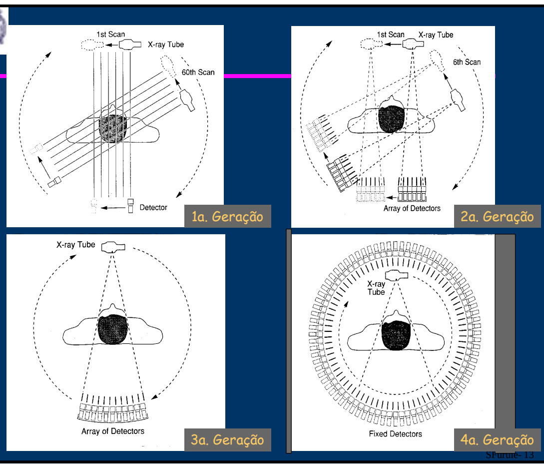
TOMOGRAFIA A PARTIR DAS PROJEÇÕES

Reconst. Tomog. a partir de projeções

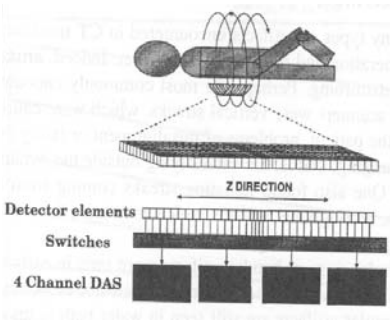
- Projection data formation
  - CT, spiral CT, multi-slice spiral CT $(0.5\ mm)^3$, .5 s
  - SPECT
  - 3D PET
- Tomographic reconstruction methods
  - ML-EM : Maximum-likelihood
  - ART : Algebraic Reconstruction Technique
  - FBP : Filtered Backprojection
  - DFM : Direct Fourier Method

Gerações de tomógrafos por proj.

- Varredura de fonte unica e rotação da fonte-detetor
- Varredura de fonte conica e rotação
- Cone beam e rotação da fonte-detetor
  - spiral CT
  - multi-slice spiral CT
- Múltiplas fontes cônicas e detetores
- Electronic beam

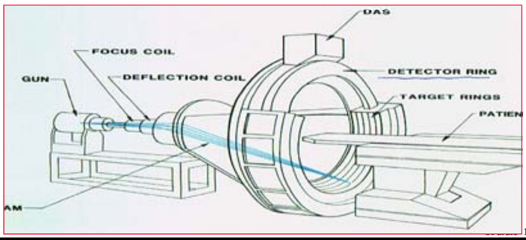


Multi-slice Spiral (helical) CT

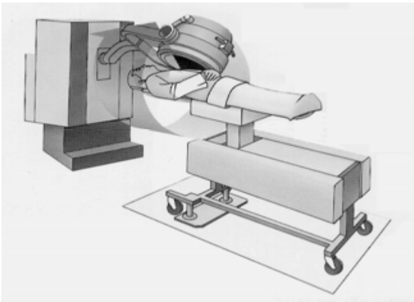


CT p/ Estruturas dinâmicas

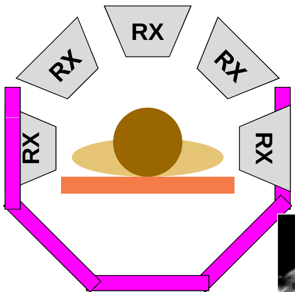
- Ultrafast CT
  - sem estruturas móveis
  - 50 ms/scan (20 cortes/s)
  - volume: 8 cm em 0.25 s

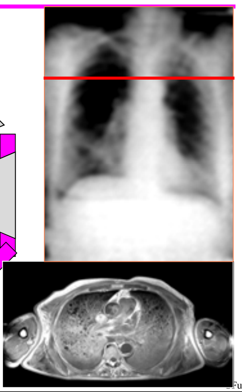


SPECT: Single Photon Emission CT

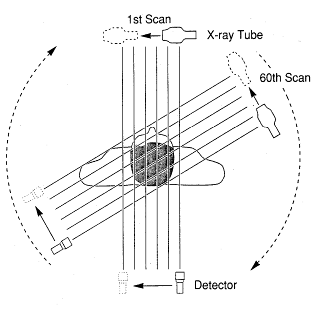


CT (raio-X): o que há de comum?

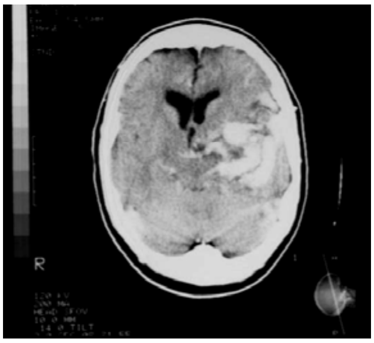




Linhas de projeção

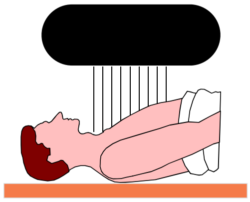


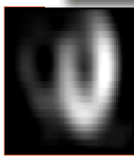
CT



MN: SPECT - o que há de comum?

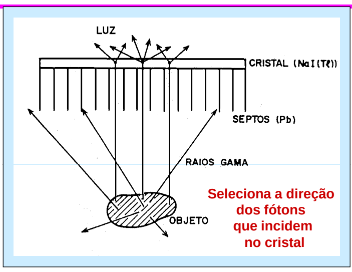
- Medicina Nuclear:SPECT

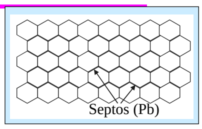




Miocárdio marcado com material radioativo (Tc)

SPECT:Colimador (septos paralelos)

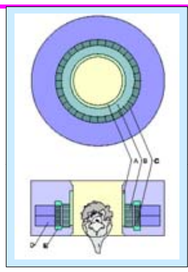


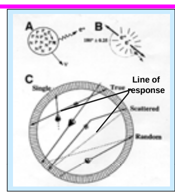


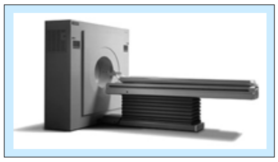
vista superior

A imagem formada no cristal é uma projeção bidimensional da distribuição tridimensional do radiofármaco no organismo

MN: PET - o que há de comum?



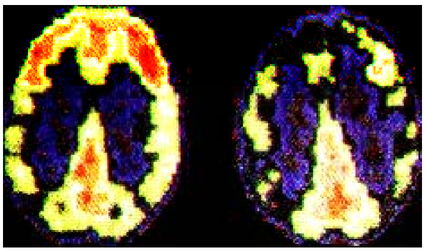




A. fontes para mapas de atenuação
B. absorvedores de fótons espalhados
C. blocos de detetores (BGO)
D. fotomultiplicadoras
E. blindagem

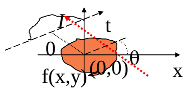
PET

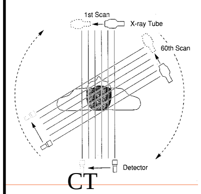
- Reconstrução Tomográfica (PET Cerebral)



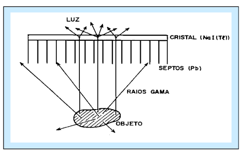
O que há de comum:

- Tomografia a partir das projeções
  - cada pixel na projeção contém informação acumulada ao longo da linha de projeção

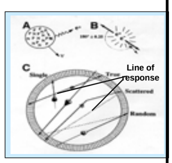




CT



SPECT



PET

O que há de diferente?

- Entre CT, SPECT e PET?

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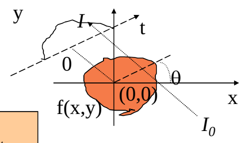
Tomografia por Transmissão (CT)

$I = I_0.\exp(-\int_L f(x,y).ds)$

$\ln(\frac{I_0}{I}) = \int_L f(x,y).ds$ (Integral de linha)



$p_i = \sum_j \mu_j.d_{ij}$

Desconhecido 1 conjunto:

$\Rightarrow$ μ_j j=1,N

Transformada de Radon 2D
(projection operator)

f(x,y) $\longleftrightarrow$ g(t,θ) (sinograma)

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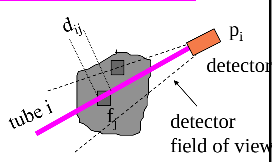
SPECT

$p_i = \sum_j f_j \underbrace{d_{ij}.A\exp(-\int_{s_{ij\to\infty}} \mu_j(s)ds)}_{h_{ij}}$



$p_i = \sum_j f_j.h_{ij}$

Desconhecidos 2 conjuntos:

$\Rightarrow$ f_j j=1,N

μ_j

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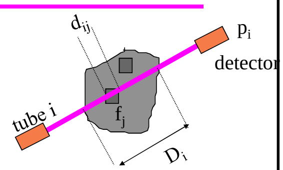
PET

$p_i = \sum_j f_j.(Ad_{ij}).\exp(-\int_{s_j} \mu_j(s)ds)$

$p_i = \exp(-\int_{D_i} \mu(s)ds).\sum_j f_j.(Ad_{ij})$



- accumulated attenuation estimation: simpler
  - Fonte externa => custo, tempo maior

$\frac{p_i}{\exp(-\int_{D_i} \mu(s)ds)} = \sum_j f_j.(Ad_{ij})$

Desconhecido 1 conjunto:

$\Rightarrow$ f_j j=1,N

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O que há de diferente?

- Entre CT, SPECT e PET?
  - CT: sist. Eq. Lineares,e 1 conjunto de incógnitas
  - SPECT: sist. Eq. Não-lineares, e 2 conjuntos de incóg.
  - PET: similar a CT se houver boa estimativa das aten.
    Ou similar a SPECT => algoritmos AA

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ALGORITMOS DE RECONSTRUÇÃO TOMOGR.

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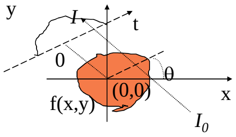
Tomografia por Transmissão (CT)

$$I = I_0 . \exp(-\int_L f(x,y).ds)$$

$$\ln(\frac{I_0}{I}) = \int_L f(x,y).ds \qquad \text{(Integral de linha)}$$

$$g(t,\theta) = R\,f = \int_L f(x,y).ds$$

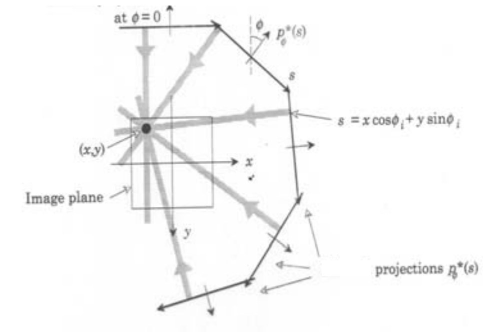
$$= \iint f(x,y).\delta(x.\cos\theta + y.\sin\theta - t).dx.dy$$



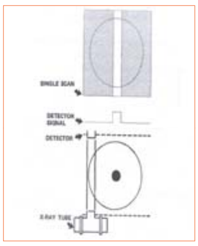
Transformada de Radon 2D
(projection operator)

f(x,y) ⟷ g(t,θ) (sinograma)

Solução?

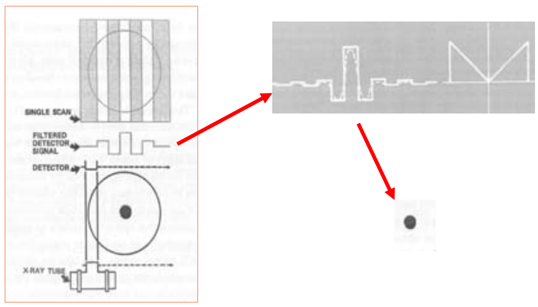


Reconstrução ingênua (backprojection)



Reprojeção simples

Reconstrução com filtered backprojection



Algébrica

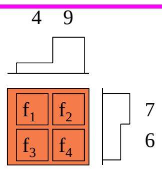


Imagem f

Problema: f |

$f_1 + f_2 = 7$

$f_3 + f_4 = 6$

$f_1 + f_3 = 4$

$f_2 + f_4 = 9$

Soluções

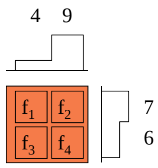
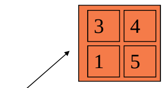
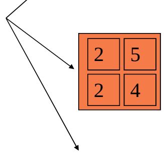


Imagem f





A . x = b

M equações com N incógnitas

Sistema indeterminado (infinitas soluções, rank < N)

Sistema inconsistente (M eq. Lin. Indep > N) => otimização

Algébrica: otimização (regularizada)

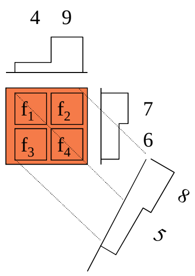


Imagem f

Problema: f |

$f_1 + f_2 \quad = 7$

$f_3 + f_4 \quad = 6$

$f_1 + f_3 \quad = 4$

$f_2 + f_4 \quad = 9$

$.5f_1 + f_3 + .5 f_4 = 5$

$.5f_1 + f_2 + .5 f_4 = 8$

Soluções (otimizada)

A . x = b

6 equações com 4 incógnitas

Sistema inconsistente (M eq. Lin. Indep > N) => otimização

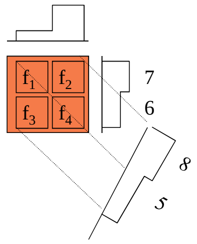
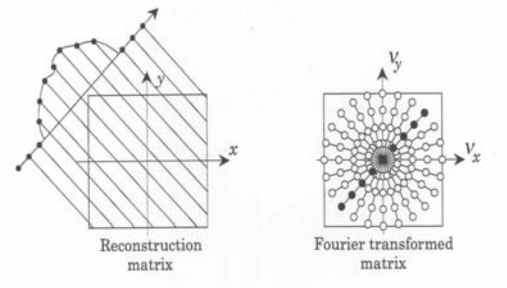


Imagem f

$$\min_{\hat{x}} | A.\hat{x} - b |^2$$
$$\hat{x} = A^{+} b$$
$$A^{+} = (A'A)^{-1} A'$$

2.5	4.6
2.0	4.1

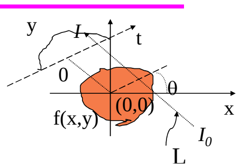
Alguma outra solução ?



Teorema do corte central

TEOREMA DA PROJEÇÃO

Tomografia por Transmissão (CT)



$$g(t,\theta) = \text{R f} = \int_L f(x,y).ds$$
$$= \iint f(x,y).\delta(x.\cos\theta + y.\sin\theta - t).dx.dy$$

Transformada de Radon 2D
(projection operator)

f(x,y) ⟷ g(t,θ) (sinograma)

Teorema da Projeção

$$g(t,\theta) = \iint f(x,y).\delta(x.\cos\theta + y.sin\theta - t).dx.dy$$

$$G(u,\theta) \equiv \iint f(x,y) \int \delta(x.\cos\theta + y.sin\theta - t).e^{-j2\pi ut} dt.dx.dy$$

$$G(u,\theta) = \iint f(x,y).e^{-j2\pi.u(x.\cos\theta + y.sin\theta)} dx.dy$$

$$\therefore G(u,\theta) = F(u.\cos\theta, u.sin\theta) \text{ => DFM}$$

Teorema da Projeção (cont.)

$Transf.Radon$ Inversa

$$f(x,y)=\iint F(u,v).e^{j2\pi.(x.u+y.v)}du.dv$$

$$coord.polares => u = w.\cos\theta \quad \text{e } v = w.sin\theta$$

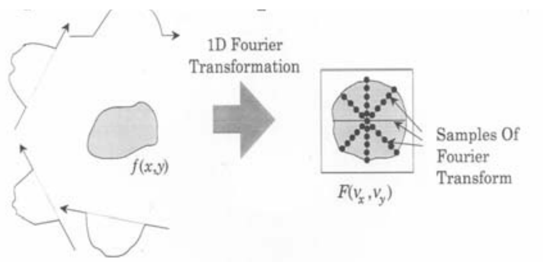
$$f(x,y)=\int_{0}^{2\pi}\int_{0}^{\infty} F_p(w,\theta).e^{j2\pi.(x.w.\cos\theta+y.wsin\theta)}w.dw.d\theta$$

$$f(x,y)=\int_{0}^{\pi}\int_{-\infty}^{\infty} F_p(w,\theta).e^{j2\pi.(x.w.\cos\theta+y.wsin\theta)}\,|\,w\,|.dw.d\theta$$

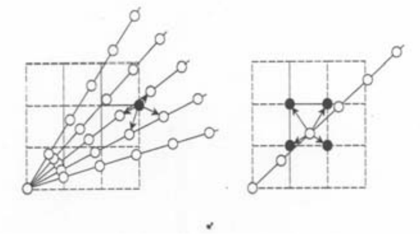
Reconstr. baseados em Transf.

- Direct Fourier Method
- Inverse Radon Transform
- Convolution Backprojection
- Filtered Backprojection
- Fan-beam
  - rebinning
  - fórmula

Ilustração DFM



DFM : interpolação em freq.



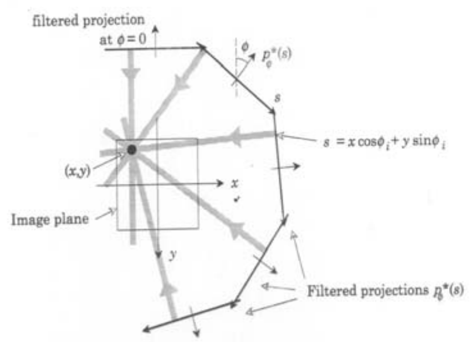
CBP, FBP

$$f(x,y)=\int_{0}^{\pi}\int_{-\infty}^{\infty} F_p(w,\theta).e^{j2\pi.(x.w.\cos\theta+y.wsin\theta)}\,|\,w\,|.dw.d\theta$$

$$f(x,y)=\int_{0}^{\pi}\{\int_{-\infty}^{\infty} F_p(w,\theta).|\,w\,|.e^{j2\pi.w.(x.\cos\theta+y.sin\theta)}dw\}.d\theta$$

$$f(x,y)=\int_{0}^{\pi}\widehat{g}(x.\cos\theta+y.sin\theta,\theta).d\theta$$

FBP



Normalização (escala Hounsfield)

$$H_{CT} = 1000 \frac{\mu - \mu_{H_2O}}{\mu_{H_2O}}$$

$$\therefore\ H_{CT}(\acute{a}gua) = 0$$
$$H_{CT}(ar) = -1000$$
$$H_{CT}(osso) \cong 1000$$
$$\mu_{H_2O} = 0.190\ cm^{-1}(70\ kev)$$

Massa branca e cinzenta: apenas alguns Hs

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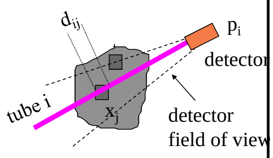
SPECT

$$p_i = \underbrace{\sum_j x_j . d_{ij} . A \exp\left(-\int_{s_{ij\to\infty}} \mu_j(s) ds\right)}_{h_{ij}}$$



$$\Rightarrow p_i = \sum_j h_{ij} . x_j$$

- Quantitative
  - EM
  - ART
- Approximate (Transform)
  - attenuation correction on projection data
  - attenuation correction on reconstructed data

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ALGORITMO DE RECONSTRUÇÃO: ART

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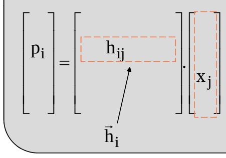
Solution: Algebraic Reconstruction

- System of linear equations
  - Huge system
  - Eg. volume: 64 x 64 x 64
    x_j j=1.. 262,144 voxels
  - Projections:
    128 views, 64x64 planes
    p_i i=1 .. 524,288 projs.
  - **H** : 524k x 262k
- Row-action methods
  - ART
  - EM

$p_i = \sum_j h_{ij} . x_j$ (all 3D projections)

$\vec{p} = \mathbf{H}.\vec{x}$ (vector notation)



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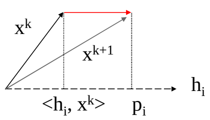
ART: Algebraic Reconst. Technique

- Noisy data
- Optimization criteria
  - Least-square solution
  - Minimum norm solution
    - row-action
    - relaxation

$$\vec{p} = \mathbf{H}.\vec{x} + \vec{n}$$

$$\vec{x}^{k+1} = \vec{x}^k + \lambda . \frac{p_i - \langle \vec{h}_i, \vec{x}^k \rangle}{||\vec{h}_i||^2} . \vec{h}_i$$



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ART 3D: Algebraic Reconst. Techn.

- Noise removal: projection data estimation
- Quantitative reconstruction
- Fast (3D)
- Simple
- General
- [H] determination
- Stop criteria

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ALGORITMO: ML-MAXIMUM LIKELIHOOD

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Statistical Solution

- Projection: Poisson noise
- Maximum likelihood
  - Expectation-maximization algorithm
  - Iterative approach
- Maximum a posteriori
  - “a priori” probability distr.

$$\vec{p} = Poisson\ (\mathbf{H}.\vec{\mathbf{x}})$$

$$\max_{\vec{x}} \Pr[\vec{p} \mid \vec{x}] \text{ (ML)}$$

$$\max_{\vec{x}} \Pr[\vec{x} \mid \vec{p}] \text{ (MAP)}$$

$$\Pr[\vec{x} \mid \vec{p}] = \frac{\Pr[\vec{p} \mid \vec{x}].\Pr[\vec{x}]}{\Pr[\vec{p}]}$$

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Expectation-maximization

- Maximum Likelihood
- ML-EM algorithm

$$\max_{\vec{x}} \Pr[\vec{p} \mid \vec{x}]$$

$$\Pr[\vec{p} \mid \vec{x}] \text{ : indep. Poisson}$$

$$x_j^{k+1} = \frac{x_j^k}{\sum_i h_{ij}} . \sum_i \frac{p_i}{< \vec{\mathbf{x}}^k, \vec{h}_i >} h_{ij}$$

$$Poisson\ (k \mid \lambda) = \frac{e^{-\lambda}.\lambda^k}{k!}$$

λ : média e variância

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ML-EM

- Handles Poisson noise
- Total count conservation
- Convergence to ML
- Expectation-maximization
  - algorithm independent of rays direction
  - quantitative approach
  - iterative
  - slow convergence
  - no stop criterion

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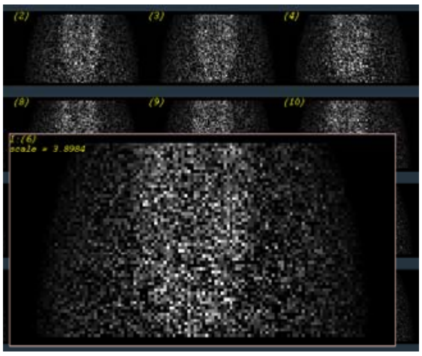
Noisy projections



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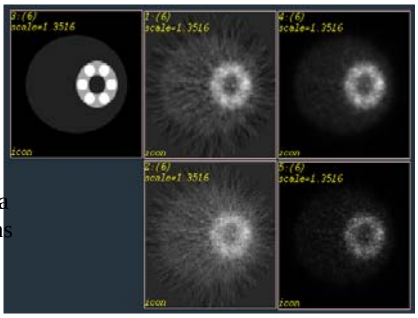
Reconstructions

ART Blob EM Blob



- Noisy data
- 5 iterations

ART Voxel EM Voxel

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Síntese

- Modelos de formação das projeções
- Reconstrução tomográfica 2D/3D
  - Aspectos gerais
  - Vantagens/desvantagens
- Métodos de reconstrução
  - Algébrico (ART)
  - Estatístico (ML-EM)
  - Analíticos
    - FBP, DFM
    - 3DRP, Favor, Cone
    - Simplificações: Rebinning

Algumas questões em aberto: CT/ECT

- SPECT/PET 3D quantitativo em tempo clínico aceitável
  - Algoritmos mais eficientes
  - Otimização de todo o processo
  - Avaliação objetiva de cada fase do processo
  - Avaliação clínica
- Correção de atenuação em PET e SPECT sem fonte externa
- PET 3D dinâmico
- Dinâmica metabólica

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